Sequential Location Games*

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Abstract

We study location games where consumers are distributed according to a density f and where market entry is costly and occurs sequentially. This permits an endogenous determination of the number of active firms, their locations and the sequence in which these locations are occupied. While in general the analysis of such games is complicated by the fact that equilibrium locations and the sequence of settlement must be determined simultaneously, we show that they can be independently derived for certain classes of densities including monotone and, under some additional restrictions, hump-shaped and U-shaped ones. For these classes we characterize the subgame perfect equilibrium outcome. Moreover, when f is monotone and concave the equilibrium locations in areas where the density is larger tend to be more profitable. When f is uniform, the number of firms entering in equilibrium is minimal. Extensions of the model allow for price competition for advertisement, tradeoffs between profits in the short and the long run, firms to operate multiple outlets, and winner-takes-it-all competition.

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1 Introduction

Explaining the determinants of product differentiation has a long tradition in economics. Theoretical research typically focuses on firms' location patterns in product space and on how these depend on the market environment. Recent empirical research aims at assessing the impact of the size of different groups of consumers on firms' choices of product characteristics. For example, should a firm cater larger (or economically more important) groups of consumers where it also expects fierce competition, or is it better off by targeting remote areas of the product spectrum where competition tends to be less intense?

The canonical framework for analyzing these issues is due to Hotelling (1929). The theoretical literature on Hotelling models can broadly be separated into three categories. First, there are the location-cum-price models where two exogenously given firms first choose locations and then prices (e.g. Hotelling, 1929; d'Aspremont, Gabszewicz, and Thisse, 1979; Anderson, Goeree, and Ramer, 1997). Second, there are the static models in which an exogenously given number of players simultaneously choose locations only (e.g. Lerner and Singer, 1937; Downs, 1957; Eaton and Lipsey, 1975; Osborne, 1995). Common to both types of models is that they are not easily amendable to the analysis of entry decisions.[1] The desire to study market entry has led Prescott and Visscher (1977, PV hereafter) to introduce a third variant of location models, to which we refer as sequential location games, where firms enter sequentially, bear a fixed setup cost and cannot relocate once they have chosen their locations.

Almost exclusively, the theoretical literature studies models with uniformly distributed consumers. The notable exception among location-cum-price models with an exogenously given number of firms is Anderson, Goeree, and Ramer (1997).[2] As for sequential location-only models, Palfrey (1984) and Callander (2005) are the only exceptions we are aware of where

[1] This is trivially true for the first approach. As for the second, it is well-known that difficulties such as non-existence of pure strategy equilibria may arise (see e.g. Eaton and Lipsey, 1975).

[2] They note (p.101) that "one assumption, which is clearly unrealistic, has been left virtually untouched by the tools of theorists. This is the condition that consumers are uniformly distributed...".

two incumbent players are concerned with deterring entry by a third one, where the underlying distributions are non-uniform, but symmetric.[3]

From a theoretical perspective, this focus on uniform distributions is not satisfactory as some of the most interesting issues cannot be tackled. For example, whether firms should first enter where there are many consumers and whether areas with more consumers are more intensively catered by firms necessarily requires a departure from the uniform distribution. Empirically, the focus on the uniform case is even more problematic since there is by now ample evidence for non-uniform consumer preferences.[4]

The point of departure of this paper is to relax the uniform assumption in sequential location games is. We extend the framework of PV by considering larger classes of densities. In a first step we develop the necessary concepts to analyze equilibrium behavior in this generalized framework. Using these concepts, we then derive some general properties of equilibrium which do not qualitatively depend on the underlying distribution. A full equilibrium characterization (i.e. how many firms enter at which locations and in which order) of this seemingly simple game is non-trivial because in general, equilibrium locations cannot be determined without knowing the sequence in which locations are occupied, which in turn depends on the equilibrium locations themselves. In this respect, a first and methodological contribution of our paper is to show that for large classes of densities, equilibrium locations and the equilibrium sequence of settlement can be determined *independently*. As a result, the equilibrium characterization for this family of sequential location games becomes tractable. In particular, we show that the subgame perfect equilibrium locations are independent of the sequence of settlement when

[3]In Loertscher and Muehlheusser (2008b), a fixed number of players enter in distinct markets, where the distribution across markets may be non-uniform.

[4]Waldfogel (2003), George and Waldfogel (2003), George and Waldfogel (2006) and Waldfogel (2007) provide evidence that preferences for media products differ vastly across ethnic groups so that aggregate distributions will be generically non-uniform. Similarly, consider the market for pharmaceutical drugs, where consumers' "preferences" are given by the prevalence of various diseases. Since some diseases are rare and others very prevalent in any given population, the distribution of consumer preferences will typically not be uniform.

the density is (i) monotonically increasing or decreasing, (ii) hump-shaped and satisfies an additional joint condition on the entry cost and the density and (iii) U-shaped and satisfies an additional symmetry condition around its minimum.

Our model exhibits the following, intuitive comparative statics properties. First, larger markets, or equivalently, markets with lower fixed costs, attract more entry and generate more product variety. This is clearly consistent with the available empirical evidence.[5] Second, firms locate closer to each other in more densely populated segments of the product spectrum, which reflects the findings in Anderson, Goeree, and Ramer (1997, p.111) that "tight density functions are a force of agglomeration". Thus, consumers with similar preferences exert positive externalities on each other. The existence and significance of such preference externalities is empirically well documented (see e.g. George and Waldfogel, 2003; Waldfogel, 2003, 2007). Third, despite the fiercer competition, equilibrium profits in these segments tend to be larger than those in less densely populated areas. Therefore, somewhat loosely speaking, firms should first enter where there are many consumers, despite the fiercer competition this involves.

Moreover, our analysis sheds new light on the uniform case by showing that it is rather special in many important respects. First, it is the distribution that induces the smallest number of active firms in equilibrium.[6] Second, as observed by PV the uniform distribution exhibits a multiplicity of subgame perfect equilibria, both with respect to locations and the sequence of settlement. This indeterminacy is unique to the uniform case insofar as the equilibrium locations and the sequence of settlement are generically pinned down for the families of densities we consider. Third, the multiplicity of equilibria under the uniform led PV to focus on a particular, symmetric equilibrium, where the sequence of settlement occurs from outside in. We show that the resulting equilibrium locations are the ones that arise when the uniform distribution

[5] See e.g. Berry and Waldfogel (1999) and Waldfogel (2003) for media markets or Hsieh and Moretti (2003) for real estate brokers.

[6] This mirrors another result in the locations-cum-price model with two exogenously given firms of Anderson, Goeree, and Ramer (1997, p.105,125), namely that the uniform puts an upper bound on the degree of equilibrium product differentiation.

is considered as the limit case of a symmetric hump-shaped density. While this result gives some justification for PV's equilibrium selection, the equilibrium sequence of settlement tends to be from inside out rather than outside in.[7]

Throughout the paper, we abstract from consumer price competition after locations have been chosen. The main reason for doing so is analytical tractability.[8] The difficulties arising in models with location-cum-price competition are well known even for the uniform case with two exogenously given firms (see e.g. d'Aspremont, Gabszewicz, and Thisse, 1979).[9] Of course, the extent to which our focusing on location choices is a good approximation to real world markets is an empirical question. One natural application of our model are media markets, with firms being newspapers, radio stations or TV broadcasters.[10] Our reading of the relevant empirical literature here is that location choice in product space is indeed of first order importance while prices are not.[11] Similarly, in on-air radio and TV broadcasting consumers are not charged any direct prices.[12]

[7]More generally, PV's claim (see their Footnote 5) that their outside-in principle would also be appropriate for non-uniform distributions finds no support for the classes of densities we consider.

[8]It is well-known that models with price competition are very sensitive to the functional form of consumers' preference costs (see e.g. d'Aspremont, Gabszewicz, and Thisse, 1979). In contrast, our results do not depend on whether these costs are, for example, linear or quadratic; all we need is that they are symmetric and monotone in distance.

[9]Unfortunately, Vogel (2008)'s innovative idea of circumventing the problem of solving for mixed strategy pricing equilibria that are off equilibrium is not readily applicable here, because one needs to find an exact bound on the profits that can be achieved in a given interval. This is true with and without price competition. Using numerical methods Neven (1987) and Götz (2005) study sequential entry followed by price competition with up to five entering firms under the assumptions of uniform distributions and quadratic transportation costs.

[10]Price competition is also naturally absent when considering product differentiation in the context of political economy (see e.g. Downs, 1957; Palfrey, 1984; Osborne, 1995; Callander, 2005).

[11]For example, in their empirical analysis of the effect of the New York Times on local newspapers' content, George and Waldfogel (2006) abstract completely from price competition. Indeed, when comparing, say, the New York Times to the New York Post, the crucial distinguishing feature tends to be their (different) locations in product space, and not price differences. Moreover, there is remarkably little variation in prices over time or across outlets and markets. According to George and Waldfogel (2000), 75% of general interest newspapers in the U.S. were sold at 50 cent per copy in 2000. This is in stark contrast with the variance in newspaper circulation.

[12]Another question in the media context concerns advertisements where firms do compete in prices and which

Apart from media markets, our model is also applicable to markets where prices are administered or where consumption decisions are made largely irrespective of prices. Pharmaceutical products are natural examples of such markets since prices are typically regulated by the government. Moreover, in many countries consumers only pay a small fraction of the price of the drugs they consume due to mandatory or private health insurance plans. Another industry where price competition is essentially absent is real estate brokerage. Brokers do not set prices, but rather commission fees that are levied on the transaction price. Empirically, these fees exhibit almost no variance across markets and over time (see e.g. Hsieh and Moretti, 2003). Thus, competition between brokers will mainly be in product location choices. Similarly, retailers who face binding retail price maintenance contracts will by and large behave as price taking firms who compete in location choice.

Media firms have most recently received a lot of attention in the two-sided markets literature. If one assumes that media firms do not compete for consumers by setting prices, then our paper also relates to this literature; see e.g. Gabszewicz, Laussel, and Sonnac (2001), Rysman (2004), Anderson and Gabszewicz (2006), Anderson and Coate (2005), Rochet and Tirole (2006) or Ambrus and Reisinger (2006).

The model permits a variety of extensions. First, interpreting firms as media companies who first compete for consumers such as readers, viewers or listeners through their location choices and then for advertisements through prices, we show that price competition for advertisements can be incorporated rather straightforwardly, provided only that advertisers' benefits from advertisements are linear in market shares.[13] Second, we consider an extension where in every

is often by far the most important source of revenue for media firms (see e.g. George and Waldfogel, 2000). In an extension (Section 6.1) we show that the model with media firms who compete first for consumers and then for advertisement revenue is easily amendable to price competition for advertisements because a firm's equilibrium profit will be linear in its market share.

[13]If consumer prices and advertisements only affect consumers decision whether to buy, view or listen to a given media product, but do not affect their decision which of these products to consume, then the model can also incorporate price competition for consumers and consumers experiencing disutility from advertisements.

period exactly one firm may enter and where profits accrue to every active firm in every period. This creates a trade-off between short-term and long-term profits, which kicks in already when only three firms enter in equilibrium, as we show in a simple example. Third, we extend the model by allowing firms to operate multiple outlets. The equilibrium locations in a model with only single-outlet firms are also equilibrium locations in a model with multi-outlet firms and are occupied by the first entering firm. Fourth, we sketch how winner-takes-it-all competition as occuring e.g. under plurality voting affects equilibrium behavior.

The remainder of the paper is organized as follows. Section 2 introduces the model. In Section 3 we develop the crucial concepts for the subsequent equilibrium analysis. Section 4 contains some general properties of equilibrium. Sections 5.1 and 5.2 analyze monotone and non-monotone densities, respectively, while Section 5.3 studies the uniform case as a limit case. Extensions are discussed in Section 6. Section 7 concludes. All proofs are in the Appendix.

2 The model

Consider a product market with a unit mass of consumers distributed along the $[0, 1]$-interval according to the cumulative distribution function $F(x)$ with density $f(x) > 0$ for all $x \in [0, 1]$. There is a large number of firms who can potentially enter the market. Firms move sequentially in an exogenously given order.[14] If firm i is given the move, it decides whether or not to enter the market. If it enters, it incurs a fixed cost $K > 0$ and chooses a location in $[0, 1]$. In either case, its decision is observed by all firms moving subsequently. Once a location is chosen, it is prohibitively costly to change it ex post.[15] Each consumer patronizes the closest firm. The profit of each active firm, gross of the entry cost K, is equal to the mass of consumers it attracts. Apart from the possibility that later entrants may face a less attractive choice set, no costs are associated with entering later. For convenience we assume that firms stay out when

[14] For the uniform case, Anderson and Engers (2001) analyze a model where the order of entry is endogenous.

[15] Such costs may include physical relocation costs, or advertisement costs to change the brand image of a firm (see e.g. PV).

indifferent.

3 Concepts

In this section, we develop several concepts which are crucial for the equilibrium analysis. We begin with the optimal location of a firm in a given interval under the assumption of no subsequent entry, and then turn to the issue of entry deterrence. Throughout the paper, we refer to an interval (L, R) as one where the locations L and R are already **occupied** by competitors, and which is **empty** in the sense that no firm is located in its interior.

3.1 Optimal locations absent further entry

Consider a firm entering in an interval (L, R).[16] Under the assumption of *no further subsequent entry* in this interval, when entering at location $x \in (L, R)$, the firm's profit is

$$\pi(x, L, R) := F\left(\frac{x+R}{2}\right) - F\left(\frac{x+L}{2}\right).$$

That is, it attracts all customers between the midpoints between its own location and the locations of its competitors to the right and left, respectively. Note that the "reach" of the firm's customer base, or its market coverage, $\Delta(L, R) := \frac{R-L}{2}$, is simply half the interval length, and thus independent of x. For this reason, choosing an optimal location within a given interval (L, R) is equivalent to finding a location x that maximizes the integral over $\Delta(L, R)$.

Definition 1 *(i)* $X^*(L, R) := \arg\max_{x \in (L,R)} \pi(x, L, R)$ *is the set of optimal locations in the interval* (L, R)*, an element of which is denoted* $x^*(L, R)$*.*[17]

(ii) For all $x^* \in X^*(L, R)$*,* $\pi^*(L, R)$ *is the firm's profit when locating optimally inside* (L, R)*.*

[16]Note that while the interval (L, R) is open by definition, firms are not a priori prohibited to choose identical locations. As shown below, however, such behavior is inconsistent with equilibrium.

[17]For example for the uniform distribution, $X^*(L, R) = (L, R)$.

(iii) $\hat{X}(\underline{z}, \overline{z}, L, R) := \arg\max_{x \in [\underline{z}, \overline{z}]} \pi(x, L, R)$ *is the set of optimal locations in the interval* (L, R) *when the choice set is restricted to some interval* $[\underline{z}, \overline{z}] \subset (L, R)$*. Elements of this set are denoted by* $\hat{x}(\underline{z}, \overline{z}, L, R)$*.*

(iv) $\hat{\pi}(\underline{z}, \overline{z}, L, R) := \pi\left(\hat{x}(L, R), L, R\right)$ *is the firm's profit when choosing one of these optimal (restricted) locations.*

(v) For arbitrarily small $\epsilon > 0$*,* $L^+ := L + \epsilon$ *and* $R^- := R - \epsilon$ *denote the smallest and the largest possible locations in* (L, R)*, respectively.*[18]

Lemma 1 *If* $x^*(L, R)$ *is interior, i.e. satisfies* $f\left(\frac{x^*+L}{2}\right) = f\left(\frac{x^*+R}{2}\right)$ *and* $f'\left(\frac{x^*+L}{2}\right) > 0 > f'\left(\frac{x^*+R}{2}\right)$*, then* $-1 < \partial x^*/\partial L < 0$ *and* $-1 < \partial x^*/\partial R < 0$ *holds.*

If $x^*(L, R)$ is determined by a first-order condition, then the firm at $x^*(L, R)$ optimally moves closer to its neighbor if that neighbor comes closer, however, it optimally moves by less than its neighbor. For example, consider the neighbor at L. By moving right the firm at L still gains costumers to its right even if its right-hand neighbor at $x^*(L, R)$ is not already there because $d(x^* + L)/dL > 0$. Similarly, $d(x^* + R)/dR > 0$ holds by Lemma 1 so that the firm at R would gain customers to its left by moving left even if the neighbor at $x^*(L, R)$ has not yet occupied his location.

Lemma 2 $\pi^*(L, R)$ *and* $\hat{\pi}(\underline{z}, \overline{z}, L, R)$ *strictly decrease in* L *and strictly increase in* R*.*

3.2 Entry-deterring locations

The following concepts are useful for addressing the issue of entry deterrence and for determining equilibrium configurations. A distinction has to be made between entry deterrence (i) with respect to an already occupied location inside (0,1), and (ii) with respect to one of the (in any equilibrium unoccupied) boundary points $\{0, 1\}$ of the product spectrum.

[18]Of course, since the interval (L, R) is open, L^+ and R^- are not well-defined in a continuous framework. We follow the standard notion in the literature where the continuous case emerges as the limit of a discrete choice set with "grid size" ϵ, where $\epsilon \to 0$.

Definition 2 *For any occupied locations* $y \in [0, \rho(1)]$ *and* $y \in [\lambda(0), 1]$*, respectively, let* $\lambda(y)$ *and* $\rho(y)$ *be such that* $\pi^*(y, \lambda(y)) = K$ *and* $\pi^*(\rho(y), y) = K$*. Moreover, denote* $\lambda_B := F^{-1}(K)$ *and* $\rho_B := F^{-1}(1-K)$*.*

Note first that $\lambda(\cdot)$ and $\rho(\cdot)$ also depend on K. Moreover, note that $\lambda(y) > y$ and $\rho(y) < y$ and that $\lambda(\cdot)$ is the inverse of $\rho(\cdot)$, i.e. $\rho(\lambda(y)) = y = \lambda(\rho(y))$.[19] Intuitively, with competitors located at y and $\lambda(y)$, an entrant would get exactly K when locating optimally in the interval $(y, \lambda(y))$ and, consequently, prefers not to enter. The intuition for $\rho(y)$ is analogous.

Because $\pi(x, L, R)$ strictly decreases in L and strictly increases in R for any $x \in (L, R)$, $\lambda(\cdot)$ and $\rho(\cdot)$ are unique. As will be shown below, for any occupied location y, $\lambda(y)$ is therefore the largest entry-deterring location to the right of y. Analogously, $\rho(y)$ is the smallest entry-deterring location to the left of y.

The second part of the definition provides an appropriate adaption of these concepts to unoccupied boundary points: If the left boundary point 0 is not occupied while a firm is located at λ_B, an entrant would just be deterred from entering in the interval $[0, \lambda_B)$.[20] An analogous argument applies to the right boundary point 1. Note that our assumption $K < \frac{1}{2}$ implies $\lambda_B < \rho_B$. If $\frac{1}{2} < K < 1 \Leftrightarrow 0 < \rho_B < \lambda_B < 1$, the first firm optimally enters anywhere in the interval $[\rho_B, \lambda_B]$ thereby forestalling further entry. If $K \geq 1 \Leftrightarrow \rho_B \leq 0 < 1 \leq \lambda_B$, the market cannot even support one firm.

Lemma 3 *(i)* $\lambda(y)$ *and* $\rho(y)$ *strictly increase in* y*.*

(ii) For any two occupied locations L, R *with* $L < R$*,*

$$\lambda(L) > R \Leftrightarrow \rho(R) < L \Leftrightarrow \pi^*(L, R) < K.$$

[19] Thus, $\lambda(y) \leq 1$ holds for all $y \leq \rho(1)$ and $\rho(y) \geq 0$ holds for all $y \geq \lambda(0)$.

[20] Note that an entrant's optimal location in this case would be λ_B^- for any distribution, since he gets the whole hinterland. Moreover, we have $\lambda_B < \lambda(0)$, because the only difference refers to whether or not the end point 0 is occupied. By definition of λ_B, this is not the case, and so a firm locating at some $x \leq \lambda_B$ gets the whole hinterland to the left of x. However, when the end point 0 is occupied as is the case by definition of $\lambda(0)$, this hinterland is shared with the firm at 0. Analogous reasoning establishes that $\rho(1) < \rho_B$.

(iii) For any occupied locations $y \in [0, \rho(1)]$ and $y \in [\lambda(0), 1]$, respectively,

$$K < F(\lambda(y)) - F(y) \leq 2K \quad \text{and} \quad K < F(y) - F(\rho(y)) \leq 2K.$$

(iv) For y given, $\lambda(y)$ is increasing, and $\rho(y)$ is decreasing in K.

Entry-deterring locations when F is uniform In the uniform case, any location in a given interval (L, R) yields the same payoff of $\frac{F(R)-F(L)}{2} = \frac{R-L}{2}$. Thus, by definition of $\lambda(y)$, $\frac{\lambda(y)-y}{2} = K$ must hold, implying $\lambda(y) - y = F(\lambda(y)) - F(y) = 2K$. Analogously, $y - \rho(y) = 2K$ holds. It then follows from part (iii) of Lemma 3 that the mass of consumers between two entry deterring locations is maximum in the uniform case. As Theorem 8 below shows, this also implies that for F uniform, the equilibrium number of active firms will be minimum.

4 Equilibrium properties

The remainder of the paper characterizes subgame perfect equilibria, to which we simply refer as "equilibria". We first derive some general properties of equilibrium, starting with an implication of Lemma 3:

Corollary 1 *Three occupied locations L, x, R, where $L < x < R$ are not consistent with equilibrium if*

$$\rho(R) \leq L < x < R \leq \lambda(L).$$

When the condition in the corollary holds, then $\pi(x, L, R) \leq K$ for all $x \in (L, R)$ follows from Lemma 3. So the firm at location x could profitably deviate by staying out of the market.

4.1 Number of entrants in a given interval

Denote by # the number of firms entering in equilibrium in a given interval (L, R).

Theorem 1 *In any equilibrium,*

(i) $\rho(R) \leq L < R \leq \lambda(L) \Leftrightarrow \# = 0$,

(ii) *(a)* $L < \rho(R) < \lambda(L) < R \Rightarrow \# \in \{1, 2\}$ *and*

(b) $\# = 1 \Rightarrow L < \rho(R) < \lambda(L) < R$,

(iii) *(a)* $L < \lambda(L) < \rho(R) < R \Rightarrow \# \geq 2$ *and*

(b) $\# > 2 \Rightarrow L < \lambda(L) < \rho(R) < R$.

Recall that because of the symmetry of $\lambda(\cdot)$ and $\rho(\cdot)$, the cases $\rho(R) < L < \lambda(L) < R$ and $L < \rho(R) < R < \lambda(L)$ cannot occur. So only the three configurations addressed in the respective parts of the theorem need to be considered. Intuitively, in part (i) the market size in a given interval is too small to support profitable entry. As the market size increases, so that we are in the case described by part (ii), at least one entrant can profitably enter in the interval. Whether this first entrant optimally forestalls further entry or invites entry by one more firm depends on the distribution of consumers. As the market size increases even further, so that we are in the case described by part (iii), the first entrant can no longer deter further entry, so that in this case at least two firms enter.

4.2 Distance between neighboring firms

Apart from the number of firms entering in equilibrium in a given interval, we can also say something about distances between firms in any equilibrium. We refer to two firms at locations L and R as **neighbors** when the interior of the interval (L, R) is empty. We start with the following corollary of Theorem 1:

Corollary 2 *In any equilibrium, two firms at locations L and R are neighbors if and only if $\pi^*(L, R) \leq K$.*

Part (i) of Theorem 1 implies that there will be no further entry if $\pi^*(L, R) \leq K$, which, as will be recalled, is equivalent to $\rho(R) \leq L \Leftrightarrow R \leq \lambda(L)$. To see that there is entry if

$\pi^*(L,R) > K$, observe first that $\pi^*(L,R) > K \Leftrightarrow \rho(R) > L \Leftrightarrow R > \lambda(L)$, for which case(s) parts (ii) and (iii) of Theorem 1 say there will be entry. Hence, L and R cannot be neighbors if $\pi^*(L,R) > K$. The import of Corollary 2 (and Theorem 1) is that there is entry in (L,R) whenever $\pi^*(L,R) > K$, so that there are no "black holes". That is, it cannot happen in equilibrium that a firm does not enter in an interval (L,R) because it correctly fears that it would subsequently not break even because of further entry whereas it would pay to enter in (L,R) were it the only entrant.

Theorem 2 *For any three neighboring equilibrium locations L, x, R satisfying $L < x < R$, the following condition must hold:*

$$\rho(x) \leq L < \rho(R) \leq x \leq \lambda(L) < R \leq \lambda(x).$$

Theorem 2 is a statement about distances between neighboring firms in any equilibrium: First, the *minimum distance* between the firms located at x and L must be strictly larger than $x - \rho(R)$. Otherwise, we would have $L > \rho(R)$ (i.e. when L is "too close"), then by Corollary 1, the firm at x does not break even. The same argument applies to the right-hand side when $R < \lambda(L)$.

Second, the *maximum distance* between the firm located at x and its neighbors at L and R, is $x - \rho(x)$ and $\lambda(x) - x$, respectively. Otherwise, by Corollary 2, there would be entry in between, contradicting that the firms at locations x and L (respectively x and R) are neighbors.

Recall that we do not a priori rule out the possibility that firms choose identical locations. However, the following implication of Theorem 2 establishes that this will not happen in equilibrium:

Corollary 3 *In any equilibrium, any location $x \in [0,1]$ is occupied by at most one firm.*

4.3 Range of product variety

Let a be the leftmost and b be the rightmost location that is occupied in equilibrium. Then:

Theorem 3 *$a \leq \lambda_B$ and $b \geq \rho_B$.*

A natural question is whether it is possible to prove the substantially stronger statement that in any equilibrium, $a = \lambda_B$ and $b = \rho_B$. Generally however, one cannot exclude the possibility that a firm might want to locate to the left (right) of λ_B (ρ_B) so as to induce its closest neighbor to locate substantially further away (which it might if the neighbor's location is given by a first order condition, see Lemma 1).

5 Equilibrium Locations and Settlement

5.1 Monotone densities

Equilibrium locations The underlying intuition for determining the equilibrium configuration for monotone densities can be best understood by considering first an interval (L, R) satisfying $L < \rho(R) < \lambda(L) < R$. From Theorem 1 we know that at least one and at most two firm(s) will enter in this interval. The next result establishes that when f is monotone, exactly one firm will enter:[21]

Lemma 4 *When f is monotone over an interval (L, R) satisfying $L < \rho(R) < \lambda(L) < R$, exactly one additional firm will enter. This firm locates at $\hat{x} = \lambda(L)$ if f is increasing, and at $\hat{x} = \rho(R)$ if f is decreasing.*

Note first again the difference between optimal locations in a given interval *absent* further entry, and optimal entry-deterring ones: Clearly, $x^*(L, R) = R^-$ which, however, would invite further entry. As a result, the first entrant optimally chooses the best entry-deterring location, $\hat{x}(\rho(R), \lambda(L), L, R) = \lambda(L)$, thereby earning profit $\hat{\pi}(\cdot) < \pi^*(\cdot)$. Note also that the entrant's optimal location $\lambda(L)$ only depends on the location of its left-hand neighbor L, but not on the location of its right-hand neighbor at R.

[21]The result is stated under more narrow conditions than necessary. It holds for any density that is quasi-concave over (L, R).

Let us now look at the more general cases where f is strictly increasing resp. decreasing for all $x \in [0,1]$. Let $\lambda^0 \equiv \lambda_B$, $\lambda^1 \equiv \lambda(\lambda_B)$, $\lambda^2 \equiv \lambda(\lambda^1)$ ...$\lambda^{k+1} \equiv \lambda(\lambda^k)$ and similarly $\rho^0 \equiv \rho_B$, $\rho^1 \equiv \rho(\rho_B)$, $\rho^2 \equiv \rho(\rho^1)$, ... $\rho^{j+1} \equiv \rho(\rho^j)$. Moreover, let $n \geq 0$ and $m \geq 0$ be the largest integers such that $\lambda^n < \rho_B$ and $\lambda_B < \rho^m$. That such n and m exist and are unique follows from the monotonicity of $\lambda(\cdot)$ and $\rho(\cdot)$.[22] Throughout we focus on the generic cases where $\lambda^k \neq \rho_B$ and $\lambda_B \neq \rho^k$ for any integer k.

Theorem 4 *When f is monotone over $[0,1]$, the set of equilibrium locations is unique.*

(i) When f is increasing, $n+2$ firms enter at locations $\{\lambda_B, \lambda^1, .., \lambda^n, \rho_B\}$.

(ii) When f is decreasing, $m+2$ firms enter at locations $\{\lambda_B, \rho^m, .., \rho^1, \rho_B\}$.

As for part (i), the optimal location of firm i is driven solely by the location of its left-hand neighbor, at location y say. As for the right-hand neighbor, either it is already there in which case, because f is increasing, firm i optimally moves to the right as far as possible without inviting further entry to its left, which is at $\lambda(y)$. When i anticipates its right-hand neighbor to be a subsequent entrant, then a fortiori $\lambda(y)$ is optimal for firm i, because its future right-hand neighbor will also optimally locate at $\lambda(\lambda(y))$, so that i pushes this firm as far to the right as possible.

It follows that it is never optimal for a firm *not* to locate at a λ-location (except for location ρ_B, of course), independent of whether its neighbors to the left and right (who will also optimally locate at λ-locations) are already there or will be future entrants. As a result, for monotone densities *the set of equilibrium locations is independent of when these locations are occupied.* Non-monotone densities will typically not have this property: In general, the set

[22]The restriction to strictly increasing (decreasing) functions can be easily relaxed since the analysis goes through if f is flat on some parts of the interval [0,1] as long as it increases (decreases) sufficiently often. To be precise, a sufficient condition is that in any interval $[\lambda^k, \lambda^{k+1}]$ there is an x such that f increases (decreases) at x. This will make sure that the best responses are unique.

of equilibrium locations will depend on the sequence in which these locations are taken and vice versa, which renders the equilibrium analysis much more complicated.

Theorem 4 also implies that the distance between equilibrium locations becomes smaller as the density increases. Hence, the more densely populated a segment of the product spectrum, the more product variety will emerge in equilibrium in this segment.

Application: Preference Externalities The present framework also exhibits what has become known as "preference externalities" (see e.g. Waldfogel, 2003; George and Waldfogel, 2003) in a very concise and natural way. To see this, assume in line with the empirical evidence presented in these studies that preferences differ according to ethnic background, so that, say, a decreasing (increasing) density describes the distribution of preferences in a population consisting of Hispanics (Whites). Because firm locations are closer together where the density is high, Hispanics have on average lower preference costs in the predominantly Hispanic area than in the predominantly White area and vice versa.

Sequence of Settlement In general, for a given distribution of consumers and order of entry, the sequence of settlement is pinned down, where the earlier entrants grasp the larger profits, except when two or more equilibrium profits are the same. Without additional information about f, however, it is generally not possible to determine the full ordering of equilibrium profits. Nonetheless, some results can be derived under fairly general conditions. In the following, we denote by $\pi(y)$ the equilibrium profit of a firm at equilibrium location y.

Theorem 5 *(i) If f is increasing, then $\pi(\rho_B) > \pi(\lambda^n)$, such that location ρ_B will be occupied prior to location λ^n in any equilibrium. When f is, in addition, weakly concave, then*

$$\pi(\lambda^{n-1}) > ... > \pi(\lambda_B)$$

holds, i.e the settlement of locations $\{\lambda_B, ..., \lambda^{n-1}\}$ occurs from the right to the left.

(ii) If f is decreasing, then $\pi(\lambda_B) > \pi(\rho^m)$, such that location λ_B will be occupied prior to location ρ^m in any equilibrium. When f is, in addition, weakly concave, then

$$\pi(\rho^{m-1}) > ... > \pi(\rho_B)$$

holds, i.e. the settlement of locations $\{\rho^{m-1}, ..., \rho_B\}$ occurs from the left to the right.

As the intuition for both parts is analogous, we confine attention to part (i): Recall first from Theorem 4 that for f increasing, the two rightmost equilibrium locations are λ^n and ρ_B, respectively. Hence, the profitability of these two locations can be unambiguously ordered. The reason is that the firm at location λ^n (ρ_B) gets K to its left (right), while, because f is increasing, there are more costumers to the right of the midpoint $\frac{\lambda^n+\rho_B}{2}$ than to the left, so that the overall profit of the firm at ρ_B is higher.

When f is in addition (weakly) concave, the same is true for all profits at (equilibrium) locations in $[0, \lambda^{n-1}]$. Hence, despite the fact that distances between neighboring firms become smaller as the density increases, firms still tend to prefer these locations to those in less densely populated segments of the product spectrum.[23] Note that concavity of f is only a sufficient condition, and the result holds as long as f is not too convex. Equally or even more importantly, the theorem holds for linear densities. In particular, it holds for any density with slope $f'(x) = \varepsilon$, where $\mid \varepsilon \mid \neq 0$ is arbitrarily small. Consequently, for any such density the order of settlement will be generically unique (i.e. for almost every K). This contrasts sharply with the uniform case, where the sequence of settlement is indeterminate since all, but at most three, equilibrium locations are equally profitable (see also Section 5.3). Note also that an outside-in principle as claimed by PV (p. 385) does not apply for strictly monotone densities: as just shown, the sequence of settlement tends to be either from right to left (for f increasing) or from left to right

[23] A potential exception is the location λ^n since its right-hand neighbor is at ρ_B and thus not determined by a λ-distance. For K large and n fixed, this distance, and the resulting profit to the right of λ^n become very small, in which case it is a rather unattractive location. An analogous argument applies to location ρ^m when f is decreasing.

(for f decreasing). Moreover, as shown in Theorem 4, the determination of the equilibrium locations themselves is driven by a sequence of λ- or ρ-functions. These are defined with respect to λ_B or ρ_B, depending on whether f is increasing or decreasing and so this process either works from left to right or from right to left, but not outside-in in the sense of PV.

5.2 Non-monotone densities

Hump shaped densities Consider now hump-shaped quasiconcave density functions such as depicted in Figures 1 and 2. Denote the single point where the density peaks by H. These densities are thus increasing for all $x < H$, and decreasing for all $x > H$. Moreover, let $r \geq 0$ be such that $f(\lambda^r)$ is still increasing while $f(\lambda^{r+1})$ is decreasing. Analogously, let $s \geq 0$ be such that $f(\rho^s)$ is decreasing and $f(\rho^{s+1})$ is increasing. Two cases emerge, depending on whether or not profitable entry is possible in the interval (λ^r, ρ^s) (see Figures 1 and 2): In the first case, the locations λ^r and ρ^s are entry-deterring, while in the second case they are not.[24]

It turns out that the second case is substantially more complicated, because the equilibrium locations and the sequence of settlement can no longer be determined separately. Intuitively, for a given number of entrants, the location "under the hump" tends to be the more profitable, the lower K.[25] Therefore, when K is sufficiently low, the first firm will indeed optimally locate under the hump. As a result, since the density is increasing in the direction of the hump, future entrants (the neighbors of the first firm, in particular) will optimally locate as close as possible without inviting further entry to their other side (i.e. they choose some λ- or ρ-location as in the case where f is monotone, see Theorem 4).

On the other hand, when K is large, the location under the hump will only be occupied at a later stage. In this case, earlier entrants might have an incentive to depart from these

[24] From the definitions of $\lambda(\cdot)$, $\rho(\cdot)$, and part (ii) of Lemma 3, it follows that $\rho^{s+1} < \lambda^r \Leftrightarrow \rho^s < \lambda^{r+1}$ holds in the first case, and $\lambda^r < \rho^{s+1} \Leftrightarrow \lambda^{r+1} < \rho^s$ in the second.

[25] Of course, whether or not a given location is attractive or not is endogenous to the game, which only goes to further stress the complications that arise.

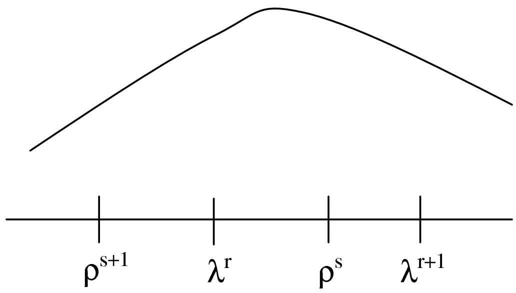

Figure 1: Interval (λ^r, ρ^s) entry-deterring (case I)

λ- and ρ-locations in order to push future entrants further away (including the one locating under the hump). As shown in Lemma 1, this is possible when optimal locations are given by a first-order condition. As a result, even when holding fixed the number of active firms, both, the equilibrium locations and the order of settlement may differ for different values of K.

In the present paper, we focus on the first case for which the equilibrium outcome can be characterized by combining our previous result for monotone densities:[26]

Theorem 6 *Let f be hump-shaped satisfying $\rho^{s+1} < \lambda^r$. Then the unique set of equilibrium locations is $\{\lambda_B, .., \lambda^r, \rho^s, .., \rho_B\}$, so that $r + s + 2$ firms enter.*

If, in addition, f is symmetric in the sense that $f(x) = f(1-x)$ for any $x \in [0,1]$, the theorem yields the equilibrium locations whenever the number of entrants is even. The reason is that, for f symmetric, an even number of entrants means "no entry under the hump", i.e. $r = s \equiv n$ and $\rho^{n+1} < \lambda^n$, so that the number of entrants is $2n + 2$.

Given Theorem 5, an albeit incomplete characterization of the sequence of settlement is also at hand:

Corollary 4 *Let f be hump-shaped quasiconcave satisfying $\rho^{s+1} < \lambda^r$. Then, the sequence of settlement of the locations $\{\lambda_B, .., \lambda^{r-1}\}$ occurs from the right to the left, and the sequence of settlement of the locations $\{\rho^{s-1}, .., \rho_B\}$ occurs from the left to the right.*

[26] We refer the interested reader to a companion paper (Loertscher and Muehlheusser, 2008a), which is exclusively devoted to the second case.

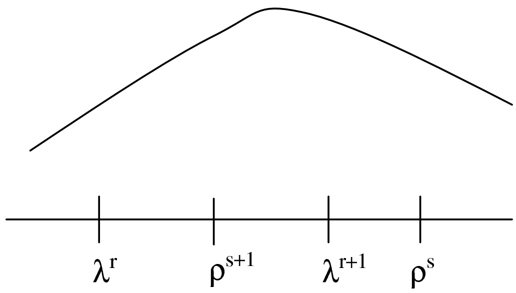

Figure 2: Interval (λ^r, ρ^s) not entry-deterring (case II)

The result follows directly from Theorem 5 and requires no separate proof. The characterization is incomplete because without additional information on f it is not possible to (i) say anything about the profits at locations λ^r and ρ^m and (ii) rank the locations to the left relative to those on the right. Note that the sequence of settlement is again not "outside-in" in the sense of PV but, essentially, "inside-out".

U-shaped densities Consider now densities which are U-shaped around some trough location M, i.e. f is decreasing for all $x < M$ and increasing for all $x > M$.

Lemma 5 *When f is U-shaped over the interval (L, R), then $x^*(L, R) \in \{L^+, R^-\}$.*

Lemma 5 has the following corollary:

Corollary 5 *The minimum point M is never occupied in equilibrium.*

Determining the full set of equilibrium locations when f is U-shaped is considerably simplified by imposing the following symmetry condition. Denote by ρ_M and λ_M the analogs to ρ_B and λ_B with respect to the trough M when M is not occupied. That is, $F(M) - F(\rho_M) = K$ and $F(\lambda_M) - F(M) = K$.

Definition 3 *The distribution F is **trough-symmetric** if*

$$F(\lambda_M) - F(M) = F(M) - F(2M - \lambda_M).$$

Observe that trough-symmetry implies that ρ_M and λ_M are at the same distance from M, i.e. $\lambda_M - M = M - \rho_M$. Notice also that any distribution that is symmetric around M is trough-symmetric. Next, let $\rho_M^1 \equiv \rho(M)$, $\rho_M^2 \equiv \rho(\rho_M^1)$, and $\rho_M^{j+1} \equiv \rho(\rho_M^j)$ and $\lambda_M^1 \equiv \lambda(M)$, $\lambda_M^2 \equiv \lambda(\lambda_M^1)$, and $\lambda_M^{k+1} \equiv \lambda(\lambda_M^k)$. Finally, let s and n be such that $\rho_M^{s+1} < \lambda_B < \rho_M^s$ and $\lambda_M^n < \rho_B < \lambda_M^{n+1}$. Clearly, n and s will only be positive integers when the fixed cost K is sufficiently small, which we are assuming for the remainder of this section.

Theorem 7 *Let F be trough-symmetric. Then in equilibrium, $r+s+4$ firms enter at locations*

$$\{\lambda_B, \rho_M^s, .., \rho_M^1, \rho_M, \lambda_M, \lambda_M^1, .., \lambda_M^n, \rho_B\}.$$

Intuitively, trough-symmetry ensures that the midpoint between firms at ρ_M and λ_M is at the trough M. By definition of ρ_M and λ_M, each firm gets profit K in this interval. Moreover, from Lemma 5, $x^*(\rho_M, \lambda_M) = \{\rho_M^+, \lambda_M^-\}$ so that an entrant optimally locating in-between would also just reap K and thus prefers not to enter.

As for the sequence of settlement, it follows directly from Theorem 5 that location λ_B (ρ_B) will be occupied prior to location ρ_M^s (λ_M^n). Moreover, when in addition f is concave on each of its two branches, Theorem 5 also applies:

Corollary 6 *Let f be trough-symmetric and concave over the intervals $[0, M]$ and $[M, 1]$, respectively. Then, the sequence of settlement of the locations $\{\lambda_M^{n-1}, .., \lambda_M\}$ occurs from the right to the left, and the sequence of settlement of the locations $\{\rho_M^{s-1}, .., \rho_M\}$ occurs from the left to the right.*

Again, the relative profitability of the locations on each branch of f depends on the exact specification. Since the sequence of settlement is, broadly speaking, from the outside in, it is in accordance with PV's claim.

5.3 Uniform distributions as limit cases

PV analyze a special case of the present model where f is uniform. They focus on (subgame perfect) equilibria where $2n+2$ firms enter at locations $\{K, .., (2n+1)K, 1-(2n+1)K, .., 1-K\}$ for $n \geq 0$. Using our notation, this can be equivalently written as[27]

$$\{\lambda_B, \lambda^1, .., \lambda^n, \rho^n, .., \rho^1, \rho_B\}.$$

They argue that the equilibrium sequence of settlement for these locations is from outside in, and if the number of active firms is odd, a final entrant enters at 1/2. An interesting question is therefore whether these equilibrium locations and the sequence of settlement emerge as the limit case of the cases previously analyzed.[28]

Monotone densities Define $f^m_\varepsilon(x) \equiv 1-\varepsilon/2+\varepsilon x$ for all $x \in [0,1]$ and $\varepsilon \in [-2,2]$. Note that $f^m_\varepsilon(x)$ is an affine function that increases in x if $\varepsilon > 0$ and decreases if $\varepsilon < 0$ and that converges to the uniform density as $\varepsilon \to 0$. Because $f^m_\varepsilon(x)$ is an affine function, it is also concave. Therefore, we know that for $\varepsilon > 0$, the equilibrium locations will be $\{\lambda_B, \lambda^1, .., \lambda^n, \rho_B\}$ and the sequence of settlement will tend to occur from right to left, while for $\varepsilon < 0$, the equilibrium locations are $\{\lambda_B, \rho^m, .., \rho^1, \rho_B\}$ and the sequence of settlement is from left to right.[29] So neither the equilibrium locations nor the equilibrium sequence of settlement correspond to those of PV.

Hump-shaped densities As discussed after Theorem 6, we can solve for the equilibrium locations with hump-shaped, symmetric densities when the number of entrants is even and thus

[27] Recall that for the uniform case, $\lambda_B = K$, $\rho_B = 1-K$, $\lambda(y) = y+2K$ and $\rho(y) = y-2K$.

[28] In the non-generic cases where $\frac{1}{K}$ is an even number, $\frac{1}{2K}$ firms enter indeed at locations $\{K, 3K, ..., \frac{1}{2}, ..., 1-3K, 1-K\}$. But since all firms then earn a profit of $2K$, the order of sequence is indeterminate, and so PV's outside-in sequence is only one out of many possible sequences.

[29] It follows from Theorem 4 that n and m are (weakly) increasing respectively decreasing in ε. Thus, for ε sufficiently large (in absolute terms), there will be more entry compared to the uniform case. However for ε sufficiently close to zero (which is the case of interest in our context) the number of active firms is the same.

equal to $2n + 2$.[30] As will be shown, PV's equilibrium locations, though not their sequence of settlement, can be obtained as the limiting case of the following hump-shaped density: Let $f^h_\varepsilon(x) \equiv 1 - \varepsilon/4 + \varepsilon x$ for $x \in [0, 1/2]$ and $f^h_\varepsilon(x) \equiv 1 + 3\varepsilon/4 - \varepsilon x$ for $x \in (1/2, 1]$ and $\varepsilon \in [0, 4]$ be the symmetric hump-shaped density with constant slopes.[31]

Before we can state the limit result as $\varepsilon \to 0$, we must make sure this limit is well defined insofar as the number of entrants remains at $2n + 2$. Theorem 8 below implies that no fewer firms will enter under f^h_ε if $2n + 2$ firms enter under the uniform. However, we also need to verify that $2n + 2$ firms enter for all $\varepsilon \in [0, \varepsilon_0]$ if $2n + 2$ firms enter for some ε_0:

Lemma 6 *If $\rho^{n+1} < \lambda^n$ holds for some $\varepsilon_0 > 0$, then it also holds for any $\varepsilon \in [0, \varepsilon_0]$.*

Lemma 6 implies a monotonicity property. As ε increases from 0 to some positive number ε_0, the equilibrium number of entrants increases monotonically (and weakly). So if the number of active firms under the uniform is $2n + 2$, this will also be true for some ε_0. This allows us to take the limit $\varepsilon \to 0$, starting from ε_0. From Theorem 6, as ε approaches zero, the equilibrium locations will indeed be the ones derived by PV. This partially corroborates the equilibrium PV focus on. It does so only partially because the equilibrium sequence of settlement is, broadly speaking, from inside out, rather than from outside in.[32]

U-shaped densities Last, consider the density $f^h_\varepsilon(x)$ for $\varepsilon \in [-4, 0]$. This is a two sided triangle distribution that is symmetric around 1/2 and hence trough-symmetric. Therefore,

[30] The two conditions for the number of active firms being equal to $2n + 2$ are (i) $\lambda^n < \rho^n$ and (ii) $\rho^{n+1} < \lambda^n$. This leads to the following condition with respect to K:

$$\frac{1}{4n + 4} < K < \frac{1}{4n + 2},$$

where the first inequality follows from (ii) and the second from (i).

[31] Note also that for $\varepsilon = 4$, $f^h_\varepsilon(x)$ is the triangle distribution. If $\varepsilon < 0$, $f^h_\varepsilon(x)$ is U-shaped (or V-shaped) and has a trough at 1/2. For $\varepsilon = -4$ it is the (trough symmetric) two-sided triangle.

[32] To be precise, the relative profitability of the locations λ^n and ρ^n will depend on ε, and so the sequence of settlement may not be strictly from inside out. However, as shown in Corollary 4, all equilibrium locations to the left (right) of λ^n (ρ^n) are the more profitable the closer they are to λ^n (ρ^n).

from Theorem 7, the equilibrium locations are determined from the minimum of $f_\varepsilon^h(x)$, i.e. from 1/2. Since each branch of the density is again a concave function, it follows from Corollary 6 that the sequence of settlement is, broadly speaking, from the outside in, which is in accordance with PV, but the equilibrium locations are not.

We conclude our discussion of the uniform case with the following result on the number of active firms in equilibrium for the uniform case:

Theorem 8 *The number of active firms in equilibrium is minimum when F is uniform.*

For an intuition, consider the uniform density as the limit case of a monotonically increasing density $f_\varepsilon^m(x)$ which makes sure that all best replies are uniquely pinned down. Now consider a firm locating at some R to the right of a firm located at L, where R is interior in the sense that its righthand neighbor will not be ρ_B. Since R^- is a best location in the interval (L, R), and indeed for any $\varepsilon > 0$ it will be the unique best location, it follows that when locating at $R = L + 2K$ a firm can get exactly K to its left without attracting further entry in (L, R). Moreover, and perhaps more importantly, by doing so, which is in its very best interest, the firm locating at R generates profits of also exactly K to the right of L for the firm at L. Iterating the argument once more, it follows that the firm at R will get K to its right as well and so gets $2K$ in total, which is the upper bound on the equilibrium profit (part (iii) of Lemma 3). Thus, in the equilibrium of the uniform case, all active firms except the ones at λ^n and ρ_B reap the maximum share of the overall industry profit. Therefore, the smallest number of active firms is supported in equilibrium.

6 Extensions

We now extend the model in several relevant ways by introducing, in turn, price competition for advertisement, a tradeoff between profits in the short and the long run, firms who are allowed to operate multiple outlets, and winner-takes-it-all competition.

6.1 Price Competition for advertisement

We now analyze price competition in the context of media markets and show that the equilibrium profits of firms are linear in market shares θ_i when firms first choose locations sequentially and then simultaneously set advertisement prices. We maintain the assumption that consumer prices are equal to zero. This encompasses the - perhaps not so - special case of free to air radio or TV stations, web radio, internet portals, free newspapers, or, increasingly so, web sites of newspapers.[33]

There is a continuum of advertisers each of which has productivity parameter s drawn independently from the continuous, strictly increasing distribution G. Each advertiser places either one add at firm i or none.[34] With θ_i as the total number of consumers (i.e. readers, viewers or listeners) attracted by firm i,[35] when placing an ad firm i at price p_i^a, the net surplus of an advertiser with productivity s is $s\theta_i - p_i^a$, and it will optimally place an ad if $s \geq p_i^a/\theta_i$.[36]

Proposition 1 *In the model with price competition for advertisement, each firm i's equilibrium profit is linear in its market share θ_i as in the basic framework.*

This is consistent with the notion that competition in, say, the newspaper market occurs with respect to location in product space, rather than consumer prices (see e.g. Waldfogel, 2003; George and Waldfogel, 2003, 2006).

[33]This model also applies for situations where the number of ads cannot be increased beyond a binding limit set by, say, policy makers.

[34]See Anderson and Coate (2005) for a related framework in which consumers also dislike advertisement.

[35]Recall that if i is at an interior location x_i with neighbors L and R, $\theta_i = F\left(\frac{x_i+R}{2}\right) - F\left(\frac{x_i+L}{2}\right)$ whereas if it has no neighbor at the left or right, $\theta_i = F\left(\frac{R+x_i}{2}\right)$ and $\theta_i = 1 - F\left(\frac{x_i+L}{2}\right)$, respectively.

[36]Note that advertisement firms can place ads in more than one media outlet. Furthermore, as each consumer patronizes at most one media outlet, the sets of consumers which advertisement firms address through different medias are disjoint. As a result, each advertisement firm separately decides for each media outlet whether or not to place an ad. See Kim and Serfes (2006) for a standard Hotelling model with prices where consumers buy multiple products.

6.2 Tradeoff between Profits in the Short and the Long Run

We have assumed throughout that though entry occurs sequentially, the only cost of late entry is that profitable locations are occupied first. An interesting modification is to consider a infinite-period model, where in every period one firm may enter, but where payoffs accrue to every active firm in every period. This induces an interesting tradeoff between short-term and long-term profits as a firm may now take a location that is not so attractive in the distant future, but that pays off in the short-run as it allows to influence the location choices of future entrants. Moreover and consistent with reality, early entrants earn a payoff for a longer period of time than late entrants.

As before, firms enter sequentially in a predeterminate order. Throughout, all firms have the same discount factor $\delta \in [0, 1]$ and face the same fixed cost of entry K. For simplicity, assume that the distribution of consumers is uniform, so that any location choice in a given interval (L, R) yields the same payoff. Because of the resulting indeterminacy of an entrant's optimal location in (L, R) when no one enters subsequently (i.e. $X^*(L, R) = (L, R)$), we follow PV and focus on equilibria where such an entrant locates at the midpoint $(L + R)/2$. Let π_i^t be the payoff firm i gets in period t upon entry. Then its discounted lifetime payoff, neglecting the fixed cost K, is $\Pi_i = \sum_{t=0}^{\infty} \delta^t \pi_i^t$.

Consider a firm who is last to enter in a given interval. This firm's per period profit is therefore constant, say π. Its discounted lifetime payoff is $\Pi = \frac{\pi}{1-\delta}$, which needs to exceed the fixed cost K in order to make entry profitable for this firm. It follows that the critical per-period profit level below which every firm will stay out is given by $\pi = (1 - \delta)K \equiv \tilde{K}$, where $\tilde{K} < K$ for all $\delta > 0$. Conversely, there will be additional entry in any given interval satisfying $\pi > \tilde{K}$. By the same logic, and because F is uniform, the firm(s) closest to the left and right bound of the spectrum will be located at $\lambda_B = \tilde{K}$ and $\rho_B = 1 - \tilde{K} = 1 - \lambda_B$.

To illustrate some basic properties of this multi-period approach, we analyze the case where, respectively, one, two or three firms enter in equilibrium, and summarize the results as follows:

Proposition 2 *In equilibrium,*

(i) for $K \geq \frac{1}{2(1-\delta)}$ *one firm will enter at location* $\frac{1}{2}$*,*

(ii) for $\frac{1}{2(1-\delta)} > K \geq \frac{1}{4(1-\delta)}$ *two firms enter at locations* $\lambda_B = \tilde{K}$ *and* $\rho_B = 1 - \tilde{K}$*, and*

(iii) for $\frac{1}{4(1-\delta)} > K \geq \frac{1}{6(1-\delta)}$ *three firms enter at locations* $\lambda_B, \frac{1}{2}$ *and* ρ_B*.*

For the sequence of settlement under (iii) there exists a critical discount factor $\tilde{\delta}$ *such that for* $\delta < \tilde{\delta}$*, firm 1 locates either at* λ_B *or* ρ_B*, firm 2 at the opposite location, and firm 3 at* $\frac{1}{2}$*, while for* $\delta > \tilde{\delta}$*, firm 1 locates at* $\frac{1}{2}$*, firm 2 either at* λ_B *or at* ρ_B*, and firm 3 at the opposite location.*

Parts (i) and (ii) are straightforward extensions of the basic framework where payoffs accrue only once (which is equivalent to assuming $\delta = 0$). Interestingly, this close analogy vanishes as the number of active firms exceeds two, as illustrated in part (iii). In particular, while the discount factor δ has no impact on the set of equilibrium locations, it does affect the sequence of settlement. Intuitively, while the location in the middle is the least attractive in the game without sequential payoff and in the stationary long run of the sequential-payoff game, this location may become optimal for early entrants. Hence, a trade-off arises between a short-run gain and a decrease in the stationary long-run profits.

More precisely, suppose firm 1 has located at $\tilde{K}$ and suppose firm 2's locates "in the middle" at some $x_2 \in (\tilde{K}, 1-\tilde{K})$ that induces the third entrant to locate at $1-\tilde{K}$ so that the locations $\{\tilde{K}, x_2, 1-\tilde{K}\}$ deter further entry. In this case, firm 2's profit will be independent of x_2 and equal to $1/2 - \tilde{K}$ after firm 3 enters. Thus, x_2 will be chosen so as to maximize firm 2's profit in the period it enters, subject to deterring further entry after firm 3 enters. But this profit is maximized with x_2 as close to firm 1's location $\tilde{K}$, since its market share will be $1-(x_2+\tilde{K})/2$, which is decreasing in x_2. So as to deter further entry in $(x_2, 1-\tilde{K})$, $(1-\tilde{K}-x_2)/2 \leq \tilde{K}$ has to hold, yielding $x_2 \geq 1-3\tilde{K}$. Thus, the optimal location of 2, if it enters in the middle, is

$x_2 = 1 - 3\tilde{K}$. But since this is less than $\frac{1}{2}$, it is a particularly unattractive choice for firm 1, and one that firm 1 may want to, and for δ relatively large optimally will, preempt by locating in the middle to begin with.

6.3 Multiple-outlet firms

Consider the following variant of a sequential location game where upon entry firms can operate multiple outlets so that, when given the move, each firm can choose to occupy as many locations as it wants to. Assume for simplicity that the density is monotone. The cost for the first outlet a firm operates is K for each firm. The fixed cost per additional outlet is positive but can be smaller than K.

Proposition 3 *The game has an equilibrium where the first firm occupies the same set of locations as derived in Theorem 4 for the single-outlet case. The first firm monopolizes the market and operates the same number of outlets in every equilibrium.*

Hence, monopoly and competitive outcomes coincide under the threat of entry. However, the multi-product monopoly engages in product proliferation because it offers more products than it would absent the threat of and the incentive to deter entry (Schmalensee, 1978; Bonanno, 1987). These results imply also that horizontal mergers should have no effects on the product variety available to consumers (see e.g. Berry and Waldfogel, 2001; Federal Communications Commission, 2001; Sweeting, 2008).

6.4 Winner-takes-it-all competition

The present framework can naturally be applied to the context of political economy for the analysis of electoral competition with an endogenous number of parties/candidates (firms). As it stands, it captures the case of *proportional representation*, where the number of each party's parliament seats is proportional to the number of votes (in our terminology, market share) it gets.

In contrast, under systems of *plurality voting* the party with the largest vote share wins the election with probability one (*winner-takes-it-all*).[37] Thus, every firm is now simultaneously competing with *all* other firms. This is in contrast to the case of proportional representation, where the payoff-relevant competitors for each party (firm) are its neighbors only.[38] Naturally, this fundamental change of the incentive structure will also affect the equilibrium properties of the model.

As before let K be the setup cost for a party or so that a firm enters if and only if the probability that it wins is no less than K, assuming equilibrium play ensues after its entry and choice of location. The firm or party with the largest market or vote share wins with probability one. If N parties tie at the top, i.e. if each of them has a maximal vote share, each of them wins with probability $1/N$. Voters (or consumers) cater the party (or firm) that is closest (non-strategic voting). Thus, the model allows for an extension of Palfrey (1984)'s analysis, where the number of potentially entering parties is fixed to three. Focussing on subgame perfect equilibria, we have the following result:

Proposition 4 *(i) If N firms enter in equilibrium, each of them wins with probability $1/N$.*

(ii) Following an off-equilibrium observation, at most $N^{off} \leq \lceil 1/K \rceil$ firms enter, and each wins with probability $1/N^{off}$.

Given this result, it is now possible to construct equilibria all of which exhibit ties among all active players.[39] Off-equilibrium behavior will typically require some concerted actions by

[37]The same is true for other winner-takes-it-all competitions such as patent races and other contests to 'conquer' markets.

[38]Clearly, in the case of a one-dimensional space as considered here, the number of neighbors is at most two. However, as already noted by Caplin and Nalebuff (1986), it will generically increase in any higher dimensional space.

[39]This feature is reminiscent of the literature on all-pay auctions (contests) with complete information (see e.g. Hillman and Riley, 1989; Baye, Kovenock, and de Vries, 1993, 1996; Che and Gale, 1998). They exhibit a similar incentives structure in that, under symmetry, all active contestants must have the same (strictly positive) probability of winning the prize, otherwise they would prefer to not to bid (i.e. stay out). As such models are usually analyzed in a static framework, equilibria are typically in mixed strategies.

those who enter to punish a deviant who deviates from his equilibrium location. Intuitively, a single firm can only punish a deviant from one side (e.g. by locating close-by to the right of the deviant), and this might not suffice to deter the deviation as the deviant might still have sufficiently large gains to his left. Therefore, to annihilate the gains from a deviation, punishments may need to be carried out pairwise (locating close-by from each side).

7 Conclusion

In this paper, we study sequential location games, in which firms enter sequentially, pay a fixed cost upon entry and where firms' payoffs are proportional to their market shares. Our analysis focuses on the impact of the underlying distribution of consumer preferences on the subgame perfect equilibrium outcome, i.e. the number of active firms, their locations and the sequence of settlement. In doing so, we extend the seminal model by Prescott and Visscher (1977), where the distribution is uniform, and we show that this model exhibits several peculiarities.

From a methodological point of view, we first show that for the class of monotone densities, the equilibrium locations and the equilibrium sequence of settlement are independent. Therefore, the equilibrium outcome can be fully characterized. Under some additional restrictions, independence of the equilibrium locations and the sequence of settlement obtains also for non-monotone densities like hump- and U-shaped ones, such that the equilibrium number of firms and their locations are then readily determined by combining the results for monotone densities.

Our model exhibits the intuitive features that larger markets and areas with higher density attract more entry, thereby giving rise to preference externalities in a natural and concise way. Moreover, if densities are monotone and concave, firms prefer to locate in high-density segments of the product spectrum (so that such locations are occupied early in the game) despite the fiercer competition this entails.

The baseline model extends naturally in various ways, including price competition for advertisement in the context of media markets, tradeoffs between profits in the short and the long run, and multi-product firms. We also discuss some of the changes in the incentive structure when competition is of the winner-takes-it-all type, such that only the firm with the largest market share prevails. Further research on this type of competition appears particularly fruitful.

Appendix

A Proofs

Proof of Lemma 1 The total differential of the first order condition $f\left(\frac{x^*+L}{2}\right) = f\left(\frac{x^*+R}{2}\right)$ with respect to L is

$$\frac{1}{2}f'\left(\frac{x^*+L}{2}\right)\left(\frac{\partial x^*}{\partial L}+1\right) = \frac{1}{2}f'\left(\frac{x^*+R}{2}\right)\frac{\partial x^*}{\partial L}.$$

This is equivalent to $\frac{\partial x^*}{\partial L} = \frac{-f'\left(\frac{x^*+L}{2}\right)}{f'\left(\frac{x^*+L}{2}\right)-f'\left(\frac{x^*+R}{2}\right)} < 0$ since $f'\left(\frac{x^*+L}{2}\right) > 0 > f'\left(\frac{x^*+R}{2}\right)$. That it is larger than minus one follows from the fact that $-f'\left(\frac{x^*+R}{2}\right) > 0$. Analogously, the total differential with respect to R is

$$\frac{1}{2}f'\left(\frac{x^*+L}{2}\right)\frac{\partial x^*}{\partial R} = \frac{1}{2}f'\left(\frac{x^*+R}{2}\right)\left(\frac{\partial x^*}{\partial R}+1\right)$$

which is equivalent to $\frac{\partial x^*}{\partial R} = \frac{f'\left(\frac{x^*+R}{2}\right)}{f'\left(\frac{x^*+L}{2}\right)-f'\left(\frac{x^*+R}{2}\right)} < 0$. That it is larger than minus one follows from the fact that $f'\left(\frac{x^*+L}{2}\right) > 0$. ■

Proof of Lemma 2 In the proof, we confine attention to π^* as the arguments regarding $\hat{\pi}$ are completely analogous.

Note first that for any location $x \in (L, R)$, $\pi(x, L, R)$ strictly decreases in L and strictly increases R:

$$\frac{\partial \pi(x,L,R)}{\partial L} = -\frac{1}{2}f\left(\frac{x+L}{2}\right) < 0 \quad \text{and} \quad \frac{\partial \pi(x,L,R)}{\partial R} = \frac{1}{2}f\left(\frac{x+R}{2}\right) > 0. \tag{1}$$

The proof for the reaction of $\pi^*(L, R)$ to changes in L and R relies on a revealed preference argument: Fix some $x^*(L, R) \in X^*(L, R)$ and suppose that the competitor to the left moves to some $L' > L$. We have to consider two cases.

Case 1: $x^*(L, R) \in X^*(L', R)$. From Eqn. (1), it follows directly that $\pi(x^*(L, R), L', R) < \pi^*(L, R)$.

Case 2: $x^*(L, R) \notin X^*(L', R)$. To see that $\pi^*(L', R) < \pi^*(L, R)$ holds, suppose otherwise that $\pi^*(L', R) \geq \pi^*(L, R)$. By definition of $x^*(L, R)$, however, $\pi^*(L, R) \geq \pi(x^*(L', R), L, R)$. Therefore, if the first inequality holds, then so does $\pi^*(L', R) \geq \pi(x^*(L', R), L, R)$. But this contradicts Eqn. 1. Completely analogous arguments apply to changes of R. ■

Proof of Lemma 3 **Part (i)** Suppose, for notational simplicity, that optimal locations are unique. By definition, when locating at $x^*(y, \lambda(y))$, an entrant gets K. When the firm to the left is instead located at some $y' > y$, $\pi^*(y', \lambda(y)) < K$ follows from Lemma 2. This Lemma also implies that $\pi^*(y', \lambda(y')) = K$ can hold only if $\lambda(y') > \lambda(y)$. A completely analogous argument establishes that $\rho(\cdot)$ is also increasing in y.

Part (ii) By construction $\lambda(\rho(R)) = R$ and by part (i) $\lambda(y)$ increases in y. Hence, $\rho(R) < L$ implies $\lambda(L) > R$. That this implies $\pi^*(L, R) < K$ follows from Definition 2.

Part (iii) $F(\lambda(y)) - F(y) > K$ and $F(y) - F(\rho(y)) > K$ follows trivially from the definition of $\lambda(\cdot)$ and $\rho(\cdot)$. The remainder of the proof for the statement with respect to $\lambda(y)$ relies on the fact that

$$F(\frac{\lambda(y) + y}{2}) - F(y) \leq K \quad \text{and} \quad F(\lambda(y)) - F(\frac{\lambda(y) + y}{2}) \leq K. \tag{2}$$

To see this, suppose to the contrary that $F(\frac{\lambda(y)+y}{2}) - F(y) > K$. Then an entrant could locate at y^+ and get $\pi(y^+, y, \lambda(y)) = F(\frac{\lambda(y)+y}{2}) - F(y) > K$ which contradicts the definition of $\lambda(\cdot)$. An analogous argument establishes the second part of (2). But now (2) implies $F(\frac{\lambda(y)+y}{2}) - F(y) + F(\lambda(y)) - F(\frac{\lambda(y)+y}{2}) = F(\lambda(y)) - F(y) \leq 2K$. The proof for the statement with respect to $\rho(y)$ is completely analogous.

Part (iv) By definition, when locating at $x^*(y, \lambda(y))$, an entrant gets K. When K increases to $K' > K$, the set of optimal locations does not change, and thus $\pi^*(y, \lambda(y)) < K'$ holds. Thus, by Lemma 2, for a given y, $\pi^*(y, \lambda(y)) = K'$ can hold only if $\lambda(y)$ increases. A completely analogous argument establishes that $\rho(y)$ decreases in K. ■

Proof of Theorem 1 **Part (i)** We first show that $\rho(R) \leq L < R \leq \lambda(L)$ implies $\# = 0$: By definition of $\lambda(L)$ and $\rho(R)$, and from Corollary 1, profitable entry in the interval (L, R) is not possible, and thus no firm will enter in equilibrium. That $\# = 0$ implies $\rho(R) \leq L < R \leq \lambda(L)$ will follow from parts (ii(a)) and (iii(a)), which imply that entry will occur whenever the condition $\rho(R) \leq L < R \leq \lambda(L)$ is not satisfied.

Part (ii) As for statement (a), label subsequent entrants by $i, i+1, i+2, ...$ We show that I) at most two firms enter in equilibrium, and II) at least one enters.

I) At most two firms enter

If the first entrant i enters at some $x_i \in [\rho(R), \lambda(L)]$, then by definition of $\lambda(\cdot)$ and $\rho(\cdot)$, there will be no further entry in the interval $[L, R]$. So consider the case where $x_i \notin [\rho(R), \lambda(L)]$, and suppose $x_i \in (L, \rho(R))$. (The case $x_i \in (\lambda(L), R)$ is completely analogous and thus omitted.) By Corollary 1, if subsequently $i+1$ enters, it must enter at some $x_{i+1} > \lambda(L)$: For $x_{i+1} \in (L, x_i]$, firm $i+1$ itself would incur a loss, for $x_{i+1} \in (x_i, \lambda(L)]$, firm i would do so. For two firms to enter, it therefore has to be the case that one, say i, locates at $x_i < \rho(R)$ and the other one at $x_{i+1} > \lambda(L)$. But now a third firm cannot profitably enter because at least one of the firms would not break even. This follows again from Corollary 1: For $x_{i+2} \in (L, x_i)$ or $x_{i+2} \in (x_{i+1}, R)$, firm $i+2$ does not break even, for $x_{i+2} \in (L, \lambda(L))$, i does not break even, and for $x_{i+2} \in (\lambda(L), x_{i+2})$, firm $i+1$ does not break even.

II) At least one firm enters

Three cases have to be considered:

Case 1: There is a $x^*(L, R) \in [\rho(R), \lambda(L)]$. In this case, the first entrant chooses this location, thereby preventing further entry. Moreover $\pi^*(L,R) > K$ since $L < \rho(R) < \lambda(L) < R$.

Case 2: There is no $x^*(L,R) \in [\rho(R), \lambda(L)]$ but $\hat{\pi}(\rho(R), \lambda(L), L, R) > K$. In this case, at least one firm will enter since $\hat{x}(\rho(R), \lambda(L), L, R)$ is a profitable and entry-deterring location. Whether one or two firms enter depends on whether the first firm i prefers an alternative location, thereby inducing subsequent entry, to $\hat{x}(\rho(R), \lambda(L), L, R)$ and thereby deterring entry.

Case 3: $\hat{\pi}(\rho(R), \lambda(L), L, R) \leq K$. Observe first that this implies $x^*(L, \lambda(L)) < \rho(R)$ and $x^*(\rho(R), R) > \lambda(L)$. We need to show that at least one firm enters, assuming equilibrium behavior by firms moving subsequently. That is, we have to show that there exists some $x_i \in (L,R)$ such that i's profit at x_i exceeds K if all subsequent firms play optimally. Let i occupy the location $x^*(L, \lambda(L))$. Observe first that there will be no subsequent entry to the left of firm i, because by Corollary 1, for any location $y \in (L, x_i)$, $\pi(y, L, x_i) < K$ holds. A necessary condition for i not to break even at $x^*(L, \lambda(L))$ is therefore that (at least) one other firm, say, $i+1$ enters to its right at some $x_{i+1} \leq \lambda(L)$. Only in this situation will i be "trapped" inside the $[L, \lambda(L)]$ interval (Corollary 1). So assume $x_{i+1} \leq \lambda(L)$. But for $i+1$ to enter at x_{i+1} in equilibrium, it must be the case that $i+1$ earns more than K either by deterring further entry or by "pushing" any subsequent entrant far enough to the right. But if $i+1$ earns more than K at x_{i+1} with $x_i > L$ to its left, then i could have chosen the location x_{i+1} itself, whereby it would have earned strictly more than $i+1$ now does. Therefore, i can guarantee itself a profit that is larger than K. Consequently, at least on firm will enter in equilibrium.

Statement (b) of part (ii) is an implication of part (i), and statement (a) of parts (ii) and (iii), respectively.

Part (iii) The proof of statement (a) relies on the validity of the following claim:

Claim: At least one firm can profitably enter either in the interval $(L, \lambda(L))$ or in the interval $(\rho(R), R)$.

We prove the claim for the case where the first entrant i enters in the interval $(L, \lambda(L))$, for the other one it is completely analogous. Suppose the first entrant i locates at $x_i = x^*(L, \lambda(L))$.

Since $x^*(L, \lambda(L)) < \lambda(L)$, there will be no more entry to the left of x_i (by Corollary 1). Let the closest firm to the right of firm i be firm $i+1$ at some location x^0_{i+1}: If $x^0_{i+1} > \lambda(L)$, then $\pi(x^*(L, \lambda(L)), L, x^0_{i+1}) > K$.

Thus, as above, the critical case is $x^0_{i+1} \leq \lambda(L)$ such that firm i would not break even (again by Corollary 1). Note that firm $i+1$ would choose such a position only if $\pi(x^0_{i+1}, x_i, x_{i+2}) > K$ where $x_{i+2} > \lambda(x_i)$ is the closest firm to the right of firm $i+1$. But then, firm i could itself locate at $x_i = x^0_{i+1}$ and earn $\pi(x^0_{i+1}, L, x_{i+2}) > \pi(x^0_{i+1}, x_i, x_{i+2}) > K$ since there will be no further entry in the interval (L, x_{i+1}). Consequently, there always exists a location in the interval $(L, \lambda(L))$ such that entry is profitable for at least one firm.

How many more firms enter depends on the location of $\lambda(x_i)$. If $\lambda(x_i) > \rho(R)$, we are in part (ii(a)), where it was shown that at least one more firm enters. If $\lambda(x_i) < \rho(R)$, then we are again in part (iii(a)) in which case at least two more firms enter.

Statement (b) of part (iii) is again an implication of part (i), and statement (a) of parts (ii) and (iii), respectively. ■

Proof of Theorem 2 From Corollary 1, if $\rho(R) \leq L$, or if $R \leq \lambda(L)$, the firm at x could profitably deviate by staying out. Moreover from Corollary 2, when the distance between the firm at x and its neighbors exceeds $x - \rho(x)$ and $\lambda(x) - x$, respectively, then there will be entry in between, contradicting that x and L (respectively x and R) are neighbors. ■

Proof of Corollary 3 As shown in Theorem 2, in any equilibrium the maximum distance between a firm at location x and its neighbors to the left and right is $x - \rho(x)$ and $\lambda(x) - x$, respectively. Moreover, as shown in the proof of part (iii) of Lemma 3, $F(\frac{\lambda(x)+x}{2}) - F(x) \leq K$ and $F(x) - F(\frac{\rho(x)+x}{2}) \leq K$, so that for the *total* profit generated at location x, $F(\frac{\lambda(x)+x}{2}) - F(\frac{\rho(x)+x}{2}) \leq 2K$ holds. Therefore, independent of how this profit is shared between the firms located at x, at most one can break even. ■

Proof of Theorem 3 The proof is straightforward and by contradiction: Assume not, i.e. assume, say, $a > \lambda_B$. Then a firm could profitably enter at λ_B and get K to its left (without attracting further entry there) and earn strictly positive profit to its right. ■

Proof of Lemma 4 We only consider the case where f is increasing, the case where it is decreasing being completely analogous.

That at least one additional firm enters follows from Theorem 1. So we are left to show that further entry deterrence is optimal for the first entrant.

Let i be this entrant and suppose to the contrary that i accommodates further entry either by choosing $x_i \in (\lambda(L), R)$ or $x_i \in (L, \rho(R))$. If $x_i \in (\lambda(L), R)$ is an equilibrium outcome, then the next entrant will deter entry (by Theorem 1). He optimally does so by choosing $\hat{x}(\rho(x_i), \lambda(L), L, x_i) = \lambda(L)$. But in this case, x_i is in between two neighbors who are in the interval $[\rho(R), R]$. By Corollary 1, i cannot break even. Hence, this cannot be optimal for i.

If, on the other hand, i chooses $x_i \in (L, \rho(R))$, the subsequently entering firm will, again, deter entry. It optimally does so by choosing $\hat{x}(\rho(R), \lambda(x_i), x_i, R) = \lambda(x_i)$. In this case, i earns strictly less than he would had he located at $\lambda(L)$ and thereby deterred entry: Both the length of the interval he captures is now $\frac{\lambda(x_i)-L}{2}$ instead of $\frac{R-L}{2}$ which he would cover when deterring entry, *and* the density over this interval is smaller than the density he would get when deterring entry. Hence, i will optimally deter entry. He optimally does so by locating at $\lambda(L)$. Note that i's profit will also be strictly higher than K, since $x^*(L, \lambda(L)) = \lambda(L)^-$ so that $\pi^*(L, \lambda(L)) = K$ and $R > \lambda(L)$ (by Lemma 2).

Proof of Theorem 4 We only prove part (i), the proof for part (ii) is completely analogous.

Existence. We first show that $\{\lambda_B, \lambda^1, .., \lambda^n, \rho_B\}$ are equilibrium locations. To that end, assume for the moment that locations λ_B and ρ_B are occupied and that all remaining firms play the following strategy: "Enter to the right of some location x_i only if its closest righthand neighbor, $i+1$, is at some $x_{i+1} > \lambda(x_i)$ and when $\lambda(x_i) < \rho_B$. If you enter to the right of x_i, enter at $\lambda(x_i)$." Call this the λ-strategy.

To see that these strategies are mutual best responses, notice first that not to enter in $[x_i, x_{i+1}]$ if $x_{i+1} \leq \lambda(x_i)$ is, obviously, a best response. Second, if x_{i+1} is the future righthand neighbor of the entering firm and if $x_{i+1} > \lambda(x_i)$, then entry at $\lambda(x_i)$ is optimal within the interval $[x_i, x_{i+1}]$, as we know from Lemma 4. (And since all better options are taken before by other firms in case there are better options, at some point some firm will enter here.) If the righthand neighbor x_{i+1} has not taken its location yet but plays the λ-strategy, then a fortiori $\lambda(x_i)$ is optimal for the entrant: Not only is it the largest location that deters entry to its left, but it will also push its righthand neighbor $i+1$ as far to the right as possible.

Moreover, under the λ-strategy, the locations λ_B and ρ_B will also be occupied: As for the leftmost location, since the density is increasing, it is optimal to move right as far as possible without inviting further entry to the left which, by definition, is at λ_B. Moreover, under the λ-strategy, the rightmost location will not affect any of the location choices of the other firms. It follows that it is optimal to locate as far left as possible without inviting further entry to the right which, again by definition, is at ρ_B.

Uniqueness. If the future left-hand and righthand neighbor to some entrant are given at, again, x_i and x_{i+1}, respectively, then $\lambda(x_i)$ is still the best response of the entrant in $[x_i, x_{i+1}]$. Observe also that it is the unique best response. So one way $\lambda(x_i)$ could not be the best response of the entrant with neighbors at x_i and x_{i+1} is that $i+1$ has not taken his location *and* threatens to locate the closer to the entrant the closer the entrant's location to $\lambda(x_i)$. Assume that the entrant believes this threat and that his best response would be to locate at some $y < \lambda(x_i)$.

To see that this threat is empty in equilibrium (i.e. even if i played his best response to this threat, the threat would in turn not be a best reply), consider the last entrant, say l, to the right of our entrant. Clearly, l's best response will be to locate at $\lambda(x_{l-1})$, where x_{l-1} is the last entrant's left-hand neighbor (which may or may not be $i+1$). Anticipating this,

$l-1$ recognizes that l's best responses increases in his own location, and thus he chooses the largest location which allows him to deter entry to his left. By iteration, we see that $i+1$'s best response is to locate at the largest location that deters entry to the left. Thus, the threat is empty and the best response is unique. ■

Proof of Theorem 5 Again, it suffices to confine attention to part (i), as the proof for part (ii) is completely analogous.

As for the comparison of locations ρ_B and λ^n, note first that the firm at ρ_B earns K to its right by definition of ρ_B. As for the firm at λ^n, note that when f is increasing, $x^*(y,\lambda(y)) = \lambda(y)^-$ so that $\pi(\lambda(y)^-, y, \lambda(y)) = K$. Thus, the firm at λ^n earns K to its left so that differences in their profits can only accrue from differences in earnings between λ^n and ρ_B. In terms of distances, both grasp exactly $\frac{\rho_B+\lambda^n}{2}$. However, the density over the share grasped by the firm at ρ_B being larger than for the share catered by the firm at λ^n, it follows that the firm at ρ_B earns strictly more.

As for the second statement in part (i) of the Theorem, notice that the equilibrium profit of the firm at location x_i is equal to the sum of two areas: To the left, it gets an area of size K, and to the right an area of size A^{i+1}, which is smaller than K. So

$$\pi(x_i) = T + A^{i+1}.$$

Let $\Delta^i := \Delta(x_{i-1}, x_1)$, i.e. half of the distance between the equilibrium locations x_{i-1} and x_i. Because $f(x)$ increases in x, $\Delta^{i+1} < \Delta^i$ holds. We are now going to show that for the areas A^i the following holds: $A^i < A^{i+1}$ for any $i \geq 1$. Since $\pi(x_i) = T + A^{i+1}$, this will then complete the proof.

Observe first that $A^i = T - C^i - D^i$. So $A^i < A^{i+1}$ will hold if we can show that $D^{i+1} < D^i$ and $C^{i+1} < C^i$ holds. Define $\tilde{f}_i(x) \equiv f(x) - f(x_i - \Delta^i)$ for $x \in [x_i - \Delta^i, x_i]$. Clearly, $\tilde{f}_i' > 0$ and $\tilde{f}_i'' \leq 0$ holds. For any $y < \Delta^{i+1}$ this implies

$$\tilde{f}_i(x_i - \Delta^i + y) \geq \tilde{f}_{i+1}(x_{i+1} - \Delta^{i+1} + y). \tag{3}$$

Observe next that

$$C^i = \int_{x_i-\Delta^i}^{x_i} \tilde{f}_i dx > \int_{x_i-\Delta^i}^{x_i-\Delta^i+\Delta^{i+1}} \tilde{f}_i dx \geq \int_{x_{i+1}-\Delta^{i+1}}^{x_{i+1}} \tilde{f}_{i+1} dx = C^{i+1}.$$

The first and last equality are identities. The first inequality is due to the fact that $\Delta^i > \Delta^{i+1}$, and the weak inequality follows from (3). Thus, $C^{i+1} < C^i$ is established.

Similarly, define $\hat{f}_i(x) \equiv f(x_i) - f(x)$ for $x \in [x_i - 2\Delta^i, x_i - 2\Delta^i]$. For any $y < \Delta^{i+1}$,

$$\hat{f}_i(x_i - 2\Delta^i + y) \geq \hat{f}_{i+1}(x_{i+1} - 2\Delta^{i+1} + y) \qquad (4)$$

holds. Observe then that

$$D^i = \int_{x_i-2\Delta^i}^{x_i-\Delta^i} \hat{f}_i dx > \int_{x_i-2\Delta^i}^{x_i-2\Delta^i+\Delta^{i+1}} \hat{f}_i dx \geq \int_{x_{i+1}-2\Delta^{i+1}}^{x_{i+1}-\Delta^{i+1}} \hat{f}_{i+1} dx = D^{i+1}.$$

■

Proof of Theorem 6 The proof is based on Theorem 4 and is straightforward. On the increasing part, all best responses are independent of the location of the right-hand neighbor. Hence $\{\lambda_B, .., \lambda^r\}$ follows. On the decreasing part, best responses are independent of the left-hand neighbor's location, whence $\{\rho^s, .., \rho_B\}$ follows. ■

Proof of Lemma 5 The length of the interval captured by the entrant is always $\frac{R-L}{2}$ independent of his location. Now let the entrant locate at one end of the interval (say, at L^+) and let him contemplate moving marginally towards the middle. Either his profit increases immediately. In this case, however, his profit will keep increasing as he moves further to the right since he keeps losing less on the left and gaining more on the right. Thus, the optimal location will be R^- in this situation. Or, the move towards the right will initially involve losses. If this is the case for all positions to the right of L^+, then he optimally locates at L^+. If eventually the profit starts increasing by moving further right, then it will increase monotonically from there onwards. Hence, the optimal location in this case will be either L^+ or R^-. ■

Proof of Corollary 5 Suppose to the contrary that in equilibrium M is occupied by, say, firm k. From Theorem 2, it follows that its left- and right-hand neighbors will be at some locations $x_L \geq \rho(M)$ and $x_R \leq \lambda(M)$, respectively. Without loss of generality, assume $f\left(\frac{x_R+M}{2}\right) \geq f\left(\frac{x_L+M}{2}\right)$. By moving marginally to the right, the profit of k increases: It gains more on the right than it loses on the left. Note that this is true independently of whether the right-hand neighbor is already there or not: If it is not there, by moving right to some $x > M$, k pushes its future right-hand neighbor to $\lambda(x)$. Even if the move to the right will attract entry to the left at $\rho(x)$, the loss due to this entry will be smaller than the gain to the left. Thus, M cannot be optimal. If $f\left(\frac{x_R+M}{2}\right) < f\left(\frac{x_L+M}{2}\right)$, analogous arguments apply for the opposite direction. ■

Proof of Theorem 7 We are going to argue that for trough-symmetric functions, ρ_M and λ_M are mutually best responses. Once this is shown, the Theorem follows immediately from the previous results on monotone densities. So, suppose some firm is located at λ_M. Observe then that any location $y \in [\rho_M, \lambda_M]$ will deter entry in between: For $y > \rho_M$, $x^*(y, \lambda_M) = \lambda_M^-$. But $\pi^* < K$ since $\frac{y+\lambda_M}{2} > M$ because of trough-symmetry. Since moving away from the middle without attracting entry is always beneficial, ρ_M dominates any interior location. Notice then that at $y = \rho_M$ the firm nets exactly K to the right, again because of trough-symmetry. Thus, this is a best response. Mutuality of best responses follows from trough-symmetry.

To see that these equilibrium locations are unique assume to the contrary that $y < \lambda_M$. Then the best replying left-hand neighbor will locate at some $z < \rho_M$. But then $y < \lambda_M$ cannot have been optimal in the first place. ■

Proof of Lemma 6 All we need to show is that an decrease in ε leads to an increase in the mass to the left of λ^n and to the right of ρ^n. Observe that both λ^n and ρ^n depend on ε and that because of symmetry there is no loss of generality if we focus on λ^n and the mass to its left. To see that the mass between two locations λ^i and λ^{i+1} increases as ε decreases, assume to the contrary that it does not. But then, because $f^h_{\varepsilon'}$ is flatter than f^h_ε for $\varepsilon' < \varepsilon$ it follows that the firm at λ^i attracts to its right more consumers for ε' than for ε while the one at λ^{i+1} attracts to its left less than for ε and thus less than K, which is a contradiction. Therefore, the mass between any two neighboring firms increases as ε decreases. Thus, if initially at ε_0 λ^n and ρ^n deter further entry in between, they will do so a fortiori for any $\varepsilon < \varepsilon_0$. ■

Proof of Theorem 8 In the uniform case, the left- and rightmost locations are λ_B and ρ_B, respectively. Moreover, the remaining interval (λ_B, ρ_B) has a mass of consumers of $F(\rho_B) - F(\lambda_B) = 1 - 2K$. As long as firms enter at λ- or ρ-distances from each other, each additional entrant reduces the remaining mass of consumers by the maximum amount $2K$ (see part (iii) of Lemma 3 and the discussion following that Lemma). Moreover, should a last, and smaller interval exist, one more firm will enter. This interval will satisfy the condition $L < \rho(R) < \lambda(L) < R$. As was shown, in part (ii) of Theorem 1, this is equal to the minimum number of firms entering in such an interval for any distribution.

For general distributions, as shown in Theorem 3, the left- and rightmost locations are $a \leq \lambda_B$ and $b \geq \rho_B$, respectively, so that the interval (a, b) has a mass of consumers of $F(b) - F(a) \geq 1 - 2K$. Moreover, from Theorem 2, firms cannot be located further away from each other than λ- or ρ-distances, so that each additional entrant will reduce the remaining mass of consumers by (weakly) less than the maximum amount $2K$.

Taken together, in the uniform case, the size of the relevant interval is minimum, and firms are located at maximum distance from each other in this interval, so that the number of active firms cannot be larger than under any other distribution. ■

Proof of Proposition 1 Total advertisement demand captured by firm i is $t_i = 1 - G(p^a_i/\theta_i)$, which leads to an inverse demand function $P^a_i(t_i) = \theta_i G^{-1}(1 - t_i)$. Since firm i's profit is $\Pi_i = p^a_i t_i$, its maximization problem can thus be expressed as $\max_{t_i} \Pi_i = \theta_i \max_{t_i} \left\{G^{-1}(1 - t_i)t_i\right\}$, where the maximum after θ_i is independent of θ_i. ■

Proof of Proposition 2 For F uniform, exactly one firm will enter and, under our assumption, locate at 1/2, if and only if $\tilde{K} \geq 1 - \tilde{K} \Leftrightarrow K \geq \frac{1}{2(1-\delta)}$ (and $\tilde{K} < 1$). If $K < \frac{1}{2(1-\delta)}$ two or more firms will enter in equilibrium. If exactly two firms enter, they locate at $\tilde{K}$ and $1-\tilde{K}$. For an additional third (and last) firm not to enter, it has to be true that the long-run stationary profit in the interval $(\tilde{K}, 1-\tilde{K})$, which is $1/2 - \tilde{K}$, be less than $\tilde{K}$. Rearranging $1/2 - \tilde{K} \leq \tilde{K}$ and substituting yields $K \geq \frac{1}{4(1-\delta)}$. Since the equilibrium locations with two entering firms are symmetric, the sequence of settlement is indeterminate. Conversely, a third firm will enter if $1/2 - \tilde{K} > \tilde{K} \Leftrightarrow K < \frac{1}{4(1-\delta)}$. The maximally entry deterring locations will be $\tilde{K}, 1/2$ and $1-\tilde{K}$, which will effectively deter entry if and only if $1/4 - \tilde{K}/2 < \tilde{K} \Leftrightarrow \tilde{K} > 1/6 \Leftrightarrow K > \frac{1}{6(1-\delta)}$.

In case (iii) the first entrant will no longer necessarily choose a location "at the bound", i.e. $\tilde{K}$ or $1-\tilde{K}$. The reason is that given that firm 1 has located at, say, $\tilde{K}$, the second entrant may choose to go into the middle, i.e. to some $x_2 \in (\tilde{K}, 1-\tilde{K})$ that induces the third entrant to take $1-\tilde{K}$ so that $\{\tilde{K}, x_2, 1-\tilde{K}\}$ deter further entry. Notice that under these assumptions, firm 2's stationary long run profit will be independent of x_2 and equal to $1/2 - \tilde{K}$ after firm 3 enters. Thus, x_2 will be chosen so as to maximize firm 2's profit in the period it enters, subject to deterring further entry after firm 3 enters. But this profit is maximized with x_2 as close to firm 1's location $\tilde{K}$ as possible, subject to deterring entry, since its market share will be $1 - (x_2 + \tilde{K})/2$, which is decreasing in x_2. So as to deter further entry in $(x_2, 1-\tilde{K})$, $(1 - \tilde{K} - x_2)/2 \leq \tilde{K}$ has to hold, yielding $x_2 \geq 1 - 3\tilde{K}$. Thus, the optimal location of 2 if it enters inside $(\tilde{K}, 1-\tilde{K})$ is $x_2 = 1 - 3\tilde{K}$. But since this is less than 1/2, this is a particularly unattractive choice for firm 1, and one that firm 1 may want to preempt by choosing a location in the middle to begin with. If 2 has located at $x_2 = 1 - 3\tilde{K}$ and 1 at $\tilde{K}$, firm 1's per period payoff from period 2 onwards is $\pi_1^2 = 1/2 - \tilde{K}$. Notice that this is equal to the stationary long-run payoff of a firm located in the middle. But since firm 1 can get strictly more than that in period 2 by locating at 1/2 (and gets 1 independent of its location in period 1), it follows that firm 1 will indeed choose to locate at 1/2 whenever firm 2 would choose $x_2 = 1 - 3\tilde{K}$

(rather than $1-\tilde{K}$) if 1 chose $\tilde{K}$. To see that firm 1's optimal location in the middle is indeed 1/2, notice that this location only matters for second period payoffs since its location in the middle does not affect first period payoffs nor third period payoffs, which are, respectively, 1 and $1/2-\tilde{K}$. As firm 2 will choose the location $x_2 \in \{\tilde{K}, 1-\tilde{K}\}$ that maximizes its market share in the period of entry, firm 1 optimally chooses x_1 so as to minimize this market share, which it does by choosing $x_1 = 1/2$.

To derive the exact conditions for either settlement pattern to occur, we need to determine firm 2's optimal choice of location.[40] Firm 2's lifetime profit Π_2^m when entering at $x_2 = 1-3\tilde{K}$ is $\Pi_2^m = 1/2+\tilde{K}+\delta[1/2-\tilde{K}]/(1-\delta)$, where $1/2+\tilde{K}$ is the payoff in the period it enters at $x_2 = 1-3\tilde{K}$ and $1/2-\tilde{K}$ is the stationary payoff of a firm in the middle. On the other hand, if firm 2 enters on the right at $1-\tilde{K}$, its lifetime payoff Π_2^r is $\Pi_2^r = 1/2+\delta[1/4+\tilde{K}/2]/(1-\delta)$, where 1/2 is the payoff in the period of entry and $1/4+\tilde{K}/2$ is the stationary per period payoff. Tedious but straightforward algebra reveals that for $\delta \in \left[\frac{4K-1}{4K}, \frac{6K-1}{6K}\right]$, $\Pi_2^m \gtrless \Pi_2^r$ if and only if $\delta \gtrless \frac{14K-1+\sqrt{36K^2-28K+1}}{20K} := \tilde{\delta}$. Thus, firm 2 prefers to enter in the 'middle' (more precisely, at $x_2 = 1-3\tilde{K} < 1/2$) if and only if $\delta > \tilde{\delta}$. Since the argument in the previous paragraph established that firm 1 will enter at 1/2 if and only if firm 2 would otherwise enter at $x_2 = 1-3\tilde{K}$, it follows that firm 1 will enter at 1/2 if $\delta > \tilde{\delta}$ and at $\tilde{K}$ otherwise. ■

Proof of Proposition 3 Whether a firm is a single- or a multi-product firm does not affect the incentives to enter for subsequent firms. Thus, the same locations that are occupied with single-outlet firms will be entry deterring with a multi-product firm. Moreover, since the equilibrium locations with multi-product firms are such that there is exactly K for grasp to one side of every firm, additional entry cannot be deterred with less outlets. This proves the first part of the statement. Multiple equilibria may arise because multiple locations can be entry deterring and the multi-product firm will be indifferent how much profit is generated at

[40]Throughout we focus on the case where 1 enters at $\tilde{K}$ when not entering inside $(\tilde{K}, 1-\tilde{K})$. The analysis of the case where it enters at $1-\tilde{K}$ is, of course, completely analogous.

which outlet. ■

Proof of Proposition 4 Part (i) is a statement about behavior along the equilibrium path. To see why it holds true, suppose it does not. But then, one firm who entered must have a larger market share than another one who entered, so that the latter's probability of winning is zero. But then, it would have been better off by staying out. Moreover, it follows that the maximum number of active firms in equilibrium is $\lceil 1/K \rceil$, where $\lceil z \rceil$ denotes the largest integer no bigger than z.

Part(ii) is a statement about off-equilibrium behavior, which can either be a location or an entry choice violating a prescribed equilibrium behavior. The second statement of part (ii) is an implication of part (i), and the first statement must hold as we are looking at equilibrium behavior in the subgame that follows after the deviation. ■

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