Webappendix: Price competition and advertisement in media markets

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This file sets up the model we sketch in Section 6 of our paper *Sequential Location Games* on price competition and advertisement in media markets.

Consider the following three-stage game. First, firms enter and locate sequentially. Let N be the number of firms who entered and label firms in increasing order of their locations, so that the locations chosen in stage 1 are $\mathbf{x} = (x_1, .., x_N)$.

Second, all firms who entered choose consumer prices $p_i \geq 0$ simultaneously. Consumers whose bliss point location is x and who buy from a media firm i located at x_i incur a travel cost of $t|x_i - x|$, where $t > 0$ measures the willingness to substitute distance for lower prices.[1] The disutility from prices paid and distances traveled is additively separable for all consumers. Moreover, consumers are not responsive to the number of ads a media outlet carries, and they buy from one firm only.[2] Notice that the locations and consumer prices entirely determine the consumer market share θ_i of each firm i.

Third, given θ_i for $i = 1, .., N$, all firms simultaneously set prices p_i^a for a continuum of advertisers with total mass $\mu > 0$. Each advertiser's productivity parameter s is independently drawn from the continuous, strictly increasing distribution G. Each advertiser places either one ad at firm i or none. When placing an ad at firm i at price p_i^a, the net surplus of an advertiser with productivity s is $s\theta_i - p_i^a$. Optimally this advertiser will place an ad at firm i if $s \geq p_i^a/\theta_i$.[3]

Solving the game backwards, each firm i's profit maximization problem in stage 3 is to choose p_i^a to maximize $\mu(1 - G(p_i^a/\theta_i)p_i^a$, which, using the substitution $q = p_i^a/\theta_i$, is equivalent to $\max_q\{(1 - G(q))q\}\mu\theta_i$. Letting $A \equiv \max_q\{(1 - G(q))q\}$, firm i's equilibrium profit in stage 3 is thus $A\mu\theta_i$, where $A > 0$ is independent of consumer market shares and only depends on G. Observe that in equilibrium all media firms attract the same number of advertisers. Because each advertiser places one ad by assumption, all media firms will have the same amount of ads. Notice also that $A\mu$ is the value of capturing the whole market. Thus, if the true fixed cost is K, then the normalized fixed cost is now $K_\mu \equiv K/(A\mu)$.

[1] Notice that this is in contrast to Gabszewicz, Laussel, and Sonnac (2001), who assume quadratic transportation costs. However, the sufficient condition that we derive below is also sufficient for quadratic transportation costs.

[2] The plausibility of this *single-homing* assumption depends on the ease with which consumers can consume multiple outlets, so that it will be better suited for newspapers than for television. See also Kim and Serfes (2006) for a standard Hotelling model with prices where consumers buy multiple products.

[3] Because of consumers' single-homing, the sets of consumers reached by advertisers through different media outlets are disjoint. As a result, media outlets compete fiercely for consumers but become monopolists vis-à-vis advertisers, which decide for each individual outlet whether or not to place an ad at the price demanded by the outlet. Naturally, this leads to *multi-homing* by advertisers.

Denoting the minimum value of the density by $\underline{f} = \min_x f(x)$, we now impose a sufficient condition that guarantees that consumer prices are zero in the stage 2 equilibrium:

Lemma 1. *In stage 2, if $\mu \geq 2t/(A\underline{f})$ holds, then the equilibrium consumer prices p_i are zero for all $i = 1, .., N$ for any set of locations $\mathbf{x}$.*

Proof of Lemma 1 The location of the consumer between firms located at x_i and x_{i+1} who charge prices p_i and p_{i+1} is $\tilde{x}_i \equiv (x_i + x_{i+1})/2 + (p_{i+1} - p_i)/(2t)$. The profit of firm i with $i = 2, .., N-1$ is then $\Pi_i = (p_i + A\mu)[F((\tilde{x}_i + \tilde{x}_{i+1})/2) - F((\tilde{x}_{i-1} + \tilde{x}_i)/2)]$. The derivative with respect to p_i is

$$\frac{\partial \Pi_i}{\partial p_i} = F((\tilde{x}_i + \tilde{x}_{i+1})/2) - F((\tilde{x}_{i-1} + \tilde{x}_i)/2) - \frac{p_i + A\mu}{2t}[f((\tilde{x}_i + \tilde{x}_{i+1})/2) + f((\tilde{x}_{i-1} + \tilde{x}_i)/2)].$$

A sufficient condition for this derivative to be negative for any $p_i \geq 0$ is $A\mu \geq t/\underline{f}$ because $F((\tilde{x}_i + \tilde{x}_{i+1})/2) - F((\tilde{x}_{i-1} + \tilde{x}_i)/2) \leq 1$. However, the sufficient conditions that guarantee that setting positive consumer prices never pays off are slightly stronger for the two firms closest to the bounds because these firms have no neighbor on one side. The partial derivative of the profit function for these firms is no greater than $1 - \frac{p_i + A\mu}{2t}\underline{f}$, which is non-positive if $A\mu \geq 2t/\underline{f}$.[4] ■

Lemma 1 and the stage 3 equilibrium imply the following result:

Proposition 1. *Assume $\mu \geq 2t/(A\underline{f})$ and keep f fixed. Then the number of active firms and the locations occupied in the subgame perfect equilibrium are the same in the augmented model of this subsection as in the subgame perfect equilibrium outcome of the sequential location game without price competition studied so far when the fixed cost is K_μ.*

References

d'Aspremont, C., J. J. Gabszewicz, and J.-F. Thisse (1979): "On Hotelling's "stability in competition"," *Econometrica*, 47, 1145–1150.

Gabszewicz, J. J., D. Laussel, and N. Sonnac (2001): "Press Advertising and the Ascent of the "Pensée Unique"," *European Economic Review*, 45, 641–651.

Kim, H., and K. Serfes (2006): "Allocation Model with Preference for Variety," *Journal of Industrial Economics*, 54(4), 569–95.

[4] A well-known problem with linear transportation costs is the non-existence of a pure strategy pricing equilibrium because firms may undercut each other's prices to deprive the neighbor of any consumers (d'Aspremont, Gabszewicz, and Thisse, 1979). However, such undercutting is not possible if all competitors set prices of 0. Thus, the existence of pure strategy pricing equilibrium is guaranteed if the sufficient condition is satisfied.