Economics and the Efficient Allocation of Spectrum Licenses*

Simon Loertscher[†] Leslie M. Marx[‡]

November 2012

Abstract

We discuss the economic literature underpinning the development of a market design approach for both primary and secondary markets for spectrum licenses and consider the practical implications for implementation.

Keywords: FCC, spectrum license auction, incentive auction, mechanism design
JEL-Classification: C72, D44, D61

*The first author thanks the Department of Treasury and Finance of Victoria and the Australian Communications and Media Authority for helpful discussions that led to the development of the ideas for this chapter. The second author thanks the U.S. Federal Communications Commission and especially Evan Kwerel and Martha Stancill for helpful discussions related to FCC spectrum license auctions. The views expressed here are those of the authors and do not necessarily reflect those of the agencies and individuals acknowledged above.

†Department of Economics, Arts West, University of Melbourne, Victoria 3010, Australia. Email: simonl@unimelb.edu.au.

‡The Fuqua School of Business, Duke University, 100 Fuqua Drive, Durham, NC 27708, USA: Email: marx@duke.edu.

1 Introduction

The development of mobile wireless technologies for voice and data, and of the mechanisms used to allocate electromagnetic spectrum for those uses, provides an insightful case study into how markets work and why market design matters for social outcomes. This chapter introduces the key concepts and theorems from economic theory and illustrates them based on the historical development of mobile wireless services. Although there is much to be learned from the experiences of countries around the globe, given the prominent role that the U.S. Federal Communications Commission (FCC) Federal Communications Commission (FCC) has played historically, we focus on the U.S. experience.

We show that under the assumption that buyers and sellers are privately informed about their valuations and costs, the distinction between primary markets and secondary markets is critical for what can be achieved with carefully designed allocation mechanisms, where by a primary market we mean a situation in which the seller (or possibly the buyer) of the assets also chooses the mechanism, and by a secondary market we mean a situation in which an entity other than a party to the transaction chooses the trading mechanism and organizes the exchange. The economics literature on mechanism design and auction theory has primarily focused on designing primary markets, notwithstanding a few notable exceptions such as Vickrey (1961), Myerson and Satterthwaite (1983), Gresik and Satterthwaite (1989) and McAfee (1992). We review the literature on primary market design and show that for the primary market an efficient allocation mechanism that does not run a deficit exists. This is in stark contrast to the known results for secondary markets, according to which such mechanisms do not exist. Moreover, we derive a new result that generalizes these impossibility results to the case of heterogeneous objects and arbitrary quasilinear utility and profit functions.

This chapter provides background on the underlying economics, including possibility and impossibility results, relevant to the design of dynamic spectrum allocation mechanisms. The basic framework is provided in Section 2 and theoretical foundations are provided in Sections 3.1 for primary markets and in 3.2 for secondary markets. The corresponding implications for market design, which may be of key interest to those approaching the problem from an engineering perspective, are provided in Section 3.3. We provide a generalization of the foundational results from the economics literature in Section 4, with associated implications for market design in Section 4.3. Finally, Section 5 reviews the Federal Communications Commission (FCC) FCC's approach to the issues raised in this chapter, including the role of results from experimental economics. Readers focused on an engineering approach to secondary market design may find the results of Sections 3.3, 4.3, and 5 to be of the greatest interest; however, we hope that the economic foundations presented in this chapter will allow one to better understand the underlying bases for the discussions in these sections.

The main part of this chapter focuses on the model with a homogenous product in which each buyer has demand for one unit and each seller has the capacity to produce one unit. With one exception, the focus of this chapter is on dominant strategy mechanisms. The exception is Proposition 3. This is the most general statement of the impossibility theorem (due to Myerson and Satterthwaite (1983)) that in the domain of Bayesian mechanisms, ex post efficient trade is not possible without running a deficit. Because the set of Bayesian mechanisms contains the set of dominant strategy mechanisms, this is a remarkably general result. Together with the fact that an efficient allocation mechanism exists for the primary market, it provides both an important rationale for taking the primary market allocation problem seriously and an explanation for why the economics literature has primarily focused on the primary market problem.

That said, there are, of course, instances in which reliance on secondary markets becomes inevitable. Perhaps the most important reason for this is technological change. For example, the increasing demand for mobile wireless services and the development of digital television make it appear highly desirable that TV broadcasters offer some of their spectrum licenses for sale to providers of mobile wireless services. This development has, for example, been the rationale for the U.S. Congress to mandate that the Federal Communications Commission (FCC) FCC set up and run a secondary market for spectrum licenses. Another important reason for the desirability of secondary markets is dispersed ownership of the assets, which makes it impossible for a single seller to design the trading mechanism. For example, this is the case for kidney exchanges (see e.g. Sönmez and Ünver (forthcoming)) and for problems of providing services such as the provision of container port drop-off and pick-up slots by terminal operators to trucking companies and the provision of child care, kindergarten and school seats to families and their children. Finally, even without dispersed ownership, it may not be in society's best interest to have the seller design the trading mechanism if the seller's interest is something other than efficiency. This is a particularly salient issue when the seller is a private entity that aims to maximize profits.

This provides motivation to examine the extent to which efficiency can be achieved in well-designed secondary markets without running a deficit. Although the Myerson-Satterthwaite Theorem Myerson-Satterthwaite Theorem and its generalizations are important and remarkable, it is equally important to note that these theorems make qualitative statements. In particular, the efficiency loss of a well-designed, centrally run exchange decreases quickly as the number of buyers and sellers increases. This result has been shown by Gresik and Satterthwaite (1989) for Bayesian mechanisms and by McAfee (1992) within the domain of dominant strategy mechanisms. We illustrate it using a version of McAfee (1992)'s second price double auction.

This chapter is organized as follows. It starts in Section 2 by defining the basic setup and notation. Then in Section 3 we present the benchmark case described by the Coase Theorem Coase Theorem (due to Coase (1960)), according to which private transactions achieve the best social outcomes, provided only the government defines and protects property rights, and transaction costs are negligible. An immediate implication of the Coase Theorem Coase Theorem is that, whenever it applies, the initial allocation of property rights is irrelevant for the efficiency of the final allocation. We illustrate the Coase Theorem Coase Theorem in an environment in which perfectly competitive markets maximize social welfare. The Coase Theorem Coase Theorem and its implications contrast with the development of mobile wireless services, which was dormant as long as lotteries (or other inefficient procedures) were used for allocating spectrum and overwhelmingly quick as soon as auctions were used to allocate spectrum in the primary market, suggesting the existence of substantial frictions. We then extend the analysis to the cases of one-sided and then two-sided private information.

Relaxing the assumptions of homogenous goods and unit demands and capacities, we introduce and analyze in Section 4 the general environment with heterogenous objects and, essentially, no assumptions on utility and profit functions other than quasilinearity. In this setup we study the celebrated Vickrey-Clarke-Groves (VCG) mechanism Vickrey-Clarke-Groves (VCG) mechanism, which is named after the authors – Vickrey (1961), Clarke (1971) and Groves (1973) – whose independent contributions led to it. The Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism is a dominant strategy mechanism that allocates efficiently any finite number of possibly heterogenous goods in the primary market without running a deficit. The Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism is a generalization of the second-price auction and provides an important theoretical benchmark for what can be achieved under these more complicated and often more realistic conditions. We extend the standard Vickrey-

Clarke-Groves (VCG) mechanism VCG mechanism to the two-sided problem that one faces in the secondary market when sellers can produce different packages of goods and buyers have heterogenous demands for these packages. This is the problem the U.S. Federal Communications Commission (FCC) Federal Communications Commission (FCC) faces for the forthcoming so called incentive auctions. We show that the two-sided Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism always runs a deficit if it achieves efficiency, which is the spirit of the Myerson-Satterthwaite Theorem Myerson-Satterthwaite Theorem and an extension of Vickrey's result to heterogenous objects.

Section 5 discusses issues of practical implementation as embraced by the Federal Communications Commission (FCC) FCC and the role for an experimental economics approach to addressing practical implementation issues. Section 6 offers concluding comments.

2 Basic model

2.1 Setup

There is a single homogeneous product, as well as a numeraire, which we interpret as money. There are n risk-neutral buyers indexed by $i \in \{1, .., n\}$, where each buyer i has value $v_i \in [\underline{v}, \bar{v}]$ for a single unit of the product and, for simplicity, unlimited budget. There are m risk-neutral sellers indexed $j \in \{1, .., m\}$, where each seller j has cost $c_j \in [\underline{c}, \bar{c}]$ to produce a single unit of the product and unlimited budget. A buyer's valuation v_i and a seller's cost c_j are sometimes also referred to as their types. The sellers' costs capture both the case where sellers have to produce the goods at some cost and the case of a pure exchange economy, in which each seller is endowed with one unit of the product and c_j is his private valuation and hence his (opportunity) cost of selling the unit.

In line with the assumptions of the Bayesian mechanism design approach, we may assume that buyer i's valuation v_i is an independent random draw from some distribution F with support $[\underline{v}, \overline{v}]$ and positive density on this support for all $i = 1, .., n$. Similarly, seller j's cost c_j may be thought of as the realization of an independent random variable with distribution G, which has support $[\underline{c}, \overline{c}]$ and positive density everywhere on the support. Let $\mathcal{V} = [\underline{v}, \overline{v}]^n$ and $\mathcal{C} = [\underline{c}, \overline{c}]^m$ be the product sets of buyers' and sellers' types. We refer to the model as one with **complete information** if both realized valuations $v = [v_1, .., v_n]$ and realized costs $c = [c_1, .., c_m]$ are known by all the agents and the mechanism designer. We say that the model is one with **incomplete information** if every agent only knows the realization of his or her own type and the commonly known distributions F and G (as well as n and m), while the mechanism designer only has the information about distributions and numbers of buyers and sellers. Lastly, the model is said to be of **one-sided private information** if only the sellers' costs (and n, m and F) are commonly known, while each buyer's type is her private information.[1] We assume that

$$\underline{c} < \overline{v} \quad \text{and} \quad \overline{v} \leq \overline{c}.$$

The first part of the assumption makes sure that trade is sometimes ex post efficient and the second one is sufficient for trade not to be always ex post efficient. Makowski and Mezzetti (1993, 1994) provide conditions under which ex post efficient trade with private information is possible without running a deficit.

[1]Of course, one could also study the converse problem with one-sided private information that is held by sellers. However, this problem is isomorphic to the one with one-sided private information held by buyers and is skipped in the interest of space.

We order (and relabel) the buyers and sellers so that $v_1 > v_2 > ... > v_n$ and $c_1 < c_2 < ... < c_m$, where we simplify by assuming no tied values or tied costs. The assumption that there are no ties is satisfied almost surely when types are drawn from continuous distribution functions.

An allocation is a $(n + m)$-dimensional vector $\Gamma = [\beta, \sigma]$ consisting of 0's and 1's. For $i \in \{1, ..., n\}$, element β_i specifies whether buyer i receives the good, and for $j \in \{1, ..., m\}$, element σ_j specifies whether seller j produces a unit of the good, with 1 meaning receive/produce. An allocation Γ is said to be feasible if $\sum_{i=1}^{n} \beta_i \leq \sum_{j=1}^{m} \sigma_j$. This implicitly assumes that goods can be freely disposed off, which for all intents and purposes is without loss of generality. Let $\mathcal{G}$ be the set of feasible allocations.

When buying a unit at price p_i, buyer i's utility when of type v_i is $u(v_i, p_i) = v_i - p_i$. Seller's j's profit with cost c_j when selling at price p_j is $\pi(c_j, p_j) = p_j - c_j$. These utility and profit functions are sometimes called quasilinear because they are linear in monetary payments.[2] We assume that agents' outside options are 0, that is, regardless of his or her type, an agent who does not trade and does not make or receive any payments has a payoff of 0.

Embodied in these definitions are further the assumptions of private values, i.e., an agent's payoff is not affected by the values or costs of other agents; and no externalities, i.e., an agent's payoff is not affected by the allocations, transfers, or utility of other agents.

2.2 Mechanisms and strategies

Let $p = [p_1, .., p_{n+m}]$ be the vector of prices of dimension $(n + m)$. A **mechanism** specifies agents' set of actions (and the order in which they choose their actions) and a feasible allocation Γ together with a price vector p. A **direct mechanism** is a mechanism that asks agents to report their types and makes the allocation and prices contingent on these reports. Formally, a direct mechanism $\langle \Gamma, p \rangle$ is a function: $\mathcal{V} \times \mathcal{C} \rightarrow \mathcal{G} \times \mathbb{R}^{n+m}$. The focus on direct mechanisms is without loss of generality because of the revelation principle, which states that whatever allocation and expected payments can be obtained as the (Bayes Nash) equilibrium outcome of some mechanism can be obtained as the (Bayes Nash) equilibrium outcome of a direct mechanism (see, for example, Krishna (2002)).[3] A **dominant strategy mechanism** is a mechanism that makes it a dominant strategy for every player to report his or her type truthfully. A Bayesian mechanism, on the other hand, is a mechanism whose allocation and payment rule are defined with respect to some Bayes Nash equilibrium under the mechanism.

Two kinds of constraints are important in mechanism design. The first one is that agents act in their own best interest. That is, a mechanism needs to make sure that agents do what they are supposed to do. For direct mechanisms, these are referred to as **incentive compatibility constraints**. Second, the mechanism should satisfy the (interim) **individual rationality** constraints that agents, once they know their types and the expected payments and allocations conditional on this information, given the mechanism, are better off participating in the mechanism than walking away. Knowing one's own type but not the types of the other players is referred to as the interim stage. The stage (which need never be reached) in which all players' types are known is called the ex post stage. Accordingly, if individual rationality constraints are satisfied given any realization of allocations and payments, the individual rationality constraints are said to be satisfied ex post. Similarly, a feasible allocation Γ^* is said to be ex post efficient if it is efficient given the realized types, that is, $\Gamma^* \in \arg\max_{\Gamma \in \mathcal{G}} \sum_{i=1}^{n} \beta_i v_i - \sum_{j=1}^{m} \sigma_j c_j$.

[2] They could be non-linear in the good that is being traded, whence the name.

[3] Recall that a Bayes Nash equilibrium of a game is a strategy profile such that every type of every player maximizes his or her expected payoff, keeping fixed the strategy profile of every other player-type, the expectation being taken with respect to the player-type's beliefs that are updated using Bayes rule.

3 Results

3.1 Efficient benchmark for complete information

In seminal work in the economics literature, Ronald Coase (Coase (1960)) put forward the idea that in a theoretical environment with complete information and no transactions costs, the initial allocation of properties rights (as long as these property rights are well defined and protected) is irrelevant because, Coase argued, agents will continue to engage in transactions as long as the allocation remains inefficient. In Coase's theoretical environment, any inefficiency in the initial allocation is eliminated through exchange in the secondary market. The general idea that, in environments with complete information, well-defined and protected property rights, and no transaction costs, efficient outcomes can be achieved through private transactions has come to be known as the Coase Theorem Coase Theorem.[4]

We begin with an illustration of an instance of the Coase Theorem Coase Theorem. For this purpose, we assume in this subsection that all values and costs are common knowledge among all the agents and also known by the mechanism designer. If $c_1 \geq v_1$, then the lowest production cost is equal to or exceeds the highest value and so there are no gains from trade. If there are at least as many buyers as sellers ($n \geq m$) and $c_m < v_m$, then efficiency requires that all sellers produce and that the m units produced be allocated to buyers $1, ..., m$.[5] Similarly, if there are more sellers than buyers ($m > n$) and $c_n < v_n$, then efficiency requires that all buyers be allocated a unit and that the n units required be produced by sellers $1, ..., n$. (More generally, these last two cases occur when $c_{\min\{m,n\}} < v_{\min\{m,n\}}$.)

In all other cases, $c_1 < v_1$ and there exists $\hat{k} \in \{2, ..., \min\{m, n\}\}$ such that $c_{\hat{k}-1} < v_{\hat{k}-1}$ and $c_{\hat{k}} \geq v_{\hat{k}}$, implying that the efficient outcome involves production by sellers $1, ..., \hat{k} - 1$ and an allocation of products to buyers $1, ..., \hat{k} - 1$. Using this definition of $\hat{k}$, we can now define

$$\kappa := \begin{cases} 0, & \text{if } c_1 \geq v_1 \\ \min\{m, n\}, & \text{if } c_{\min\{m,n\}} < v_{\min\{m,n\}} \\ \hat{k} - 1, & \text{otherwise,} \end{cases}$$

so that the efficient outcome is for sellers $j \leq \kappa$ to produce and for buyers $i \leq \kappa$ to receive one unit each.

Coase Theorem

3.1.1 The Coase Theorem

One way to view the market in this environment is to assume the existence of a "two-sided" Walrasian auctioneer accepting bids and asks for the product. The Walrasian equilibrium in this model involves a price

$$p \in \begin{cases} [v_1, c_1], & \text{if } \kappa = 0 \\ [c_{\min\{m,n\}}, v_{\min\{m,n\}}], & \text{if } \kappa = \min\{m, n\} \\ [\max\{c_\kappa, v_{\kappa+1}\}, \min\{c_{\kappa+1}, v_\kappa\}], & \text{otherwise.} \end{cases}$$

First, note that by the definition of κ and our ordering of the costs and values, the price p is well defined. If $\kappa = 0$, then $v_1 \leq c_1$; if $\kappa = \min\{m, n\}$, then $c_{\min\{m,n\}} < v_{\min\{m,n\}}$; and otherwise

[4] There is no single formal statement of the Coase Theorem Coase Theorem. For example, see Mas-Colell, Whinston, and Green (1995).

[5] In slight abuse of everyday language, we mean that sellers sell their units when we say that "sellers produce", regardless of whether they actually physically produce the units or only sell units of goods or assets they are endowed with.

$\max\{c_\kappa, v_{\kappa+1}\} \leq \min\{c_{\kappa+1}, v_\kappa\}$, where we define c_{m+1} to be $\bar{c}$ and v_{n+1} to be $\underline{v}$. Second, given such a price p, there is an equilibrium in which only buyers $i \leq \kappa$ wish to purchase, implying demand of κ. (If the price is set at its lower bound and that lower bound is $v_{\kappa+1}$, then buyer $\kappa+1$ is indifferent between purchasing and not.) Third, there is an equilibrium in which only sellers $j \leq \kappa$ wish to produce, implying supply of κ. (If the price is set at its upper bound and that upper bound is $c_{\kappa+1}$, then seller $\kappa+1$ is indifferent between producing and not.) Thus, a market price p as defined above implements the efficient outcome.

In the sense described here, the competitive market equilibrium delivers the efficient outcome.

Proposition 1 *The competitive equilibrium for the environment with unit demand and supply and complete information is efficient.*

The efficiency of the competitive equilibrium does not rely on which of the first κ sellers trades with which of the first κ buyers, only that it is the first κ sellers who produce and the first κ buyers who are ultimately allocated the products.

The so-called Efficient rationing "efficient rationing rule" allocates the product of the least-cost seller to the highest-valuing buyer, and so on, so that for $i \in \{1, ..., \kappa\}$, seller i trades with buyer i; see Shubik (1959), Beckmann (1965), Levitan and Shubik (1972), Kreps and Scheinkman (1983), and Davidson and Deneckere (1986) for more on rationing rules. For example, if we assume sellers charge differential prices for their products, then the demand for a low-priced seller's product may be greater than 1. Once that seller's product has been purchased by a buyer, the residual demand facing the remaining sellers will depend on which buyer made that first purchase. Under the Efficient rationing efficient rationing rule, the residual demand at any price is simply the original demand minus 1 (as long as that quantity is nonnegative). Efficient rationing Efficient rationing obtains under the assumption that buyers are able to costlessly resell products to each other. In a game in which sellers first set prices and then buyers state whether they are willing to purchase from each seller, then Efficient rationing efficient rationing delivers the efficient outcome as a Nash equilibrium of the game.

Under the alternative rationing rule of Random rationing "random rationing" each buyer is equally likely to be given the opportunity to trade with the least-cost seller. That is, all buyers with values greater than the price of the least-cost seller are equally likely to trade with that seller. Under the Random rationing random rationing rule, the allocation is not necessarily efficient because a buyer with value less than v_κ might be allocated a unit.

One can define games in which Efficient rationing efficient rationing arises in a game involving bilateral bargaining. For example, suppose the market is organized as a series of bilateral negotiations, each delivering the Nash bargaining outcome.[6] First, buyer 1 and seller 1 engage in bilateral bargaining. If they do not come to agreement, then neither trades. Then buyer 2 and seller 2 do the same, and this continues until there are either no more buyers or no more sellers. In this game, if $v_1 > c_1$, the outcome of the first Nash bargaining game is for seller 1 to produce the good and sell it to buyer 1 for a payment of $\frac{v_1+c_1}{2}$. The outcome of this sequence of Nash bargaining games is for the first κ buyers and sellers to trade, but no others. Thus, Efficient rationing efficient rationing and the efficient outcome is achieved.

Other sequences of negotiations are possible as well, including potentially those that involve buyers other than the first κ acquiring product initially, but then later selling to the higher valuing buyers.

[6]Under Nash bargaining a buyer of type v and a seller of type c trade at the price $p(v, c)$ that maximizes $(v - p)(p - c)$ over p, which is $p(v, c) = (v + c)/2$ provided $v \geq c$. Otherwise, they do not trade.

The Walrasian equilibrium and the differential pricing game with Efficient rationing efficient rationing produce outcomes that are in the "core" in the sense that no subset of agents can profitably deviate from the specified outcome. Furthermore, only efficient allocations are in the core; see Shapley and Shubik (1971). For example, if an outcome involved production by a seller with cost greater than c_κ and no production by seller κ, then a coalition involving the seller with cost greater than c_κ, seller κ, and the purchaser of the product from the seller with cost greater than c_κ could profitably deviate by sharing in the efficiency gains associated with moving production to seller κ. Finally, every core outcome has the property that all traders receive payoffs equal to those in one and the same Walrasian equilibrium p for every $p \in [\max\{c_\kappa, v_{\kappa+1}\}, \min\{c_{\kappa+1}, v_\kappa\}]$, assuming for simplicity that the efficient quantity κ is neither 0 nor $\min\{n, m\}$. This can easily be seen by noting that if two buyers pay differential prices, then the buyer paying the higher price can form a profitable deviating coalition with the lower priced seller. Thus payoffs must be as if all trades occurred at the same price, and any such price must be a Walrasian equilibrium price.

The result of the Coase Theorem Coase Theorem is that in an environment with complete information, as long as property rights are well defined and protected, and as long as there are no transactions costs, it should not matter precisely what the market processes are. As long as any inefficiency remains, there are mutually beneficial trades that can be made, and one would expect those to be realized.

3.1.2 Implications for market design

As described by Milgrom (2004) (pp.19-20), critics of an auction approach to allocating spectrum licenses have argued based on the Coase Theorem Coase Theorem, saying:

> [O]nce the licenses are issued, parties will naturally buy, sell, and swap them to correct any inefficiencies in the initial allocation. Regardless of how license rights are distributed initially, the final allocation of rights will take care of itself. Some critics have gone even farther, arguing on this basis that the only proper object of the government is to raise as much money as possible in the sale, because it should not and cannot control the final allocation.

However, the evidence suggests that the market for spectrum licenses does not satisfy all the requirements of the Coase Theorem Coase Theorem. As described by Milgrom (2004), p.40:

> The history of the US wireless telephone service offers direct evidence that the fragmented and inefficient initial distribution of rights was not quickly correctable by market transactions. Despite demands from consumers for nationwide networks and the demonstrated successes of similarly wide networks in Europe, such networks were slow to develop in the United States.

In order to understand the market design issues for spectrum licenses, we must begin by understanding the ways in which deviations from the perfect world of the Coase Theorem Coase Theorem potentially affect the efficiency of market outcomes. We begin in the next subsection by considering the simple adjustment to the environment just considered to allow the values of buyers and costs of sellers to be the agents' own private information.

3.2 Results for private information and strategic interaction

We begin by providing an example of an efficient mechanism for the case in which there is a single seller that acts as the mechanism designer with the goal of maximizing efficiency. Then we examine the case where sellers must be incentivized to participate and reveal their privately held information, in which case the Myerson-Satterthwaite Theorem Myerson-Satterthwaite Theorem implies that the outcome is generally not efficient. We then examine the implications for the design of primary market versus secondary market institutions.

3.2.1 Efficient mechanisms generate a surplus for one-sided private information

In order to illustrate the existence of an efficient mechanism for a primary market, we simplify the above environment by assuming there is a single seller that can produce m units at increasing marginal cost c_j for $j \in \{1, ..., m\}$. But we adjust the above environment so that the values of the buyers $v_1, ..., v_n$ are the private information of the individual buyers. In this environment, there exists an efficient mechanism that never runs a deficit.

To see this, consider a mechanism in which buyers submit reports of their values and order the reports as $r_1 \geq r_2 \geq ... \geq r_n$. Let the mechanism identify the set of efficient trades as described above based on the vector of reports r (rather than the true values as above) and the seller's marginal costs. The mechanism identifies the number κ of units to allocated based on reports and costs, and those units are allocated to buyers with reports $r_1, ..., r_\kappa$. Each buyer receiving a unit of the good pays the same amount $p = \max \{c_\kappa, r_{\kappa+1}\}$ to the seller.

It follows from the analysis above that if buyers truthfully report their values, this mechanism is efficient. The result that no buyer has an incentive to misreport her true value follows from the usual second-price auction logic. Suppose that buyer i considers reporting $r_i > v_i$ rather than v_i. This change only affects the outcome for buyer i if buyer i does not receive a unit when she reports v_i but does receive a unit when she reports r_i, in which case the change results in buyer i being allocated a unit at a price greater than her value for the unit. Thus, the buyer prefers to report truthfully rather than any amount greater than her value.

To see that no buyer has an incentive to underreport her value, suppose that buyer i considers reporting $r_i < v_i$ rather than v_i. This change only affects the outcome for buyer i if buyer i receives a unit when she reports v_i but does not receive a unit when she reports r_i, in which case the change results in buyer i not receiving a unit when she would have acquired a unit at a price less than her value, giving her positive surplus. Thus, it is a weakly dominant strategy for buyers to report truthfully.[7]

Proposition 2 *In an environment with a single multi-unit seller and unit demand buyers with private information, there exists an ex post efficient, incentive compatible, and individually rational mechanism.*

Proposition 2 establishes that efficiency can be achieved when there are no incentive issues for the seller, such as might be the case in the primary market for spectrum licenses where government is the seller. However, as we show next, this result does not extend to the case of secondary markets in which sellers must be incentivized as well.

[7] It is also worth noting that nothing in this argument hinges on the assumption that buyers are risk neutral because it extends straightforwardly to the case where buyers' utility functions $u(v_i, p)$ are increasing functions of the monetary payoff $v_i - p$, regardless of the sign of the second derivative of these functions.

3.2.2 Efficient mechanisms run a deficit for two-sided private information

In order to examine the effects of having private information for both buyers and sellers, consider the environment above, but assume that each buyer's value and each seller's cost is the agent's own private information. As we show, incentives for buyers and sellers to engage in strategic behavior are unavoidable whenever values are private.

Vickrey (1961) deserves credit for having first presented an impossibility result for market making in the domain of dominant strategy mechanisms. Vickrey states, "When it comes to markets where the amounts which each trader might buy or sell are not predetermined but are to be determined by the negotiating procedure along with the amount to be paid, the prospects for achieving an optimum allocation of resources become much dimmer. A theoretical method exists, to be sure, which involves essentially paying each seller for his supply an amount equal to what he could extract as a perfectly discriminating monopolist faced with a demand curve constructed by subtracting the total supply of his competing suppliers from the total demand, and symmetrically for purchase. But ... the method is far too expensive in terms of the inflow of public funds that would be required ...." (Vickrey (1961) p.29). We present and prove a more general version of this result below.

In the domain of Bayesian mechanism, the seminal impossibility result of Myerson and Satterthwaite (1983), which was originally stated for one buyer and one seller, extends to this environment and establishes that under weak conditions there does not exist a mechanism that is ex post efficient, incentive compatible, interim individually rational and that does not run a deficit.

In what follows, we assume $\underline{v} < \bar{c}$, so that we avoid the trivial case in which it is always efficient for all buyers and sellers to trade.

Consider the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism, whose operation in this context requires that the sellers report their costs and that the buyers report their values.[8] Given the reported cost and values, the mechanism would implement the efficient allocation given the reports. Assuming truthful reporting, each buyer $i \leq \kappa$ receives a unit and pays $\max\{c_\kappa, v_{\kappa+1}\}$, and each seller $j \leq \kappa$ produces a unit and receives $\min\{c_{\kappa+1}, v_\kappa\}$. For reasons that are analogous to those in the setup with one-sided private information, truthful reporting is a weakly dominant strategy for every buyer and every seller irrespective of their types.

Participation in this mechanism is clearly individually rational. A buyer with value $\underline{v}$ has expected surplus of zero and a buyer's expected surplus is increasing in her value. A seller with cost $\bar{c}$ has expected surplus of zero and a seller's expected surplus is decreasing in his cost.

If there exists some other efficient mechanism that is also incentive compatible, then by the revenue equivalence theorem[9] there is a constant ξ_b such that the expected payment for any buyer b with value v under this mechanism differs from the expected payment under the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism by ξ_b. Similarly, there is a constant ξ_s such that the expected receipts for any seller s with cost c under this mechanism differs from that under the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism by ξ_s. If this other mechanism is also individually rational, it must be that $\xi_b \leq 0$ because otherwise the buyer with value $\underline{v}$ would have negative expected surplus, and it must be that $\xi_s \geq 0$ because otherwise a seller with cost $\bar{c}$ would have negative expected surplus, contradicting individual rationality.

[8]Section 4 contains the general description of the two-sided Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism.

[9]See Proposition 14.1 in Krishna (2002) for a statement of the revenue equivalence theorem for the multi-unit case.

Thus, in this other mechanism, the sum of buyers' payments minus the sum of sellers' receipts must be weakly lower than in the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism.

This gives us the following lemma, which is a straightforward extension of Proposition 5.5 in Krishna (2002).

Lemma 1 *Consider the model with incomplete information. Among all allocation mechanisms that are efficient, incentive compatible, and individually rational, the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism maximizes the sum of buyers' payments minus the sum of sellers' receipts.*

Using Lemma 1, we can prove a generalized version of the Myerson-Satterthwaite Theorem Myerson-Satterthwaite Theorem.

Proposition 3 *In the model with incomplete information, there is no mechanism that is efficient, incentive compatible, individually rational, and at the same time does not run a deficit.*

Proof. Consider the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism described above. The total amount received by sellers, $\kappa \min \{c_{\kappa+1}, v_\kappa\}$, is weakly greater than the total payments made by the buyers, $\kappa \max \{c_\kappa, v_{\kappa+1}\}$, and strictly greater in general given our assumption that $\underline{v} < \bar{c}$. Thus, in general, the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism runs a deficit. Using Lemma 1, if the VCG runs a deficit, then every other incentive compatible and individually rational mechanism also runs a deficit. Thus, there does not exist an efficient mechanism that is incentive compatible, individually rational, and balances the budget for all realizations of values and costs. Q.E.D.

3.2.3 For large markets approximately efficient mechanisms generate a surplus for two-sided private information

As a contrast to Proposition 3, Gresik and Satterthwaite (1989) show that in the limit as a market becomes large, the expected inefficiency associated with the optimally designed mechanism decreases; see also Rustichini, Satterthwaite, and Williams (1994), Tatur (2005), and Cripps and Swinkels (2006) for limit results with large double auctions. Intuitively, as the market becomes large, each trader has no effect on the market clearing price and so no incentive to disguise private information, allowing the efficient allocation to be obtained.

To provide a specific example of an almost efficient two-sided mechanism that does not run a deficit, we provide a slightly simplified version of the dominant strategy mechanism proposed by McAfee (1992).

Buyers and sellers simultaneously submit bids b_i and s_j to the clearinghouse. Given submitted bids, the clearinghouse induces the Walrasian quantity minus one, i.e. $\kappa - 1$, to be traded, κ being defined with respect to the submitted bids. Buyers who trade pay the price $p_B = \max\{b_\kappa, s_{\kappa-1}\}$ to the clearinghouse. Sellers who trade receive the price $p_S = \min\{b_{\kappa-1}, s_\kappa\}$. All other buyers and sellers pay and receive nothing.

By the usual arguments, each buyer and seller has a weakly dominant strategy to bid truthfully, i.e. $b_i = v_i$ and $s_j = c_j$ are weakly dominant strategies, because no agent can ever affect the price he or she pays or receives. The bids only affect whether or not an agent trades. By bidding truthfully, agents can make sure that they trade in exactly those instances in which it is in their best interest. For a buyer i that is to trade if and only if $v_i \geq p_B$ and for a seller j this is to trade if and only if $c_j \leq p_S$.

Given truthful bidding, $p_B = \max\{v_\kappa, c_{\kappa-1}\} = v_\kappa$ and $p_S = \min\{v_{\kappa-1}, c_\kappa\} = c_\kappa$. By the definition of κ, $v_\kappa > c_\kappa$, whence it follows that $p_B > p_S$, and so the mechanism generates a surplus.

For given v and c, the efficiency loss is small in the sense that the only loss is the failure to trade by the buyer-seller pair whose contribution to overall welfare is smallest amongst the κ pairs that would trade under an efficient allocation. Moreover, as the number of buyers and sellers becomes large, this efficiency loss becomes smaller in expectation by a law of large numbers argument.

3.3 Implications for the design of primary and secondary markets

As shown above, the existence of second-price auctions suggests that efficiently functioning primary markets for spectrum licenses are possible. The efficiency result holds in an environment in which all available units for sale and all potential buyers of those units participate in a centralized mechanism. This suggests a role for a market designer in the primary market.

Turning to secondary markets, the Myerson-Satterthwaite Theorem Myerson-Satterthwaite Theorem implies that efficiency without running a deficit is not possible for secondary markets. As stated by Milgrom (2004) p.21:

> [E]ven in the simplest case with just a single license for sale, there exists *no* mechanism that will reliably untangle an initial misallocation. ... According to a famous result in mechanism design theory – the Myerson-Satterthwaite Theorem Myerson-Satterthwaite theorem – there is no way to design a bargaining protocol that avoids this problem: delays or failures are inevitable in private bargaining if the good starts out in the wrong hands. (emphasis in the original)

The key implication of these results is that we cannot rely on competitive markets to produce efficient outcomes in secondary markets.

Nonetheless, there is potentially an important role for market design approaches to improve the efficiency of secondary markets. This is clear from the results of McAfee (1992) and others showing that approximate efficiency can be achieved in large secondary markets. Because one might not expect large secondary markets to arise in a typical environment with dispersed ownership, these results highlight the potentially valuable role a market designer can play by centralizing secondary markets to achieve a larger market size.

Given the pace of technological change in our use of spectrum, it is inevitable that secondary markets must play an important role. The Federal Communications Commission (FCC) FCC and Congress recognize this in their current work to design "incentive auctions" to facilitate a move to a more efficient allocation of spectrum licenses. As stated by the Federal Communications Commission (FCC) FCC:

> "Commercial uses of spectrum change over time with the development of new technologies. ... The use of incentive auctions is one of the ways we can help meet the dramatic rise in demand for mobile broadband. Current licensees – like over-the-air-broadcasters – would have the option to contribute spectrum for auction in exchange for a portion of the proceeds. ... The use of incentive auctions to repurchase spectrum is a complex and important undertaking that will help ensure America's continued leadership in wireless innovation. The best economists, engineers and other experts are hard at work at the Commission to get the job done

right – with openness and transparency."[10]

4 Generalization

In this section, we introduce the setup for the general two-sided Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism, whose standard one-sided form is due to the independent contributions of Vickrey (1961), Clarke (1971) and Groves (1973). We show that the two-sided Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism provides agents with dominant strategies, allocates goods efficiently when the agents play these strategies, and always runs a deficit when the agents play their dominant strategies. Lastly, we look at a number of simple mechanisms the two-sided Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism specializes to under appropriate restrictions.

4.1 Model

For the general model, we move away from the assumption of a single homogeneous product and assume there are $K \geq 1$ potentially heterogenous objects that the sellers can produce. These K objects can be bundled into $2^K - 1$ different packages (excluding the nil package). All buyers i have valuations v_i^k for each possible package k and all sellers j have costs c_j^k for producing any package k with $k = 1, .., 2^K - 1$, $i = 1, .., n$ and $j = 1, .., m$. So buyer i's valuations can be summarized as a $2^K - 1$ dimensional vector $v_i = [v_i^1, .., v_i^{2^K-1}]$, whose k-th element is i's valuation for package k, and seller j's costs are a $2^K - 1$ dimensional vector $c_j = [c_j^1, .., c_j^{2^K-1}]$, whose k-th element is j's cost for producing package k. All agents (i.e. buyers and sellers) have quasilinear preferences, linear in payments. So if buyer i is allocated package k and pays the price p, her net payoff is $v_i^k - p$. Similarly, if seller j produces package k and is paid p, his net payoff is $p - c_j^k$. We set the valuation of every buyer for receiving the nil package and the cost of every seller of producing the nil package to 0.

We refer to v_i and c_j as buyer i's and seller j 's type and assume that each agent's type is his or her private information, that types are independent across agents, that $v_i^k \in [\underline{v}_i^k, \overline{v}_i^k]$ for all $k = 1, .., 2^K - 1$, so that $v_i \in \times_{k=1}^{2^K-1}[\underline{v}_i^k, \overline{v}_i^k] := \mathcal{V}_i$ for all $i = 1, .., n$, and that $c_j^k \in [\underline{c}_j^k, \overline{c}_j^k]$ for all $k = 1, .., 2^K - 1$, so that $c_j \in \times_{k=1}^{2^K-1}[\underline{c}_j, \overline{c}_j] := \mathcal{C}_j$ for all $j = 1, .., m$. Let $\underline{v}_i = [\underline{v}_i^1, .., \underline{v}_i^{2^K-1}]$ and $\overline{c}_j = [\overline{c}_j^1, .., \overline{c}_j^{2^K-1}]$ denote buyer i's lowest and seller j's highest types, respectively.

The assumption that types are drawn from rectangular sets allows us to speak of an agent's highest and lowest possible type without any ambiguity. It is, for example, an appropriate assumption if the v_i^k's are independent draws from some distributions F_i^k, or, perhaps more realistically, if v_i is drawn from a distribution F_i with full support on $\mathcal{V}_i$, and analogously for sellers. Clearly, this assumption comes at the cost of some loss of generality because it rules out the case where a buyer may never have the lowest possible (or highest possible) valuations for all objects. This assumption is usually not made in the literature on one-sided Vickrey-Clarke-Groves (VCG) mechanism VCG mechanisms because there the (implicit or explicit) assumption is that the seller's cost is less than every buyer's valuation for every package, which is typically normalized to 0. This structure also does not need to be imposed in the two-sided setup where each buyer i has a maximal willingness to pay $v_i \in [\underline{v}_i, \overline{v}_i]$ for one unit only and each seller j has a capacity to produce one unit only at cost $c_j \in [\underline{c}_j, \overline{c}_j]$, which has been analyzed in a variety of setups (see e.g. Myerson and Satterthwaite (1983), Gresik and

[10]Federal Communications Commission website, http://www.fcc.gov/topic/incentive-auctions, accessed June 27, 2012.

Satterthwaite (1989), Krishna (2002)). It is necessary here because the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism requires that we can unambiguously identify a lowest type for every buyer and a highest (or least efficient) type for every seller.[11] We assume that types are private information of the agents but that $\mathcal{V}_i$ and $\mathcal{C}_j$ are known by the mechanism designer for $i = 1, .., n$ and $j = n+1, .., n+m$.

In the case of spectrum licenses, and in many other instances, it is sensible to assume that packages can be re-packaged at zero cost to society if trade is centralized. Therefore, no congruence between packages produced and packages delivered is required.[12]

4.1.1 Setup

Let $v = (v_1, .., v_n)$ and $c = (c_1, .., c_m)$. An allocation is a $K \times (n+m)$-matrix $\Gamma = [\beta, \sigma]$ consisting of 0's and 1's. The h-th column of Γ, denoted Γ_h, specifies which goods buyer h receives for $h = 1, .., n$ and which goods seller h produces for $h = n+1, .., n+m$ (with 1 meaning receive/produce). That is, $\beta_{hi} = 1(0)$ means that buyer i gets (does not get) good h and $\sigma_{hj} = 1(0)$ means that seller j produces (does not produce) good h. An allocation Γ is said to be feasible if for every $h = 1, .., K$

$$\sum_{i=1}^{n} \beta_{hi} \leq \sum_{j=n+1}^{n+m} \sigma_{hj}. \tag{1}$$

Observe that there are 2^K possible Γ_h's, so every non-zero Γ_h corresponds to some specific package. Letting Γ_h correspond to package h, we let $v_i(\Gamma_h) \equiv v_i^h$ and $c_j(\Gamma_h) \equiv c_j^h$ and $\underline{v}_i(\Gamma_h) \equiv \underline{v}_i^h$ and $\overline{c}_j(\Gamma_h) \equiv \overline{c}_j^h$ in slight abuse of notation. Letting $\mathcal{G}$ be the set of feasible allocations and $W(\Gamma, v, c) := \sum_{i=1}^{n} v_i(\Gamma_h) - \sum_{j=1}^{m} c_j(\Gamma_h)$ be social welfare at allocation Γ given v and c, the (or a) welfare maximizing allocation for given v and c is denoted $\Gamma^*(v, c)$ and satisfies

$$\gamma^*(v, c) \in \arg\max_{\Gamma \in \mathcal{G}} W(\Gamma, v, c). \tag{2}$$

4.1.2 Mechanisms and strategies

Like the standard (one-sided) Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism, the two-sided Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism is a direct mechanism that asks all agents to submit bids on all possible packages, where the space of an agent's possible bids is identical to the space of his or her possible types. In any direct mechanism, an allocation rule is a matrix valued function $\Gamma : \mathbb{R}_+^{(n+m)(2^K-1)} \to \mathcal{G}$ that maps agents' bids into feasible allocations. An allocation rule $\Gamma(v, c)$ is said to be efficient if $\Gamma(v, c) = \Gamma^*(v, c)$ for all $(v, c) \in \mathcal{V} \times \mathcal{C}$, where $\mathcal{V} = \times_{i=1}^{n} \mathcal{V}_i$ and $\mathcal{C} = \times_{j=1}^{m} \mathcal{C}_j$.

And like the one-sided Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism, the two-sided Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism uses an efficient allocation rule. That is, if the bids are v and c, the allocation is $\Gamma^*(v, c)$. [13] Letting v_{-i} be the bids of

[11] To be more precise, Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism does not exactly require this since we could also define payments of agent h by excluding h (see below); however, if we did so, then the mechanism would necessarily always run a deficit. So the assumption is necessary only if we want to allow for the possibility that the mechanism does run a surplus under some conditions.

[12] If, on the other hand, sellers and buyers trade directly, then efficiency may dictate that the packages produced be identical to the packages delivered or, at least, that the packages buyers receive are unions of packages produced.

[13] If there are multiple welfare maximizing allocations, the mechanism can pick any of those.

buyers other than i , the payment $p_i^B(v,c)$ that buyer i has to make given bids (v,c) is

$$p_i^B(v,c) = W(\Gamma^*(\underline{v}_i, v_{-i}, c), \underline{v}_i, v_{-i}, c) - W_{-i}(\Gamma^*(v,c), v, c), \tag{3}$$

where $W_{-i}(\Gamma^*(v,c),v,c) = W(\Gamma^*(v,c),v,c) - v_i(\Gamma_i^*(v,c))$. Notice that these payments look somewhat different from the way in which they are usually described, where buyer i's payment would be: maximal social welfare excluding buyer i (i.e. where the welfare maximizing allocation is chosen as if i did not exist) minus the social welfare excluding buyer i at the welfare maximizing allocation that includes buyer i (see e.g. Milgrom (2004), p.49). The difference is due to the fact that the lowest types $\underline{v}_i$ are not necessarily 0 here, whereas if they are 0 (as mentioned, this is the standard assumption in one-sided setups), then there is obviously no difference between the two. However, in two-sided setups, the distinction matters with regards to revenue. Intuitively, this makes sense: The mechanism does have to provide the incentives to all agents of revealing their types, and they could always pretend to be the least efficient type.

Similarly, the payment seller j gets when the bids are v and c is

$$p_j^S(v,c) = W_{-j}(\Gamma^*(v,c), v, c) - W(\Gamma^*(\overline{c}_j, v, c_{-j}), \overline{c}_j, v, c_{-j}), \tag{4}$$

where c_{-j} is the collection of the bids of all sellers other than j and $W_{-j}(\Gamma^*(v,c),v,c) = W(\Gamma^*(v,c),v,c) + c_j(\Gamma_j^*(v,c))$.

For reasons mirroring those in one-sided setups, the two-sided Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism provides each agent with dominant strategies. To see this, notice that buyer i's payoff when her type is v_i and her bid is b_i while every one else bids v_{-i} and c is

$$\begin{aligned} v_i(\Gamma^*(b_i, v_{-i}, c)) &- p_i^B(b_i, v_{-i}, c) \\ &= W(\Gamma^*(b_i, v_{-i}, c), v, c)) - W(\Gamma^*(\underline{v}_i, v_{-i}, c), \underline{v}_i, v_{-i}, c), \end{aligned} \tag{5}$$

which is maximized at $b_i = v_i$ since $\Gamma^*(b_i, v_{-i}, c)$ is the maximizer of $W(\Gamma, b_i, v_{-i}, c)$ rather than of $W(\Gamma, v, c)$ and since the last term is independent of b_i. Analogously, seller j's payoff when his type is c_j and his bid is s_j while every one else bids v and c_{-j} is

$$\begin{aligned} p_j^S(v, s_j, c_{-j}) &- c_j(\Gamma_j^*(v, s_j, c_{-j})) \\ &= W(\Gamma^*(v, s_j, c_{-j}), v, c)) - W(\Gamma^*(v, \overline{c}_j, c_{-j}), v, \overline{c}_j, c_{-j}), \end{aligned} \tag{6}$$

which is maximal at $s_j = c_j$.

4.2 Results

In the results presented below, we find it useful in terms of establishing notation to first state results for two-sided private information and then specialize to one-sided private information.

4.2.1 Efficient mechanisms run a deficit for two-sided private information

Denote, respectively, buyer i's and seller j's marginal contribution to welfare at (v,c) by

$$MW_i(v,c) := W(\Gamma^*(v,c), v, c) - W(\Gamma^*(\underline{v}_i, v_{-i}, c), \underline{v}_i, v_{-i}, c)$$

and

$$MW_j(v,c) := W(\Gamma^*(v,c), v, c) - W(\Gamma^*(v, \overline{c}_j, c_{-j}), v, \overline{c}_j, c_{-j}).$$

We now assume that

$$\sum_{i=1}^{n} MW_i(v,c) + \sum_{j=1}^{m} MW_j(v,c) > W(\Gamma^*(v,c),v,c). \quad (7)$$

Condition (7) typically holds in two-sided setups with unit demand and unit capacities. For example, in the classical setup with one buyer and one seller and a single unit with $v > c$ and $\underline{v} = \underline{c}$ and $\overline{v} = \overline{c}$, $W(\Gamma^*(v,c),v,c) = v - c$ and $MW_i(v,c) = v - c = MW_j(v,c)$, so $MW_i(v,c) + MW_j(v,c) = 2W(\Gamma^*(v,c),v,c)$.[14] More generally, in the setup with M sellers with unit capacities and N buyers with unit demand with $\underline{v} = \underline{c}$ and $\overline{v} = \overline{c}$, we have $\sum_{i=1}^{N} MW_i(v,c) + \sum_{j=1}^{M} MW_j(v,c) = W(\Gamma^*(v,c),v,c) + \kappa[\min\{c_{\kappa+1}, v_\kappa\} - \max\{c_\kappa, v_{\kappa+1}\}]$, where κ is the efficient quantity and c_j the j-lowest cost and v_i the i-th highest valuation. Assuming continuous distributions, this is strictly larger than $W(\Gamma^*(v,c),c,c)$ with probability 1 whenever $\kappa > 0$. Note that in a one-sided setup such as a single unit auction, the sum of the marginal contributions to welfare of the buyers, $\sum_{i=1}^{n} MW_i(v)$, is less than maximum welfare $W(\Gamma^*(v),v)$ because $W(\Gamma^*(v),v) = v_1$ and $\sum_{i=1}^{n} MW_i(v) = v_1 - v_2$. An analogous statement will hold for one-sided allocation problems that only involve sellers. Intuitively, as observed by Shapley and Shubik (1971) agents on the same side of the market are substitutes to each other while agents from different sides are complements for each other. Based on a result of Shapley (1962), Loertscher, Marx, and Wilkening (2013) show that the inequality in (7) is never reversed in the assignment game of Shapley and Shubik (1971), where buyers perceive sellers as heterogenous and where all buyers have unit demand and all sellers have unit capacities, provided only the least efficient type of a buyer and the least efficient seller type optimally never trade. To see that condition (7) can hold for very general preferences and costs in two-sided setups, consider the case with $M = 1$ seller and assume that $\overline{c} \geq \overline{v}_i$ for all i, where $\overline{c}$ is the highest possible cost for the seller S. Then $MW_S(v,c) = W(\Gamma^*(v,c),v,c)$ because if the seller had the highest possible cost draw the optimal allocation would involve no production at all. On top of that, $\sum_{i=1}^{n} MW_i(v,c) > 0$ will hold with probability 1 whenever $W(\Gamma^*(v,c),v,c) > 0$, and so condition (7) will be satisfied.

We can now state a generalization of Vickrey (1961)'s observation that for homogenous goods efficient market making with a dominant strategy mechanism is only possible by running a deficit.

Proposition 4 *Under condition (7), the two-sided Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism runs a deficit each time it is run and some trade is ex post efficient.*

[14] This is due to the two-sided nature of the market making problem. In a second price sales auction of a single object with n buyers and $MW_i(v) := W(\Gamma^*(v),v) - W(\Gamma^*(\underline{v}_i, v_{-i}), \underline{v}_i, v_{-i})$, $\sum_{i=1}^{n} MW_i(v) = v_1 - v_2 < W(\Gamma^*(v),v) = v_1$, where v_i is the i-th highest draw. Accordingly, revenue $R(v)$ is $R(v) = W(\Gamma^*(v),v) - \sum_{i=1}^{n} MW_i(v) = v_1 - (v_1 - v_2) = v_2$.

Proof. Given bids (v,c) the revenue under the mechanism is

$$\begin{aligned}
R(v,c) &= \sum_{i=1}^{n} p_i^B(v,c) - \sum_{j=1}^{m} p_j^S(v,c) = W(\Gamma^*(v,c),v,c) \\
&- \sum_{i=1}^{n} \{W(\Gamma^*(v,c),v,c) - W(\Gamma^*(\underline{v}_i,v_{-i},c),\underline{v}_i,v_{-i},c)\} \\
&- \sum_{j=1}^{m} \{W(\Gamma^*(v,c),v,c) - W(\Gamma^*(v,\overline{c}_j,c_{-j}),v,\overline{c}_j,c_{-j})\} \\
&= W(\Gamma^*(v,c),v,c) - \left[\sum_{i=1}^{n} MW_i(v,c) + \sum_{j=1}^{m} MW_j(v,c)\right] < 0,
\end{aligned}$$

where the inequality is due to (7). Q.E.D.

To fix ideas, let us revisit the case where buyers have unit demand, sellers have unit capacities, and all goods are homogenous. Under the Walrasian allocation rule, we have

$$W(\Gamma^*(v,c),v,c)) = \sum_{h=1}^{\kappa} v_h - \sum_{j=1}^{\kappa} c_j$$

and, for $v_{\kappa+1} \leq c_\kappa$,

$$W(\Gamma^*(\underline{v}_i,v_{-i},c),\underline{v}_i,v_{-i},c) = \sum_{h\neq i}^{\kappa} v_h - \sum_{j=1}^{\kappa-1} c_j$$

for any $i \leq \kappa$. Accordingly,

$$p_i^B(v,c) = W(\Gamma^*(\underline{v}_i,v_{-i},c),\underline{v}_i,v_{-i},c)) - W(\Gamma^*(v,c),v,c) + v_i = c_\kappa. \tag{8}$$

On the other hand, if $v_{\kappa+1} > c_\kappa$,

$$W(\Gamma^*(\underline{v}_i,v_{-i},c),\underline{v}_i,v_{-i},c) = \sum_{h\neq i}^{\kappa+1} v_h - \sum_{j=1}^{\kappa} c_j$$

and hence

$$p_i^B(v,c) = W(\Gamma^*(\underline{v}_i,v_{-i},c),\underline{v}_i,v_{-i},c)) - W(\Gamma^*(v,c),v,c) + v_i = v_{\kappa+1}. \tag{9}$$

Summarizing,

$$p_i^B(v,c) = \max\{c_\kappa, v_{\kappa+1}\}, \tag{10}$$

and, analogously,

$$p_j^S(v,c) = \min\{c_{\kappa+1}, v_\kappa\} \tag{11}$$

for any i and j that trade.

This double-auction has uniform prices for all buyers and for all sellers who trade. Let $p^B(v,c) = \max\{c_\kappa, v_{\kappa+1}\}$ be the price trading buyers pay and $p^S(v,c) = \min\{c_{\kappa+1}, v_\kappa\}$ be the price all sellers who produce pay. Observe that by construction, $p^S(v,c) > p^B(v,c)$. Therefore, the mechanism runs a deficit whenever trade is efficient.

4.2.2 Efficient mechanisms generate a surplus for one-sided private information

Let us next assume that sellers' costs are known, so that there is no need to give incentives to sellers to reveal their information. This does not affect buyers' incentive problem, and so each buyer who buys is still asked to pay $p_i^B(v,c)$. However, sellers who trade do not have to be paid more than their cost, and so the mechanism never runs a deficit.

Again to fix ideas, let us revisit the case where buyers have unit demand, sellers have unit capacities, and all goods are homogenous. In this case, each buyer who buys is asked to pay $p^B(v,c) = \max\{c_\kappa, v_{\kappa+1}\}$ and sellers who produce are not paid more than c_κ, and so the mechanism never runs a deficit. In the case with one seller, one object, and a cost $c_1 = 0$, the object is sold to the buyer who values it the most at the second-highest bid. Given our convention of relabeling agents, the object is sold to buyer 1 at price v_2. Notice that $W(\Gamma^*(v,c),v,c) = v_1$ and $W(\Gamma^*(\underline{v}_i, v_{-i}, c), v, c) = v_1$ for all $i \geq 2$, while $W(\Gamma^*(\underline{v}_1, v_{-1}, c), v, c) = v_2$. Therefore, revenue $R(v,c)$ is $R(v,c) = v_1 - (v_1 - v_2) = v_2$ as it should be for a second-price auction.

4.3 Implications flowing from the general case

It is remarkable that there exists a mechanism, namely the generalized Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism described above, that generates efficient outcomes in primary markets (with a surplus) and in secondary markets (with a deficit).

The efficiency of the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism is achieved at a potentially significant cost in terms of complexity because, in general, efficiency requires full combinatorial bidding, with each bidder submitting $2^K - 1$ bids. Even in moderately complex environments, say, with $K = 10$ objects, evaluating every single package accurately and placing an appropriate bid on it will in general impose too heavy a burden on bidders whose rationality is inevitably bounded. An additional concern is the computational burden the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism imposes on the mechanism designer, who has to determine the winners and the bidder-specific payments. However, in some cases, a designer might have information about the likely structure of buyer preferences, in which case it might be possible with little or no reduction in efficiency to reduce the dimensionality of the bids by limiting the set of packages on which bids may be submitted. (See, for example, the discussion of Hierarchical package bidding (HPB) hierarchical package bidding in Section 5.1.)

Even if practical consideration limit the real-world applicability of the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism, it continues to be useful as a theoretical benchmark and goal. It provides a benchmark for evaluating proposed designs that might have more straightforward implementation or, for secondary markets, that overcome the issue that the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism runs a deficit. A natural and open question is how McAfee (1992)'s almost efficient dominant strategy mechanism that never runs a deficit can be extended to this setup.

In the next section we address these issues further by considering the issue of practical implementation.

5 Practical implementation

As described above, even when it is theoretically possible to construct an efficient mechanism, there can be practical hurdles to implementation.

An early advocate of the use of auctions by the Federal Communications Commission (FCC) FCC was Coase (1959), who made the decidedly Coasian argument:

> There is no reason why users of radio frequencies should not be in the same position as other businessmen. There would not appear, for example, to be any need to regulate the relations between users of the same frequency. Once the rights of potential users have been determined initially, the rearrangement of rights could be left to the market. The simplest way of doing this would undoubtedly be to dispose of the use of a frequency to the highest bidder, thus leaving the subdivision of the use of the frequency to subsequent market transactions. (Coase (1959), p.30)

The U.S. Communications Act of 1934 (as amended by the Telecom Act of 1996) gives the Federal Communications Commission (FCC) FCC the legal authority to auction spectrum licenses. The language of the Act suggests that efficiency concerns should dominate revenue concerns in the Federal Communications Commission (FCC) FCC's auction design choices. Section 309(j) of the Act states that one objective of the auctions is "recovery for the public of a portion of the value of the public spectrum," but it also states that "the Commission may not base a finding of public interest, convenience, and necessity solely or predominantly on the expectation of Federal revenues."Thus, from the beginning, the Federal Communications Commission (FCC) FCC faced the conflicting objectives of revenue versus efficiency, [15] although efficiency concerns have been predominant in the design concerns of the Federal Communications Commission (FCC) FCC.

In this section we discuss practical implementation considerations, with a focus on primary markets, which is where the economics literature, including theoretical and experimental literature, and practice have been most developed and advanced. Issues will be similar (and possibly exacerbated) for the market making problem for a secondary market with two-sided private information.

In a theoretical sense, the problem of designing an efficient mechanism for the primary market is solved in the concept of the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism. However, the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism has proved challenging in terms of its practical implementation; see Ausubel and Milgrom (2006) and Rothkopf (2007). For efficiency to obtain, Vickrey-Clarke-Groves (VCG) mechanism VCG mechanisms require bidders to reveal their true values, but bidders may have concerns that doing so would disadvantage them in future competition. The Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism is vulnerable to Collusion collusion by a coalition of losing bidders, and history has shown that bidder Collusion collusion in spectrum license auctions is a realistic concern. The Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism is vulnerable to the use of multiple bidding identities by a single bidder, which is something that is currently permitted in Federal Communications Commission (FCC) FCC auctions, and given the possibility for complex contractual relationships among bidders, is something that might be difficult to prohibit. There are potential political repercussions for the seller if submitted bids in a Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism reveal that the winner's willingness to pay was greatly more than the actual payment. In addition, the Federal Communications Commission (FCC) FCC's revenue from Vickrey-Clarke-Groves (VCG) mechanism a VCG mechanism could be low or even zero. Finally, the efficiency of the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism is obtained by allowing combinatorial bidding, but given the number of different

[15] Jehiel and Moldovanu (2001) show that there is a conflict between efficiency and revenue maximization in multi-object auctions even with symmetric bidders.

spectrum licenses, the computational complexity associated with winner determination may be insurmountable, and, perhaps more importantly, it is unrealistic to expect bidders to be able to provide reports of their values for all of the potentially very large number of possible packages.[16]

In practice, the Federal Communications Commission (FCC) FCC has had to develop more suitable designs that accommodate these concerns, at least to some extent, while keeping in mind the objectives of efficiency and, to a lesser degree, revenue. The use of open ascending auctions reduces the complexity concerns for both the auctioneer and bidders, and is not problematic for substitute preferences, but creates "exposure risk" for bidders who perceive different objects as complements because these bidders risk receiving only parts of a package and overpaying what they receive because the package would be of more value to them than the sum of its components. It seems very plausible that bidders may have such preferences, for example, for various collections of spectrum licenses.

Concerns regarding the exposure risk problem have focused attention on the need for some kind of package bidding. Such combinatorial auction designs create issues of their own, such as bid shading or demand reduction (that is, bidders strategically bidding less than their valuations), leaving designers once again in the world of the second best. There is little theoretical guidance available associated with designs such as combinatorial clock auctions and Hierarchical package bidding (HPB) hierarchical package bidding because they are rarely addressed in the theoretical literature, presumably based on the perception that they are typically not theoretically tractable. Instead, design has relied heavily on experiments, economic intuition, and practical experience.

It does not seem very plausible that secondary markets can or should be relied upon to correct for deficiencies associated with primary market allocations; indeed, it has been argued that combining primary and secondary markets may be a source of inefficiency by inducing speculative bidding (Garratt and Tröger (2006)) and facilitating Collusion collusion (Garratt, Tröger, and Zheng (2009)).

In Section 5.1 we describe some of the choices made by the FCC in face of these challenges. In Section 5.2 we describe the role that experimental economics can play in guiding practical responses to resolving tradeoffs in market design.

5.1 FCC approach

The Federal Communications Commission (FCC) FCC acted as a primary market designer for spectrum licenses, holding auctions for those licenses since 1994; see McMillan (1994), McAfee and McMillan (1996), Cramton (1997), Kwerel and Rosston (2000), and Marx (2006). When initially developing its auction design, the Federal Communications Commission (FCC) FCC had to address the issues that it need to auction a large number of heterogeneous licenses where there were potentially strong complementarities as well as substitutability among various licenses. Existing auction mechanisms of the time did not appear well suited to address these challenges.

The Federal Communications Commission (FCC) FCC needed a mechanism that would allow bidders to shift bids between licenses they viewed as substitutes as prices changed and to bid simultaneously on licenses they viewed as complementary. As described above, a buyer's

[16] Lucking-Reiley (2000) mentions that, even in the absence of complexity, second price auctions the seller may face the problem that the bidders do not trust him to reveal the second highest bid truthfully (which was an issue with stamp collection auctions in the middle of the 20the century).

willingness to pay might be defined over packages of licenses and not independently over individual licenses.

Even in cases where the potential complexity of combinatorial bidding is not a concern and the efficient outcome can be induced by a Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism, as pointed out by Ausubel and Milgrom (2006) and Rothkopf (2007), there are practical implementation problems associated with the Vickrey-Clarke-Groves (VCG) mechanism VCG mechanism. For example, Vickrey-Clarke-Groves (VCG) mechanism VCG mechanisms require that bidders reveal their true values. Bidders may not want to do this if, for example, it weakens their bargaining position in future transactions, and the Federal Communications Commission (FCC) FCC may not be able to maintain the secrecy of bids because of the Freedom of Information Act. In addition, as stated in Rothkopf (2007) p.195, "[i]n government sales of extremely valuable assets, the political repercussions of revealing the gap between large offers and small revenue could be a dominant concern."

Ultimately, based on substantial input from Federal Communications Commission (FCC) FCC and academic economists, the FCC developed a simultaneous multiple round auction (SMR) simultaneous multiple round (SMR) auction, also known as a simultaneous ascending auction. This basic auction format, with various modifications and extensions,[17] continues to be used today. As described by the Federal Communications Commission (FCC) FCC:

> In a simultaneous multiple round auction (SMR) simultaneous multiple-round (SMR) auction, all licenses are available for bidding throughout the entire auction, thus the term "simultaneous." Unlike most auctions in which bidding is continuous, simultaneous multiple round auction (SMR) SMR auctions have discrete, successive rounds, with the length of each round announced in advance by the Commission. After each round closes, round results are processed and made public. Only then do bidders learn about the bids placed by other bidders. This provides information about the value of the licenses to all bidders and increases the likelihood that the licenses will be assigned to the bidders who value them the most. The period between auction rounds also allows bidders to take stock of, and perhaps adjust, their bidding strategies. In an simultaneous multiple round auction (SMR) SMR auction, there is no preset number of rounds. Bidding continues, round after round, until a round occurs in which all bidder activity ceases. That round becomes the closing round of the auction.[18]

The Federal Communications Commission (FCC) FCC has allowed limited combinatorial bidding at certain auctions. The Federal Communications Commission (FCC) FCC recognizes the benefits of what it refers to as package bidding: "This approach allows bidders to better express the value of any synergies (benefits from combining complementary items) that may exist among licenses and to avoid the risk of winning only part of a desired set."[19] But the Federal Communications Commission (FCC) FCC has balanced these benefits with the potential costs of complexity by limiting the set of packages on which bidders may submit bids,

[17] A number of relatively minor modifications to the original design have been made to address susceptibility to Collusion collusion by bidders. See, e.g., Brusco and Lopomo (2002), Cramton and Schwartz (2000, 2002), Kwasnica and Sherstyuk (2001), McAfee and McMillan (1996), and Marx (2006). Regarding the use of a contingent re-auction format, see Brusco, Lopomo, and Marx (2011).

[18] FCC website, http://wireless.fcc.gov/auctions/default.htm?job=about_auctions&page=2, accessed June 28, 2012.

[19] FCC website, http://wireless.fcc.gov/auctions/default.htm?job=about_auctions&page=2, accessed June 28, 2012.

generally to a set of packages with a hierarchical structure Hierarchical package bidding (HPB) ("hierarchical package bidding" or HPB).[20] The Federal Communications Commission (FCC) FCC used the SMR-HPB format in Auction 73 for C-block licenses, allowing bidding on three package ("50 States," "Atlantic," and "Pacific") as well as the individual licenses.[21] Under some reasonable restrictions on preferences, it may be possible to maintain efficiency with only restricted package bidding, such as Hierarchical package bidding (HPB) hierarchical package bidding.

The Federal Communications Commission (FCC) FCC has also used an auction design whereby the auction mechanism itself selected among multiple competing band plans. This differs from the Federal Communications Commission (FCC) FCC's standard approach of predetermining the single band plan under which licenses will be offered at auction.[22]

Wilson (1987) criticizes the mechanism design approach for focusing on the development of optimal designs for particular environments, often involving parameters that are unlikely to be known by the designer in practice, rather than identifying simple mechanisms that perform reasonably well across a variety of environments that might be encountered. Overall, the Federal Communications Commission (FCC) FCC's approach has been to do the best possible to design a primary market that balances efficiency and complexity concerns, while also recovering some of the value of the spectrum resource for taxpayers. The Federal Communications Commission (FCC) FCC has relied on secondary market transactions to address remaining inefficiencies in the initial allocation and to address the dynamic nature of the efficient allocation as demand, technology, and the set of market participants evolve over time.

5.2 Experimental approach

Given the inability of theory to provide an efficient, easily implementable mechanism that does not run a deficit, the Federal Communications Commission (FCC) FCC has requested, commissioned, and relied upon economic experiments to guide its decision making in Spectrum license spectrum license market design.

Economic experiments typically involve the use of cash to motivate subjects (often university students) to participate in an exercise that is designed to reflect some aspect of real-world markets under study. The details of the exercise would typically be transparent and carefully explained to the subjects. Subjects would make choices and be rewarded in a way designed to elicit the type of behavior one might expect from market participants motivated by profit. Data collected from these experiments, which are generally repeated many times, sometimes with the same subjects and sometimes with new subjects, is used to generate insights as to the behavior one would expect in real-world settings.[23]

For example, the Federal Communications Commission (FCC) FCC relied on experimental work by Goeree and Holt (2010) in determining whether and in what form to allow limited package bidding in FCC Auction 73 (700 MHz Auction).[24] Goeree and Holt (2010) describe their work as providing a "wind tunnel" test of three alternative auction formats under con-

[20] The hierarchical structure of HPB was suggested byRothkopf, Pekec, and Harstad (1998). The pricing mechanism for HPB was proposed by Goeree and Holt (2010).

[21] FCC website, Auction 73 Procedures Public Notice, available at http://hraunfoss.fcc.gov/edocs_public/attachmatch/DA-07-4171A1.pdf, accessed June 28, 2012.

[22] FCC website, Auction 65 Procedures Public Notice, available at http://hraunfoss.fcc.gov/edocs_public/attachmatch/DA-06-299A1.pdf, accessed June 28, 2012.

[23] See Kagel and Roth (1995) for an introduction to the methodology of experimental economics.

[24] See the FCC's Procedures Public Notice for Auction 73, available at http://hraunfoss.fcc.gov/edocs_public/attachmatch/DA-07-4171A1.pdf), accessed July 11, 2012.

sideration.

Experimental work by Brunner, Goeree, Holt, and Ledyard (2010) raise cautions regarding a package bidding format proposed by the Federal Communications Commission (FCC) FCC, referred to as SMRPB: “We find clear differences among the package formats ... both in terms of efficiency and seller revenue. ... The SMRPB auction performed worse than the other combinatorial formats, which is one of the main reasons why the Federal Communications Commission (FCC) FCC has decided not to implement SMRPB procedure for package bidding.” (Brunner, Goeree, Holt, and Ledyard (2010) p.1)

Experimental work by Banks, Olson, Porter, Rassenti, and Smith (2003) allows those researchers to suggest improvements to the details of the Federal Communications Commission (FCC) FCC’s standard simultaneous multiple round auction (SMR) SMR auction format. They also find that when license values are superadditive, the combinatorial auction outperforms the Federal Communications Commission (FCC) FCC’s standard auction format in terms of efficiency,[25] but at a cost to bidders in terms of the time required to complete the auction. In addition, they report results suggesting the combinatorial auction format considered might not perform well in certain “stress test” scenarios. Other economic experimental work that is relevant to Spectrum license spectrum license market design includes Banks, Ledyard, and Porter (1989), Porter, Rassenti, Roopnarine, and Smith (2003), Kwasnica and Sherstyuk (2001), and DeMartini, Kwasnica, Ledyard, and Porter (2005).

6 Conclusions

In this chapter, we provide the economic underpinnings for a government role in allocating spectrum licenses in the primary market and the issues that arise when one considers a role for a market maker in the secondary market. We examine when economic theory suggests market design could usefully play a role and what that role might be.

The results presented here are limited in that we focus on the case of private values.[26] In addition, we focus on dominant strategy mechanisms rather than considering the broader class of Bayesian mechanisms. However, the impossibility result of Myerson and Satterthwaite for efficient secondary market design that does not run a deficit also holds for Bayesian mechanisms, and so the result is stronger than what we have presented here.

Finally, given the swift pace of technological innovation in mobile wireless technologies, it is clear that dynamic issues are a key concern. We have focused on static mechanism design, although there is a growing literature on dynamic mechanism design; see, e.g., Bergemann and Said (2011). As our understanding of the issues and possibilities associated with dynamic mechanism design advances, there may be new possibilities to improve Spectrum license spectrum license allocation mechanisms, although one might expect that issues of complexity and other weaknesses of theoretically desirable designs will mean a continued reliance on basic economic foundations together with results of experiments, economic intuition, and practical experience.

References

Ausubel, L., and P. Milgrom (2006): “The Lovely but Lonely Vickrey Auction,” in *Combinatorial Auctions*, ed. by P. Cramton, Y. Shoham, and R. Steinberg, pp. 17–40. Cambridge

[25] Brunner, Goeree, Holt, and Ledyard (2010) also find efficiency benefits from combinatorial bidding.

[26] The limitation of the private values assumption is highlighted in Jehiel, Moldovanu, and Stacchetti (1996) and Jehiel and Moldovanu (1999).

University Press, Cambridge and London.

Banks, J., J. Ledyard, and D. Porter (1989): "Allocating Uncertain and Unresponsive Resources: An Experimental Approach," *RAND Journal of Economics*, 20(1), 1–25.

Banks, J. S., M. Olson, D. Porter, S. Rassenti, and V. Smith (2003): "Theory, Experiment and the Federal Communications Commission Spectrum Auctions," *Journal of Economic Behavior and Organization*, 51(3), 303–350.

Beckmann, M. (1965): "Edgeworth-Bertrand Duopoly Revisited," in *Operations Research Verfahren*, ed. by R. Henn, vol. III, pp. 55 – 68. Verlag Anton Hain, Meisenheim.

Bergemann, D., and M. Said (2011): "Dynamic Auctions: A Survey," *Wiley Encyclopedia of Operations Research and Management Science*.

Brunner, C., J. K. Goeree, C. A. Holt, and J. O. Ledyard (2010): "An Experimental Test of Flexible Combinatorial Spectrum Auction Formats," *American Economic Journal: Microeconomics*, 2(1), 39–57.

Brusco, S., and G. Lopomo (2002): "Collusion Via Signaling in Simultaneous Ascending Bid Auctions with Multiple Objects and Complementarities," *The Review of Economic Studies*, 69(2), 407–436.

Brusco, S., G. Lopomo, and L. M. Marx (2011): "The Economics of Contingent Re-Auctions.," *American Economic Journal: Microeconomics*, 3(2), 165–193.

Clarke, E. (1971): "Multipart Pricing of Public Goods," *Public Choice*, 11(1), 17–33.

Coase, R. (1959): "The Federal Communications Commission," *Journal of Law and Economics*, 2, 1–40.

——— (1960): "The Problem of Social Cost," *Journal of Law and Economics*, 3, 1–44.

Cramton, P. (1997): "The FCC spectrum auctions: An early assessment," *Journal of Economics and Management Strategy*, 6(3), 431–495.

Cramton, P., and J. A. Schwartz (2000): "Collusive Bidding: Lessons from the FCC Spectrum Auctions," *Journal of Regulatory Economics*, 17, 229–252.

——— (2002): "Collusive Bidding in the FCC Spectrum Auctions.," *Contributions to Economic Analysis and Policy (Article 11)*, 1(1), 1–18.

Cripps, M. W., and J. M. Swinkels (2006): "Efficiency of Large Double Auctions," *Econometrica*, 74(1), pp. 47–92.

Davidson, C., and R. A. Deneckere (1986): "Long-run competition in capacity, short-run competition in price, and the Cournot model," *RAND Journal of Economics*, 17(3), 404–415.

DeMartini, C., A. Kwasnica, J. Ledyard, and D. Porter (2005): "A New and Improved Design For Multi-Object Iterative Auctions," *Management Science*, 3(51), 419–434.

Garratt, R., and T. Tröger (2006): "Speculation in Standard Auctions with Resale," *Econometrica*, 74(3), pp. 753–769.

GARRATT, R. J., T. TRÖGER, AND C. Z. ZHENG (2009): "Collusion via Resale," *Econometrica*, 77(4), pp. 1095–1136.

GOEREE, J., AND C. HOLT (2010): "Hierarchical Package Bidding: A Paper & Pencil Combinatorial Auction," *Games and Economic Behavior*, pp. 146–169.

GRESIK, T., AND M. SATTERTHWAITE (1989): "The rate at which a simple market converges to efficiency as the number of traders increases: An asymptotic result for optimal trading mechanisms," *Journal of Economic Theory*, 48(1), 304–332.

GROVES, T. (1973): "Incentives in Teams," *Econometrica*, 41(4), 617–631.

JEHIEL, P., AND B. MOLDOVANU (1999): "Resale Markets and the Assignment of Property Rights.," *Review of Economic Studies*, 66(4), 971 – 991.

——— (2001): "A Note of Revenue Maximization and Efficiency in Multi-object Auctions.," *Economics Bulletin*, 3(2), 1 – 5.

JEHIEL, P., B. MOLDOVANU, AND E. STACCHETTI (1996): "How (Not) to Sell Nuclear Weapons.," *American Economic Review*, 86(4), 814 – 829.

KAGEL, J. H., AND A. E. ROTH (eds.) (1995): *The Handbook of Experimental Economics*. Princeton University Press.

KREPS, D. M., AND J. A. SCHEINKMAN (1983): "Quantity Precommitment and Bertrand competition yield Cournot outcomes," *Bell Journal of Economics*, 14(2), 326–337.

KRISHNA, V. (2002): *Auction Theory*. Elsevier Science, Academic Press.

KWASNICA, A. M., AND K. SHERSTYUK (2001): "Collusion via Signaling in Multiple Object Auctions with Complementarities: An Experimental Test," *Working Paper, Penn State University*.

KWEREL, E., AND G. L. ROSSTON (2000): "An Insiders' View of FCC Spectrum Auctions.," *Journal of Regulatory Economics*, 17(3), 253–289.

LEVITAN, R., AND M. SHUBIK (1972): "Price Duopoly and Capacity Constraints," *International Economic Review*, 13(1), 111–122.

LOERTSCHER, S., L. MARX, AND T. WILKENING (2013): "A Long Way Coming: Designing Centralized Markets with Privately Informed Buyers and Sellers," *Mimeo, Duke University and University of Melbourne*.

LUCKING-REILEY, D. (2000): "Vickrey Auctions in Practice: From Nineteenth-Century Philately to Twenty-First-Century E-Commerce.," *Journal of Economic Perspectives*, 14(3), 183 – 192.

MAKOWSKI, L., AND C. MEZZETTI (1993): "The Possibility of Efficient Mechanisms for Trading and Indivisible Object," *Journal of Economic Theory*, 59(2), 451–465.

MAKOWSKI, L., AND C. MEZZETTI (1994): "Bayesian and Weakly Robust First Best Mechanisms: Characterizations," *Journal of Economic Theory*, 64(2), 500 – 519.

MARX, L. M. (2006): "Economics at the Federal Communications Commission," *Review of Industrial Organization*, 29, 349–368.

Mas-Colell, A., M. D. Whinston, and J. R. Green (1995): *Microeconomic Theory*. New York: Oxford University Press.

McAfee, P., and J. McMillan (1996): "Analyzing the Airwaves Auctions," *Journal of Economic Perspectives*, 10(1), 159–175.

McAfee, R. P. (1992): "A Dominant Strategy Double Auction," *Journal of Economic Theory*, 56(2), 434–450.

McMillan, J. (1994): "Selling Spectrum Rights," *Journal of Economic Perspectives*, 8(3), 145–162.

Milgrom, P. (2004): *Putting Auction Theory to Work*. Cambridge University Press.

Myerson, R., and M. Satterthwaite (1983): "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, 29(2), 265–281.

Porter, D., S. Rassenti, A. Roopnarine, and V. Smith (2003): "Combinatorial auction design," *Proceedings of the National Academy of Sciences of the United States of America*, 100(19), 11153–11157.

Rothkopf, M. (2007): "Thirteen Reasons Why the Vickrey-Clarke-Groves Process Is Not Practical.," *Operations Research*, 55, 191–197.

Rothkopf, M., A. Pekec, and R. Harstad (1998): "Computationally Manageable Combinational Auctions," *Management Science*, 44(8), 1131–1147.

Rustichini, A., M. A. Satterthwaite, and S. R. Williams (1994): "Convergence to Efficiency in a Simple Market with Incomplete Information," *Econometrica*, 62(5), pp. 1041–1063.

Shapley, L. (1962): "Complements and substitutes in the optimal assignment problem," *Naval Research Logistics Quarterly*, 9, 45–48.

Shapley, L., and M. Shubik (1971): "The assignment game I: The core," *International Journal of Game Theory*, 1(1), 111–130.

Shubik, M. (1959): *Strategy and Market Structure*. New York: John Wiley.

Sönmez, T., and U. Ünver (forthcoming): "Market Design for Kidney Exchange," in *Oxford Handbook of Market Design*, ed. by Z. Neeman, M. Niederle, and N. Vulkan. Oxford University Press, Oxford.

Tatur, T. (2005): "On the Trade off between Deficit and Inefficiency and the Double Auction with a Fixed Transaction Fee," *Econometrica*, 73(2), pp. 517–570.

Vickrey, W. (1961): "Counterspeculation, Auction, and Competitive Sealed Tenders," *Journal of Finance*, 16, 8–37.

Wilson, R. (1987): "Bidding," *The New Palgrave of Economics*, 1, 238 – 242.