Fee-Setting Intermediaries: On Real Estate Agents, Stockbrokers, and Auction Houses*

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November 30, 2011

Abstract

Mechanisms according to which intermediaries charge commission fees and let sellers set prices are prevalent in practice. We analyze such fee-setting mechanisms within a dynamic random-matching model, in which, in every period, privately informed buyers and sellers are randomly matched with intermediaries. We show that every intermediary can achieve the maximum profit with a fee-setting mechanism. Consequently, fee setting is an equilibrium outcome. We derive the conditions for optimal fees to be linear and show that equilibrium fees decrease and become more linear in rematching frequency. The model fits many stylized facts observed in real estate brokerage.

Keywords: brokers, applied mechanism design, linear commission fees, optimal indirect mechanisms, auction houses.

JEL-Classification: C72, C78, L13

*We want to thank Alp Atakan, Yuelan Chen, Daniele Condorelli, Eddie Dekel, Winand Emons, Péter Eső, Ekaterina Goldfayn, Julio González-Díaz, Philipp Kircher, Stephan Lauermann, Ferenc Niedermayer, François Ortalo-Magné, Marco Ottaviani, Martin Peitz, Bernard Salanié, Mark Satterthwaite, Armin Schmutzler, Art Shneyerov, Dan Spulber, Adam Szeidl, Ernst-Ludwig von Thadden, Cédric Wasser, Asher Wolinsky, Abdullah Yavas, the participants of SED 2008 in Ann Arbor, the Midwest Economic Theory Meeting 2008 in Columbus, IIOC 2009 in Boston, SFB TR15 Meeting 2010 in Caputh and seminars at the Universities of Basel, Bern, Bristol, Mannheim, Melbourne, Michigan, Munich, New South Wales, Pennsylvania, Queensland, Sydney, Zurich, and Columbia, Concordia, Deakin, Georgetown and Northwestern University for their helpful comments. The second author gratefully acknowledges the hospitality of the Managerial Economics and Decision Sciences and Management and Strategy Departments at Northwestern University's Kellogg School of Management. Financial support through a research grant by the Faculty of Business and Economics at the University of Melbourne is also gratefully acknowledged.

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1 Introduction

Many markets are organized by intermediaries, and many of these intermediaries neither buy nor sell the goods whose exchange they enable. Instead, they set percentage fees to be levied on the price, which is subsequently set by the seller. Buyers then either accept or reject the price. If the mechanism involves an auction, in which the seller sets a reserve price, the buyers bid in the auction, and the fee is levied on the realized price. We call such mechanisms "fee-setting mechanisms".

Real estate brokers, stockbrokers, art galleries, and auction houses or auction sites are just a few examples of fee-setting intermediaries. Real estate brokers in the United States typically charge 5 to 6 per cent of the transaction price. Commission fees by art galleries are said to be in the range of 30 to 50 per cent. The auction houses Sotheby's and Christie's use a regressive fee structure and so does eBay.[1] Other areas, in which fee-setting mechanisms are frequently used include, stock brokerage, share-cropping in agriculture, contracts between authors and publishing companies, and the arrangements by which retailers charge producers a percentage on the revenue their products generate. Similarly, electronic payment systems and credit cards charge percentage fees. Percentage fees are also used in a slightly different environment in investment banking[2] and by labor market intermediaries, in particular by head hunters.

Industries, in which fee-setting mechanisms are predominantly used are quite sizeable. For example, the sales generated by Sotheby's in 2007 alone exceeded USD 4 billion.[3]

[1] The marginal rate at Sotheby's is 25 per cent for items with prices of up to USD 20,000, 25 per cent for between USD 20,000 and USD 500,000 and 12 per cent beyond that. At eBay (ebay.com, accessed on May 5, 2008) the marginal fee on the closing price is 8.75 per cent for below USD 25, 3.5 per cent for between USD 25 and USD 1000, and 1.5 per cent for above USD 1000. Sotheby's and Christie's charged a linear fee of 20 per cent prior to being investigated by the U.S. Department of Justice, convicted of collusive behavior, and induced to change the fee structure. Similarly, real estate brokerage has come under scrutiny by the U.S. Department of Justice (Department of Justice, 2007). There is a widespread, though rarely explicit, suspicion that, in particular, the almost complete invariance of broker commission fees reflects collusive behavior.

[2] Underwriters on initial public offerings in the U.S. charge, in most cases, exactly 7 per cent (see Chen and Ritter, 2000). Stoughton, Wu, and Zechner (2011) analyze competition between percentage fee-setting intermediaries in investment management, taking as given that intermediaries use percentage fees.

[3] See Sotheby's Annual Report 2007, p. 28, available at Sothebys.com. The fee structure was reported in the New York Times (2008).

The annual operating revenue of eBay was more than USD 7.5 billion in 2007, and Christie's annual sales in 2006 exceeded USD 4.5 billion.[4] The real estate brokerage industry in the U.S. generates annual sales beyond USD 1000 billion and commission fees of more than USD 60 billion per year.[5] Credit card companies are also big business. For example, MasterCard's annual revenue in 2007 exceeded USD 4 billion.[6]

Despite their widespread use and economic significance, fee-setting mechanisms have received very little attention in the theoretical economic literature. In particular, no prior analysis of the optimality of fee setting and the structure of fees from a mechanism design perspective exists. The purpose of this paper is to provide insight into why intermediaries choose to set commission fees and, when they do, what determines the size and form of those fees.

This paper makes two main contributions. First, we set up a dynamic random-matching model with a continuum of buyers, sellers and intermediaries. We derive the exchange mechanism of every intermediary as the endogenous solution of a mechanism design problem that depends on the distributions of the types of buyers and sellers, which are, in turn, endogenous to the choice of mechanisms by all intermediaries. Second, we analyze and characterize fee-setting mechanisms in the stage game of the dynamic model, which can be seen as a one period model on its own.

The following is a description of the dynamic model. Buyers and sellers have private information about their valuations for an indivisible homogeneous good. In every period, one buyer, one seller, and one intermediary are randomly matched. Intermediaries are free to choose the trading mechanism anew in every period. The equilibrium mechanism used by the intermediaries in the market determines the option value of future trade and hence the endogenous reservation values of buyers and sellers. The distributions of these values in turn determine the best response mechanism of an intermediary. Focusing on a steady-state equilibrium in which every intermediary uses the same stationary mechanism, we show that the equilibrium mechanism does not vary with the number of in-

[4] See www.marketwatch.com and www.sgallery.net, respectively.

[5] See Rutherford, Springer, and Yavas (2005).

[6] MasterCard Worldwide, Annual Report 2007. For more evidence on credit card charges, see Shy and Wang (2011).

termediaries under standard assumptions on the matching technology (Atakan, 2006a,b; Shneyerov and Wong, 2010), which is consistent with empirical observations (see for example Hsieh and Moretti, 2003). Equilibrium fees decrease as the matching frequency increases, or equivalently the period length between subsequent rematchings decreases.

In an extension, we study inefficient free entry by intermediaries whose opportunity costs of entry and levels of ability may be heterogeneous. We show also that, in the dynamic model, a direct seller (who sells without a broker) may charge a higher price than an indirect seller (who sells through a broker). This provides an explanation for an empirically observed regularity (see Rutherford, Springer, and Yavas, 2005; Levitt and Syverson, 2008; Hendel, Nevo, and Ortalo-Magné, 2009).

Our paper contributes to the large and growing literature on intermediation, including Rubinstein and Wolinsky (1987), Gehrig (1993), Yavas (1992), Spulber (1996, 1999), Rust and Hall (2003), Duffie, Garleau, and Peddersen (2005), Jullien and Mariotti (2006), Hagiu (2007), Shy and Wang (2011), and Antràs and Costinot (forthcoming), by adding a mechanism design perspective to the notion of (dynamic) random matching, present in most of these papers.[7] Apart from Myerson and Satterthwaite (1983) the only authors applying mechanism design to intermediation we are aware of are Spulber (1988), Jullien and Mariotti (2006), and Matros and Zapechelnyuk (2008). Our paper differs from these because it uses dynamic random matching, involves multiple competing sellers and intermediaries, and makes predictions on price dispersion, fee structures, and time on market.

As we add intermediaries to a dynamic random-matching model with incomplete information similar to Wolinsky (1988), Satterthwaite and Shneyerov (2007, 2008), and Atakan (2006b), our paper also relates to this strand of literature.[8] Insofar as the inter-

[7]Yavas (1992) and Hagiu (2007) provide explanations of when intermediaries may use percentage fees and when they may set prices. Hagiu's argument relies on the presence and nature of network externalities. Yavas' explanation depends, among other things, on agents' search intensity, which is endogenous in his model. Shy and Wang (2011) analyze whether credit card companies are better off with percentage fees than with fixed per transaction fees. Jullien and Mariotti (2006) derive an intermediary-optimal trading mechanism in the context of a common value auction with two buyers and one seller, which is similar to the stage game mechanism in our model.

[8]Without modeling the intermediary Coles and Muutho (1998) provide a dynamic matching model on an intermediary platform, in which prices are either determined via Nash bargaining or via Bertrand-type price competition, the bargaining procedure depending on the degree of competition. For an empirical

mediaries in our dynamic model are competing mechanism designers, this paper is related to the work of McAfee (1993), who studies mechanism design by competing sellers.[9]

Our paper also contributes to the literature on real estate economics. Most of the literature analyzing real estate brokerage remains in the principal-agent framework, in which the seller, and occasionally the buyer, is the principal and the broker the agent. Arnold (1992) offers a theoretical discussion of this topic, while Rutherford, Springer, and Yavas (2005) as well as Levitt and Syverson (2008) conduct empirical work. However, the use of percentage fees by intermediaries is hard to explain within a principal-agent framework.[10] In contrast, our dynamic model not only provides an explanation for why intermediaries use fee-setting mechanisms, but is also consistent with a number of stylized facts in real estate markets, as discussed in Section 4.2 below.

In the static model (or, equivalently the stage game of the dynamic model), proportional fees are profit-maximizing mechanisms for brokers when the seller's supply function exhibits a constant price elasticity. Furthermore, these profit-maximizing fees are lower the higher the elasticity of supply is. Of course, this raises the questions of under what conditions constant elasticities are a reasonable approximation for the optimal mechanism, and what determines the magnitude of the elasticity of supply. These questions are addressed in the dynamic model, in which we show that dynamics has a tendency to increase the elasticity of supply and – under certain conditions that are spelled out below – to make the constant elasticity a better approximation. We provide analytical results for the stage game. For the dynamic model we derive analytical results for the partial first-order effect of a move from a static to a dynamic model. We further provide numerical results for the equilibrium total effect of more dynamics. However, the questions of what ratio of the highest possible profits can be achieved with proportional fees

analysis of search and matching in the housing market, see Genesove and Han (2010).

[9]McAfee (1993, p.1304) notes that [his] "paper falls far short of a real theory of equilibrium institutions partly because it places the design of institutions in the hands of the sellers. A more satisfactory approach requires explicit modelling of the role of intermediaries, or auctioneers, who compete among each other for both buyers and sellers."

[10]The quantitative appropriateness of percentage fees as an incentive device for brokers acting on a seller's behalf has recently been questioned by Rutherford, Springer, and Yavas (2005) and Levitt and Syverson (2008). For brokers who act on the behalf of a buyer, percentage fees that are levied on the transaction price cannot be explained as an incentive device.

is an empirical one. We address this in our empirical companion paper (Loertscher and Niedermayer, 2011), which provides strong empirical support for the theoretical model.

The remainder of this paper is structured to reflect this line of reasoning. Section 2 introduces the basic model. Section 3 derives equilibrium results for the static and the dynamic model. Section 4 provides additional results that are of particular relevance to real estate brokerage, and discusses the model's connection with empirical stylized facts from this industry. Section 5 discusses alternative applications and mechanisms, including auction houses, stockbrokers, price posting and slotting allowances. Section 6 concludes the paper. All proofs are provided in the Appendix, and so is additional material that is not intended for publication.

2 The Model

We study the infinite horizon model of which Figure 1 illustrates one period. In every period, a mass 1 of buyers and a mass 1 of sellers consider entering the market. The valuation $\tilde{v}$ for the good of an entering buyer $\tilde{v}$ is drawn from the distribution $\tilde{F}_0$ with strictly positive density $\tilde{f}_0$ on the support $[\underline{\tilde{v}}_0, \bar{\tilde{v}}_0]$. Each entering seller has one unit of an indivisible good. His cost of selling the good $\tilde{c}$ is drawn from $\tilde{G}_0$ with $\tilde{g}_0 > 0$ on $[\underline{\tilde{c}}_0, \bar{\tilde{c}}_0]$.[11] Both valuations and costs are private information, whereas the distributions are common knowledge. We refer to $\tilde{v}$ and $\tilde{c}$ as a buyer's and seller's static type (that is, static valuation and cost, respectively).

Buyers enter a pool with mass σ. The endogenous distribution of the valuations of buyers in this pool is $\tilde{F}(\tilde{v})$ with support $[\underline{\tilde{v}}, \bar{\tilde{v}}]$. Similarly, sellers enter a pool of mass σ, and the (endogenous) distribution of their costs is $\tilde{G}(\tilde{c})$ with support $[\underline{\tilde{c}}, \bar{\tilde{c}}]$. We restrict our attention to equilibria in which the market is in steady state, that is, the traders entering the market (pool) have the same mass and distribution of types as those who leave. We assume that buyers and sellers who are certain that they cannot trade do

[11] Throughout the paper, we refer to a buyer as a "she" and to a seller and an intermediary as a "he". For brevity, a seller's valuation of the good, or his opportunity cost of selling the good, is called his cost. This makes explicit that the model also applies to settings where the good has to be produced by the seller at a cost.

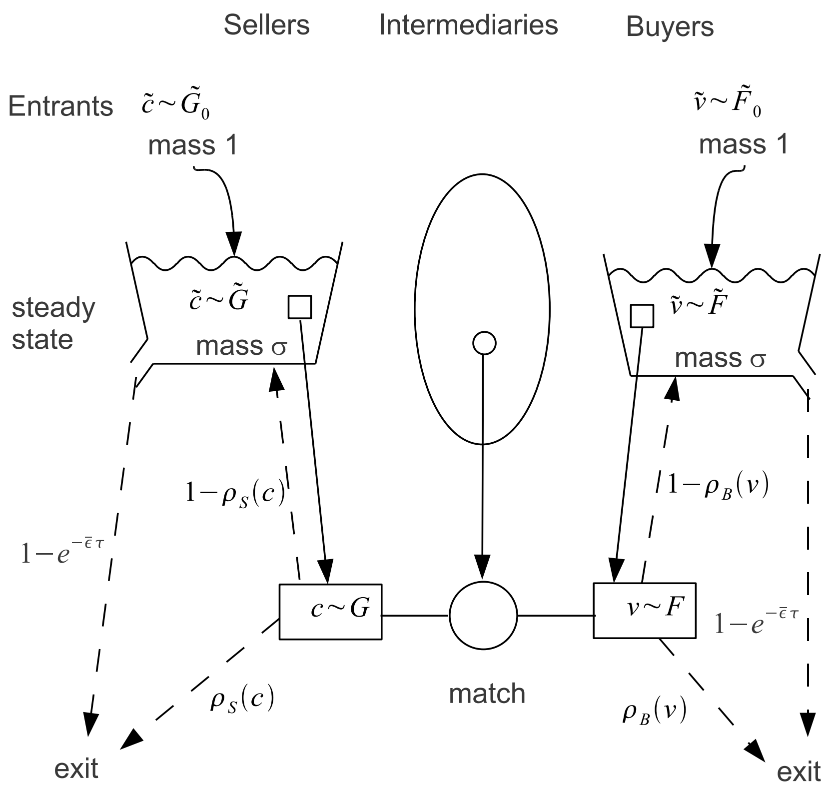

Figure 1: Market in steady state. In each period, mass 1 of traders with distributions $\tilde{F}_0$ and $\tilde{G}_0$ enter the market and join pools with distributions $\tilde{F}$ and $\tilde{G}$. Traders have dynamic type distributions F and G. With probabilities $\rho_B(v)$ and $\rho_S(c)$, they leave the market because they trade, with probability $1-e^{-\bar{\varepsilon}\tau}$, they leave the market for exogenous reasons.

not enter the market.[12] Therefore, $\tilde{G}$ and $\tilde{F}$ are the steady-state cumulative distribution functions. Their densities are denoted with $\tilde{f}$ and $\tilde{g}$. There is an unlimited supply of intermediaries standing ready to offer their services. In each period, each buyer, each seller, and an intermediary are uniform random matched[13] in a triple consisting of one member from each of the three groups. All agents are risk neutral, and preferences are quasilinear, that is, when trading at price p a buyer whose static valuation is $\tilde{v}$ gets the instantaneous net utility $\tilde{v} - p$ and a seller with static cost $\tilde{c}$ enjoys a net payoff of $p - \tilde{c}$.

Buyers and sellers who do not trade stay in the market with the exogenous probability $e^{-\bar{\varepsilon}\tau}$ until the next period, where τ represents the length of a period and $\bar{\varepsilon}$ is the flow

[12]This assumption is in line with the literature and equivalent to assuming that the inequalities in Condition 1 below are binding in steady state.

[13]This matching technology is essentially the same as in Atakan (2006b). It differs from Satterthwaite and Shneyerov (2007, 2008) who assume a seller is matched with zero, one, or many buyers.

exit rate. With probability $1 - e^{-\overline{\varepsilon}\tau}$, a trader drops out of the market and has utility 0. For simplicity, assume that intermediaries stay in the market forever.[14] Future utility is discounted with the factor $e^{-(\overline{\delta}-\overline{\varepsilon})\tau}$, where $\overline{\delta} - \overline{\varepsilon}$ is the pure rate of time preference. Therefore, $e^{-\overline{\delta}\tau}$ is the total discount factor. The period length τ can be interpreted as the extent of frictions in the market or as a parameter of the degree of competition: the shorter the time of a new match after a failed trade, the more fiercely intermediaries compete. To simplify notation, we also write $\varepsilon := e^{-\overline{\varepsilon}\tau}$ for the probability of remaining and $\delta := e^{-\overline{\delta}\tau}$ for the total discount factor.

In the dynamic model, buyers and sellers can trade in the future with positive probability if trade fails in the present. Therefore, for given mechanisms employed by intermediaries a buyer whose static type is $\tilde{v}$ has a maximal willingness to pay $B(\tilde{v}) = \tilde{v} - \delta W_B(\tilde{v})$, where $W_B(\tilde{v})$ is the buyer's option value of continuing. Following the literature, we call this lowered willingness to pay interchangeably dynamic valuation or dynamic type. Similarly, a seller with static type $\tilde{c}$ will have a dynamic cost $S(\tilde{c}) = \tilde{c} + \delta W_S(\tilde{c})$.[15] In the following, we will use dynamic valuation of the buyer $v = B(\tilde{v})$ and the dynamic cost of the seller $c = S(\tilde{c})$, which of course remain to be determined, to derive the endogenous dynamic distributions $F(v)$ and $G(c)$ with densities f and g. The dynamic distributions have to satisfy $F(B(\tilde{v})) = \tilde{F}(\tilde{v})$ and $G(S(\tilde{c})) = \tilde{G}(\tilde{c})$.

We assume that intermediaries have all the instantaneous bargaining power. That is, matched to a buyer and seller in a given period an intermediary chooses a mechanism that maximizes his expected profit subject to the buyer's and seller's individual rationality and incentive compatibility constraints. The relevant distributions an intermediary has to take into account when designing his exchange mechanism are the distributions of buyers' and sellers' dynamic types F and G. Note that from the point of view of an intermediary this is equivalent to a one-shot game because the probability that he will meet the same buyer or the same seller in a subsequent period is zero and because he takes

[14] Alternatively, and without affecting any of the results, we could assume that intermediaries arrive and exit each period randomly.

[15] Put differently, the fact that there is a future drives a positive (negative) wedge between a buyer's (seller's) static type and her (his) dynamic type. Since it can be shown that both B and S are strictly increasing for active traders in any equilibrium, we can work directly with the dynamic types v and c.

the mechanisms offered by other intermediaries in subsequent periods as given. Though the individual rationality and incentive compatibility constraints are endogenous to the game, they are exogenous for every intermediary.

Summarizing, there are three types of distribution in our model: static entrant $\tilde{F}_0$ and $\tilde{G}_0$, static steady-state $\tilde{F}$ and $\tilde{G}$, and dynamic steady-state F and G. The distributions $F(v)$ and $G(c)$ and the mechanisms employed by the intermediaries determine the probabilities $\rho_B(v)$ and $\rho_S(c)$ that a buyer v and a seller c trade in a given period. Since the exogenous per period exit probability is $1 - e^{-\bar{\varepsilon}\tau}$, the probabilities of staying in the market from one period to the next are $(1-\rho_B(v))e^{-\bar{\varepsilon}\tau}$ and $(1-\rho_S(c))e^{-\bar{\varepsilon}\tau}$. The steady-state mass of agents in the market is σ and the mass of entrants is 1. In a steady-state, the mass and distribution of entering buyers and sellers has to be equal, formally

$$\tilde{f}_0(\tilde{v}) = \sigma[1-(1-\rho_B(B(\tilde{v})))e^{-\bar{\varepsilon}\tau})]\tilde{f}(\tilde{v}) \quad \text{and} \quad \tilde{g}_0(\tilde{c}) = \sigma[1-(1-\rho_S(S(\tilde{c})))e^{-\bar{\varepsilon}\tau})]\tilde{g}(\tilde{c}). \quad (1)$$

3 Equilibrium

We now turn to the equilibrium analysis of the model. We begin with the stage game, which corresponds to a static, one-shot game, in which traders drop out after one period ($\delta = \epsilon = 0$). The stage game can be seen as a static model and is of interest of its own.

3.1 The Stage Game

In the stage game, the distributions of costs and valuations are the exogenously given primitives of the model. The intermediary is a monopolist who faces a buyer and a seller whose valuation v and cost c for a homogenous good of known quality is private information and independently drawn from distributions F and G, which have strictly positive densities f and g with supports $[\underline{v}, \bar{v}]$ and $[\underline{c}, \overline{c}]$, respectively. We assume that the intermediary has all the bargaining power and chooses the trading mechanism, subject to the individual rationality and incentive compatibility constraints of the buyer and the seller. Informally, a mechanism is the following. First, the mechanism designer (here the intermediary) offers a menu of possible actions to the seller and the buyer, announcing for each combination of actions the payments a participant pays or receives and whether the good is exchanged. Observing this menu, both seller and buyer then pick actions that are mutual best responses.[16] For the example of fee setting, the seller's choice of actions is the choice of which price to ask for. The buyer chooses between the actions of accepting or rejecting the offer.

The intermediary's expected profit under some mechanism is the expected equilibrium payment by the buyer minus the expected equilibrium payment to the seller. A mechanism is intermediary-optimal if there is no mechanism that gives strictly higher expected profits to the intermediary, subject to the constraints that the buyer and seller want to participate and that the strategies of the buyer and the seller constitute a (Bayes) Nash equilibrium of the game.

Denote by $\Phi(v) := v - (1 - F(v))/f(v)$ the buyer's virtual valuation function. Analogously, let the seller's *virtual cost function* be denoted as $\Gamma(c) := c + G(c)/g(c)$. The intermediary's mechanism design problem is said to be regular if both virtual type functions are increasing. To simplify notation, in particular dealing with the inverses Φ^{-1} and Γ^{-1}, we focus on equilibria that satisfy the following two conditions. First,

Condition 1. $\Phi(\bar{v}) \leq \Gamma(\bar{c})$ *and* $\Phi(\underline{v}) \leq \Gamma(\underline{c})$.

Second, F and G satisfy Myerson's regularity condition, that is, Φ and Γ are increasing.[17] As it will be clear later on, Condition 1 means that we focus on purely non-full-trade equilibria, that is, equilibria in which no-one trades with probability one. The general mechanism design problem, for which these conditions do not necessarily hold, can also be dealt with by standard techniques such as ironing and a slight modification of the definition of the inverses (see Appendix B).

Fee-Setting Mechanisms The focus of this paper is on the following type of indirect mechanisms, which we call *fee-setting* mechanisms.

The intermediary first announces a fee function $\omega(.)$ that determines the amount the intermediary receives upon successful sale at price p, leaving $p - \omega(p)$ to the seller. Then

[16] A more detailed explanation of these concepts is, for example, provided by Krishna (2002, Chapter 5). Myerson and Satterthwaite (1983) introduce the formal mechanism design problem for an intermediary who faces one buyer and one seller with private information and independent types.

[17] This implies a convex optimization problem for the intermediary.

the seller sets the price p, knowing ω and his own cost c. Finally, observing p and her own valuation v, the buyer accepts or rejects the offer p and the game ends. If the buyer accepts, the seller receives the net price $p - \omega(p)$ and the intermediary collects the fee $\omega(p)$. Notice that in a fee-setting mechanism, payments occur only if there is trade. Further, the buyer will accept if and only if $v \geq p$.

Our assumption that the seller pays the fee is without loss of generality, that is, how the intermediary's fee $\omega(p)$ is allocated between the buyer and the seller does not matter. We use the convention that an equilibrium is *essentially unique* if all seller and buyer types who trade with positive probability in some equilibrium take the same action in every equilibrium.[18] Similarly, a fee structure is said to be *essentially unique* if it is uniquely pinned down for all those prices that are accepted by some type of buyer with positive probability.

The Simple Economics of Optimal Intermediation The buyer's virtual valuation $\Phi(v)$ can be interpreted as the marginal revenue of increasing the probability of trade and the seller's virtual cost as marginal cost.[19] Therefore, the intermediary wants the seller and the buyer to trade if and only if marginal revenue exceeds marginal cost, that is, whenever $\Phi(v) \geq \Gamma(c)$. As shown by Myerson and Satterthwaite (1983), this is the optimal allocation rule for the intermediary.[20] Due to payoff equivalence (see, for example, Krishna, 2002), once the allocation rule is determined, the equilibrium expected payoffs for all player types are determined up to an additive constant. It is in the intermediary's interest to minimize this constant, subject to the individual rationality constraint that all types of buyers and sellers are willing to participate in the

[18]Put differently, an equilibrium is essentially unique if equilibria only differ with respect to actions of types who never trade in any equilibrium.

[19]This intuition is due to Bulow and Roberts (1989): interpret the probability that $\tilde{V} \geq v$ and $\tilde{C} \leq c$ as quantity demanded and supplied q, that is, $q := 1 - F(v)$ and $q := G(c)$. Thus, the inverse demand and supply functions are $v = F^{-1}(1-q)$ and $c = G^{-1}(q)$, yielding $R(q) := qF^{-1}(1-q)$ and $C(q) := qG^{-1}(q)$ as revenue and cost functions, respectively. Taking derivative w.r.t. q and substituting back in yields $R'(q) = \Phi(v)$ and $C'(q) = \Gamma(c)$.

[20]Myerson and Satterthwaite (1983) are almost exclusively cited for their impossibility results, rather than their results on profit-maximizing intermediaries. A notable exception is Spulber (1999, Chapter 7), who compares the optimal direct mechanism of Myerson and Satterthwaite with price posting by the intermediary.

mechanism. Therefore, under the intermediary optimal mechanism, the worst off agents are indifferent between participating and not. In other words, the lowest valuation buyer $\underline{v}$ and the least efficient seller $\overline{c}$ receive expected payoffs of zero. See Lemma 1 in Appendix A for a summary and formalization of these results.

Intermediary Optimal Fee-Setting Mechanism We now show that an intermediary optimal fee-setting mechanism exists. For notational ease, we define $P(c) := \Phi^{-1}(\Gamma(c))$. If a fee function $\omega(p)$ exists such that the seller of type c optimally sets the price $p = P(c)$, the fee-setting mechanism with the fee $\omega(p)$ will be intermediary optimal, since it will induce trade if and only if $\Phi(v) \geq \Gamma(c)$.

An intuitive derivation of the optimal fee-setting mechanism can be obtained by first looking at a dominant strategy direct mechanism implementation.[21] The dominant strategy implementation is that the intermediary asks agents to report their types and allows trade if and only if $v \geq P(c)$ (or equivalently $c \leq P^{-1}(v)$). If trade occurs, the buyer pays $P(c)$ and the seller receives $P^{-1}(v)$.[22] Therefore, conditional on trade, the seller of type c receives $E_v[P^{-1}(v)|v \geq P(c)]$ in expectations over v. Since the seller is risk neutral, the intermediary could alternatively pay the seller the expected value as the net price $P(c)-\omega(P(c))$. Equating the net price $P(c)-\omega(P(c))$ with the seller's expected payoff and replacing $P(c)$ with p gives the optimal fee as stated in Proposition 1.

Proposition 1. *An intermediary optimal fee-setting mechanism exists and is essentially unique. The optimal fee function is*

$$\omega(p) = p - E_v[P^{-1}(v) \mid v \geq p] \tag{2}$$

for $p < \bar{v}$ *and an arbitrary* $\omega(p) > \overline{v} - p$ *for* $p > \bar{v}$*. A seller with cost* $c \leq P^{-1}(\overline{v})$ *sets the price* $p = P(c)$ *and a buyer with valuation* v *accepts if and only if* $p \leq v$*. Moreover, the*

[21] A direct mechanism requires participants to report their valuations to the mechanism designer who will take actions for them rather than their having to take actions themselves. A dominant strategy direct mechanism means that it is a dominant strategy (that is, optimal independently of the other agent's actions) for every participant to report his type truthfully.

[22] The buyer gets a take-it-or-leave-it offer at price $P(c)$. Accepting the offer if and only if $v \geq P(c)$ is clearly a dominant strategy. Similar arguments apply for the seller. This dominant strategy implementation is already mentioned in Myerson and Satterthwaite (1983) after Theorem 4.

Nash equilibrium of the stage game implied by this fee-setting mechanism is essentially unique.

Notice that the expected profit of a seller of type $c \leq P^{-1}(\overline{v})$ who sets the price p is $(E_v[P^{-1}(v) \mid v \geq p] - c)(1 - F(p))$. Since the unique maximizer p of this profit function satisfies $P^{-1}(p) = c$, the fee function in (2) induces the intermediary optimal allocation rule. Moreover, because the best response of any seller who trades with positive probability is unique, the equilibrium is essentially unique, and so is the intermediary optimal fee function.

Note that this fee-setting mechanism is optimal in the general class of mechanisms, that is, the intermediary cannot do better by any other mechanism than fee setting. Taking the derivative of (2) and rearranging by partial integration reveals that the marginal fee ω' can never be higher than 100 per cent. This is of course what one would expect from incentive compatibility. Interestingly, $\omega'(p) < 0$ is not impossible a priori.[23] Further, we can derive meaningful lower and upper bounds for the fee function ω. Define the convex hull $\underline{\Gamma}$ of the function Γ as the largest convex function below Γ (see Myerson (1981) for a formal definition). Similarly, let $\overline{\Gamma}$ be the "concave hull" of Γ, that is, the lowest concave function above Γ. In addition, for any $\alpha > 1$ let $G_\alpha(c) := G(c)^\alpha$ be a power transformation of the distribution $G(c)$ and denote the virtual cost function associated with this transformed function by Γ_α. Observe that $\Gamma_\alpha(c) = (1 - \frac{1}{\alpha})c + \frac{1}{\alpha}\Gamma(c)$. One natural interpretation for $G_\alpha(c)$ in a static model is that the seller's opportunity cost of selling c is due to a (simple take-it-or-leave-it) price offer that he receives from a source outside the brokerage market.[24] If one price offer is drawn from $G(c)$, then the best offer of α independently and identically distributed offers is drawn from $G_\alpha(c)$. Observe also that the price elasticity of supply is $\eta_s(c) := cg(c)/G(c)$.[25] From this angle, the power transformation of a distribution can be seen as a proportional rescaling of its

[23] Notice that this does not violate incentive compatibility of the seller because the net price $p - \omega(p)$ is only one part of the seller's payoff, the probability of of sale $1 - F(p)$ being the other one.

[24] For a static model with $\epsilon = \delta = 0$ the endogenous distribution $G(c)$ and the exogenous distribution $\tilde{G}_0(c)$ coincide (save for issues of entry). Therefore, comparative statics with respect to $G(c)$ has a direct interpretation.

[25] To see this, interpret the probability that a seller sells at price p as "quantity" $q(p) := G(p)$. The price elasticity of supply at price p is thus defined as $\eta_s(p) := q'(p)p/q(p) = pg(p)/G(p)$.

elasticity, since $\eta_{s,\alpha}(c) := cg_\alpha(c)/G_\alpha(c) = \alpha\eta_s(c)$, where $\eta_{s,\alpha}(c)$ is the elasticity of supply associated with $G_\alpha(c)$.

Proposition 2. *(i) The fee function ω has the convex lower bound $p - \underline{\Gamma}^{-1}(p)$ and the concave upper bound $p - \overline{\Gamma}^{-1}(p)$, that is, $p - \underline{\Gamma}^{-1}(p) \leq \omega(p) \leq p - \overline{\Gamma}^{-1}(p)$. (ii) The difference between the bounds $[p - \overline{\Gamma}_\alpha^{-1}(p)] - [p - \underline{\Gamma}_\alpha^{-1}(p)]$ decreases with α and goes to 0 as $\alpha \to \infty$ for all p. Further, $|\Gamma_\alpha''(c)|$ decreases with α.*

Proposition 2 shows that increases in α make supply more elastic and at the same time make Γ_α more linear in the sense of reducing the difference between the convex lower bound and concave upper bound of the optimal fee function associated with Γ_α. In the analysis of the dynamic model below, we identify explicit factors that have a similar effect as do increases in α in the static model.

Auction theory offers a clear economic interpretation for the inverse virtual cost function $\Gamma^{-1}(p)$, since this is the optimal reservation price set in a procurement auction by a buyer with valuation p. Equivalently, this is the price a monopsonist with valuation p would offer to a seller with supply function $G(c)$. If the valuation v of the buyer were public rather than private, the broker would charge the buyer the gross price $p = v$ and pay the seller the net price $\Gamma^{-1}(p)$, resulting in a fee $p - \Gamma^{-1}(p)$. The net price $p - \omega(p)$ is between $\overline{\Gamma}^{-1}(p)$ and $\underline{\Gamma}^{-1}(p)$ by Proposition 2. If Γ is convex, then $\underline{\Gamma} = \Gamma$ and the net price is below $\Gamma^{-1}(p)$, therefore, the seller receives less than with certainty about the buyer's valuation. Consequently, fees $\omega(p)$ are higher than with public information about v. The opposite holds for Γ concave: fees $\omega(p)$ are lower than with public information.

Note that the lower and upper bounds for the fee function $\omega(p)$ provided by Proposition 2 are independent of F. Therefore, the impact of F on $\omega(p)$ is limited to where $\omega(p)$ lies within the bounds. This limited effect of F on $\omega(p)$ suggests a closer look at the supply side, which is what we do now. Keeping F fixed, we let ω_α be the fee associated with the distribution G_α, and we denote by ω and η_s ($\hat{\omega}$ and $\hat{\eta}_s$) the fee and elasticity of supply associated with the distribution G ($\hat{G}$), respectively. Proposition 3 shows that fees decrease, moving toward zero, as elasticity increases.

Proposition 3. *Keep F fixed. (i) The fee $\omega_\alpha(p)$ decreases with α and goes to zero as $\alpha \to \infty$ for all p. (ii) Consider two markets with identical distributions of buyers F, in which the sellers' distributions are G and $\hat{G}$, respectively. If $\eta_s(c) < \hat{\eta}_s(c)$ for all c, then $\omega(p) > \hat{\omega}(p)$ for all p.*

Define $\Gamma_L(c)$ as the linear function connecting $(\underline{c}, \Gamma(\underline{c}))$ and $(\overline{c}, \Gamma(\overline{c}))$. Both $\Gamma^{-1}(p)$ and $\Gamma_L^{-1}(p)$ are in the interval $[\underline{\Gamma}^{-1}(p), \overline{\Gamma}^{-1}(p)]$ for all p. Therefore, as α increases and the gap between the bounds decreases, the upper bound for the distance between $\omega(p)$ and the (linear) function $p - \Gamma_L^{-1}(p)$ decreases. Note further that for Γ linear, $\underline{\Gamma}$ and $\overline{\Gamma}$ coincide, and so do the bounds for ω. Hence, the linear bounds imply a linear ω. The linearity of Γ is actually a necessary and sufficient condition for the linearity of the optimal fee, as shown in the next proposition.

Proposition 1 also implies that the intermediary can achieve his maximal expected profit without knowing or making use of the buyer's valuation when determining payments in case of trade. The buyer's valuation is only needed to determine whether the good is traded. However, since in general this depends on the buyer's distribution as well, the optimal fee-setting mechanism will in general depend on F. Therefore, it is rather striking that for a certain family of distributions of seller's types, namely all those that exhibit virtual costs that are linear in c, the optimal fee charged by the intermediary is independent of F and linear:

Proposition 4 (Optimality of Linear Fee Mechanisms)**.** *The following are equivalent statements:*

(i) a linear fee mechanism is optimal, that is, $\omega(p) = \xi p + \zeta$ is intermediary optimal,

(ii) c is drawn from a generalized power distribution $G(c) = \left(\frac{c-\underline{c}}{\overline{c}-\underline{c}}\right)^\alpha$ with $\alpha > 0$,

where $\xi = 1/(\alpha + 1)$ and $\zeta = -\underline{c}/(\alpha + 1)$ holds.

For example, if G is uniform on $[0, 1]$, then $\Gamma(c) = 2c$ and $\omega(p) = p/2$. To get 6 per cent as the optimal fee, one needs a power distribution with $\alpha \approx 15$, which can be seen as a transformation of a uniform distribution G with $\underline{c} = 0$ into $G_\alpha = G^\alpha$. However, beginning with a non-uniform rather than a uniform G will not make that much of a difference, since $\Gamma_\alpha(c) = (1/15)\Gamma(c) + (14/15)c$ is a convex combination of $\Gamma(c)$ and c that puts most of the weight on c.

A power distribution allows for a simple economic interpretation of the driving force behind the fees. Note that a power distribution with support $[0,1]$ – that is, $G(c) = c^\alpha$ – corresponds to a constant elasticity of supply of sellers $\eta_s(c) = \alpha$ for all c. Proposition 4 tells us that the more elastic the supply, the lower the fees charged to the seller. Some algebra reveals that for support $[0,1]$ the expression for the optimal fee $\xi = 1/(1+\alpha)$ from Proposition 4 corresponds to a procurement version of the standard Lerner formula (that is, for a monopsonist rather than a monopolist)[26]

$$\frac{MR - P}{P} = \frac{1}{\eta_s},$$

with marginal revenue MR being the gross price charged to the buyer ($MR = p$) and the procurement price P being the net price paid to the seller ($P = (1-\xi)p$). If we assume further that c stems from the best of n outside price offers that are independent draws from a power distribution $G(c) = c^{\alpha_0}$ in a static model, we get a simple and intuitive relation between the number of outside offers n, the elasticity of supply and the level of fees: the best of the n outside offers will be drawn from $c^{n\alpha_0}$, whose elasticity is $\eta_s(c) = n\alpha_0$. Therefore, the optimal fee is $\xi = 1/(n\alpha_0 + 1)$. As the number of outside offers n increases, so does the elasticity η_s, which leads to lower fees ξ. Note that while the intuition is clearest for power distributions, Proposition 2 (ii) shows that there is an underlying principle that goes beyond power distributions: the more outside offers sellers get, the lower the fees.

The optimal intersect $\zeta = -\underline{c}/(1+\alpha)$ can be seen as a means of making sure that a seller with the lowest cost $\underline{c}$ would get a zero fee if he were to set a price equal to his cost (that is, $\omega(\underline{c}) = \xi\underline{c} + \zeta = 0$.)

It is remarkable that for power distributions of the seller's cost, the elasticity of supply (together with $\underline{c}$) alone determine the fee, with the buyer's distribution F having

[26] The standard Lerner formula $(P - MC)/P = -1/\eta_d$ with marginal costs MC and elasticity of demand η_d is derived from the monopolist's problem $\max_P (P - MC)D(P)$ for demand $D(P)$. The corresponding "procurement Lerner formula" $(MR - P)/P = 1/\eta_s$ is derived from the monopsonist's problem $\max_P (MR - P)S(P)$ with supply $S(P)$.

no impact on fees. One may wonder whether the independence of fees from F can hold if the distribution G is not a power distribution. In fact, it cannot, as shown in the next proposition.

Proposition 5. *If a fee function ω is optimal for a given G and for an arbitrary regular F, then the fee has to be linear and G has to be a generalized power distribution.*

By a logic similar to Proposition 2, one can show that an overall increase in the elasticity of supply $\eta_s(c)$, or an overall increase in the (absolute) elasticity of demand $|\eta_d(v)|$ leads to an overall decrease in the price $P(c)$, where $\eta_d(v) = -vf(v)/(1 - F(v))$. The effect of a change of $\eta_d(v)$ on the fee is ambiguous and, as noted above, for a constant η_s, η_d has no influence on fees at all.

Not surprisingly, analogous results can be obtained for mechanisms, in which the buyer sets the price and the fee is conditioned on this price. For instance, it is optimal for the intermediary to let the buyer set the price and charge the fee $\omega_B(p) = E_c[P(c)|c \leq p]-p$, which induces the buyer to set the price $p = P^{-1}(v)$. For $F(v) = 1-[(v-\underline{v})/(\bar{v}-\underline{v})]^{\beta}$, the fee will be linear and independent of the seller's distribution.[27]

It is also worth mentioning that with an appropriately chosen fee structure the socially optimal mechanism that maximizes expected gains from trade subject to incentive compatibility and individual rationality constraints can be implemented with a fee-setting mechanism if F and G have monotone hazard rates. This complements the finding of Myerson and Satterthwaite (1983) that for F and G uniform the double-auction of Chatterjee and Samuelson (1983) has an equilibrium that implements the socially optimal mechanism. Moreover, fee-setting mechanisms satisfy individual rationality constraints ex post because a buyer never accepts a price above v and a seller who sells with positive probability never sets a price that would leave him with a negative surplus.

The static model on its own (that is, $\delta = \epsilon = 0$) analyzed so far provides a notion of how the elasticity of the seller's supply affects fees. However, it abstracts away from some aspects relevant to many real-world brokerage markets. It also leaves a few questions

[27]Observe that for $\underline{c} = \underline{v} = 0$ and $\overline{c} = \overline{v} = 1$, $G(c) = c^{\alpha}$ and $F(v) = 1-(1-v)^{\beta}$ are beta-distributions with shape parameters $\beta = 1$ and $\alpha = 1$, respectively. The corresponding virtual valuation and virtual cost functions are $\Phi(v) = ((\beta+1)v-1)/\beta$ and $\Gamma(c) = c(\alpha+1)/\alpha$.

unanswered, which at first sight seem unrelated to the aspects that are being abstracted away from. In particular, the static model neglects the possibility of delay that arises when a buyer, who does not like the price a seller offers to her, delays trade and searches for another offer. In a similar vein, the static model does not reflect the fact that brokers face competition, since traders have the option to move to another broker should the current broker's mechanism attempt to extract too much rent from them. The static model explains the structure and the level of fees with the elasticity of supply, which is not directly observable: linear fees are a good approximation for brokers if constant elasticities of supply are a good approximation, and fees are low if the elasticity is high. However, why should we believe that constant elasticities are a reasonable approximation? What causes supply to be more or less elastic? The dynamic model (that is, with $\delta > 0$ and $\epsilon > 0$) provides a parsimonious way to incorporate competition between brokers and the possibility of delay of trade. Interestingly, and perhaps surprisingly, it also provides answers to the open questions just raised and thus complements the answers in Propositions 2 and 3, which are based on power transformations in a static model.

3.2 The Dynamic Game

We now turn to the analysis of the dynamic game with random matching. In every period every intermediary who is matched to a buyer and a seller first announces a fee ω that is a function of the price the seller will set. Then the seller sets a price p. If the buyer accepts, he pays p, the seller receives the net price $p-\omega(p)$, and the intermediary collects the fee $\omega(p)$. If there is trade, the net utility of the seller with cost $\tilde{c}$ is $p-\omega(p)-\tilde{c}$, the net utility of a buyer of type $\tilde{v}$ is $\tilde{v}-p$, and both traders leave the market. Throughout the analysis of the dynamic game, we focus on an equilibrium in which the population of intermediaries choose time invariant mechanisms, although we do not restrict the individual intermediary to choosing a time-invariant mechanism.[28]

We first determine the dynamic equilibrium strategies of buyers and sellers, taking the mechanism(s) intermediaries use as fixed. Following a similar logic to Satterthwaite

[28]However, given that all others are using time-invariant mechanisms, an individual intermediary's best response mechanism will be time invariant as well in a steady-state market equilibrium.

and Shneyerov (2007), we first consider the discounted utility $W_B(\tilde{v}, v)$ of a buyer with static valuation $\tilde{v}$ who cannot commit to reject an offer below her dynamic valuation v

$$W_B(\tilde{v}, v) = \rho_B(v)(\tilde{v} - D_B(v)) + (1 - \rho_B(v))\delta W_B(\tilde{v}, v), \tag{3}$$

where $\rho_B(v)$ is the probability of trade implied by the mechanism chosen by the intermediaries and $D_B(v) := E_c[P(c)|P(c) \leq v]$ is the buyer's expected payment. Rearranging yields $W_B(\tilde{v}, v) = (\tilde{v} - D_B(v))P_B(v)$, where $P_B(v) := \rho_B(v)/(1 - (1 - \rho_B(v))\delta)$, in Satterthwaite and Shneyerov (2007)'s terminology, is "the discounted ultimate probability of trade".

Assuming that the buyer plays a steady-state strategy (that is, the maximal price v that she is willing to accept is the same in each period), her "interim utility" is

$$W_B(\tilde{v}) = \sup_v (\tilde{v} - D_B(v))P_B(v) = (\tilde{v} - D_B(B(\tilde{v})))P_B(B(\tilde{v})),$$

where we have substituted $B(\tilde{v})$ for v. By the same logic as Satterthwaite and Shneyerov (2007)'s Lemma 3 – that is, using Milgrom and Segal (2002)'s generalized versions of the envelope theorem – $W_B(\tilde{v}) = W_B(\underline{\tilde{v}}) + \int_{\underline{\tilde{v}}}^{\tilde{v}} P_B(B(x))dx$. Since the lowest valuation buyer is indifferent toward participating, $W_B(\underline{\tilde{v}}) = 0$. A buyer will accept an offer if the price is below her dynamic valuation $v = B(\tilde{v}) = \tilde{v} - \delta W_B(\tilde{v})$. Combining this with $W_B(\underline{\tilde{v}}) = 0$, we get $B(\tilde{v}) = \tilde{v} - \delta \int_{\underline{\tilde{v}}}^{\tilde{v}} P_B(B(x))dx$ and the differential equation

$$B'(\tilde{v}) = 1 - \delta P_B(B(\tilde{v})). \tag{4}$$

Note that the dynamic valuation approaches the static valuation (that is, $B(\tilde{v}) \to \tilde{v}$) as the waiting times between trading opportunities become infinitely long (that is, as $\tau \to \infty$) implying $\delta = e^{-\overline{\delta}\tau} \to 0$, which means that the game reduces to the one-shot game analyzed in Section 3.1. Further, observe that $B'(\tilde{v}) = (1-\delta)/(1-(1-\rho_B(\tilde{v}))\delta) \in [1 - \delta, 1]$. Therefore, the difference $\tilde{v} - B(\tilde{v})$ between static and dynamic valuation increases in $\tilde{v}$ for $\delta > 0$.

A similar analysis can be carried out for the seller. For expositional clarity, assume that the intermediary uses the dominant strategy implementation described in Myerson and Satterthwaite (1983) (and above Proposition 1).[29] For this mechanism, the same logic applies for the seller as for the buyer: he accepts any offer that is above his dynamic cost. By the same procedure we get $S(\tilde{c}) = \tilde{c} + \delta W_S(\tilde{c})$, where $W_S(\tilde{c}) = \int_{\tilde{c}}^{\bar{\tilde{c}}} P_S(S(x))dx$, and

$$S'(\tilde{c}) = 1 - \delta P_S(S(\tilde{c})), \tag{5}$$

and where $P_S(c) := \rho_S(c)/(1-(1-\rho_S(c))\delta)$ is the ultimate discounted probability of trade for a seller. Observe that (5) implies that the difference between dynamic and static types, $S(\tilde{c}) - \tilde{c}$, is increasing in $\tilde{c}$ for $\delta > 0$.

Every intermediary has measure zero and takes the distribution F and G as exogenously given. Therefore, the probabilities of trade will be given by the intermediary optimal allocation rule (applied to the dynamic types):

$$\rho_B(v) = G(\Gamma^{-1}(\Phi(v))), \qquad \rho_S(c) = 1 - F(\Phi^{-1}(\Gamma(c))), \tag{6}$$

Dynamic type distributions are linked to static type distributions by

$$F(B(\tilde{v})) = \tilde{F}(\tilde{v}), \qquad G(S(\tilde{c})) = \tilde{G}(\tilde{c}). \tag{7}$$

The differential equation system (4), (5), (6), (7) characterizes the dynamic types $v = B(\tilde{v})$, $c = S(\tilde{c})$ and their distributions F and G for any given steady-state distributions $\tilde{F}$ and $\tilde{G}$. The relation of these distributions to the distributions of the entrants $\tilde{F}_0$ and $\tilde{G}_0$ is determined via equations (1). This allows for the derivation of the optimal fee function ω, by obtaining F and G from the underlying steady-state static distributions. As noted by Burdett and Coles (1999), one can divide the problem of analyzing the equilibrium in a search model into two subproblems. First, the partial equilibrium problem, that is, traders behave optimally given the endogenous distributions in the market. Second, the market equilibrium condition, which determines the endogenous distributions in the market, given the optimal behavior of agents.

Using this terminology, we can state the following corollary to Proposition 1:

[29] That is, there is trade if and only if $\Phi(v) \geq \Gamma(c)$, in the case of trade the buyer pays $\Phi^{-1}(\Gamma(c))$ and the seller gets $\Gamma^{-1}(\Phi(v))$)

Corollary 1. *For any chosen strategies of other intermediaries, selecting a fee-setting mechanism is a best response for an intermediary. The optimal fee is given by $\omega(p)$ in Proposition 1, in which F and G are the endogenous dynamic steady-state distributions. Consequently, if there is a market equilibrium in any strategies, there is also an equilibrium, in which all intermediaries choose fee-setting mechanisms.*

As is well known in the literature, it is hard to show the existence and to find a characterization of a market equilibrium if the types of agents are private and their distributions endogenous. One way out is to get rid of the endogeneity of distributions by assuming that exiting traders are replaced by clones with the same type as is done, for example, by Wolinsky (1988). An alternative, which is pursued by Satterthwaite and Shneyerov (2007, 2008), is to consider small frictions close to zero, which allows one to focus on full-trade equilibria, in which every offer is accepted. While these assumptions are justified for the questions addressed in the abovementioned papers, they would not be appropriate for our purposes.[30] The reason is that we want to consider markets with large search frictions and endogenous cumulation (rather than replacement by clones) as a driving force that shapes the level and structure of fees.

To be able to make statements on the effects of dynamics in the presence of large search frictions we choose two approaches. First, we start out with a static model (that is, large frictions, $\delta = \epsilon = 0$) and analyze the effect of a small increase of ϵ. Second, we solve the market equilibrium conditions numerically, using our analytical results on the partial equilibrium behavior.

Note that the endogenous distributions F and G are closer to what one observes empirically than the exogenous distributions $\tilde{F}_0$ and $\tilde{G}_0$. Therefore, it is relevant for empirical work that it can be shown that given the endogenous distributions F and G there exist exogenous distributions $\tilde{F}_0$ and $\tilde{G}_0$ such that F and G constitute a market equilibrium. $\tilde{F}_0$ and $\tilde{G}_0$ are unique for the sets of active traders $[\underline{\tilde{v}}, \overline{\tilde{v}_0}]$ and $[\underline{\tilde{c}_0}, \overline{\tilde{c}}]$.

[30] Nöldeke and Tröger (2009) provide an existence proof of the market equilibrium with linear search technology and non-transferable utility in a matching market. They note that "the existence of steady state equilibria in heterogeneous agent matching models with a linear search technology has so far only been established for some simple examples with discrete type distributions".

3.3 Convergence to Linearity

Proposition 4 has established that linear fees are optimal for the intermediary if the seller's cost is distributed according to a power distribution. The dynamic aspects of the model offer an intuitive explanation for why power distributions may be good approximations for the endogenously determined distributions of sellers' steady-state dynamic types. As sellers with high costs offer their goods at higher prices, these relatively inefficient goods take more time to sell. Consequently, they accumulate in the market. Therefore, the steady-state seller distribution (of static types) will have more weight on the right end and less on the left end than their entrant distribution. As we illustrate next, this makes a power distribution an increasingly effective approximation of the endogenous steady-state distribution the larger is the compound discount factor δ and the rematching probability ϵ.

Closed-form solutions to the differential equations (4), (5), (6) can be obtained whenever $G(c) = ((\alpha+1)c/\alpha)^\alpha$ and $F(v) = 1 - ((\beta+1)/\beta)^\beta (1-v)^\beta$ with supports $[0, \alpha/(\alpha+1)]$ and $[1/(\beta+1), 1]$, respectively, provided α and β are positive integers.[31] These closed-form solutions may prove useful for applied research, for example, to empirically determine the size of profit maximizing percentage fees when the researcher's interest is not on how well such simple (but not necessarily optimal) mechanisms perform.[32] However, a purpose of our current paper is determining how well simple percentage fees perform. Assuming that $G(c)$ is a power distribution would thus defy this purpose. We first use perturbation analysis to characterize the first-order partial effects of the model becoming more dynamic. Then, we solve the equilibrium numerically to illustrate the total effect of increasing dynamics.

[31] The probabilities of trade are $\rho_B(v) = \left(\frac{\alpha+1}{\alpha}v - \frac{1}{\alpha}\right)^\beta$ and $\rho_S(c) = \left(1 - \frac{\beta+1}{\beta}c\right)^\alpha$. The transformations from static to dynamic types B and S are given by the roots of the polynomials $B(\tilde{v}) + \frac{\delta}{1-\delta}\frac{\alpha}{\alpha+1}\frac{1}{\beta+1}\left(\frac{\alpha+1}{\alpha}B(\tilde{v})\right)^{\beta+1} - \tilde{v} + \text{const} = 0$ and $S(\tilde{c}) - \frac{\delta}{1-\delta}\frac{\beta}{\beta+1}\frac{1}{\alpha+1}\left(1 - \frac{\beta+1}{\beta}S(\tilde{c})\right)^{\alpha+1} - \tilde{c} + \text{const} = 0$.

[32] Assume that Φ and Γ are linear (with $G(c) = [(c - \underline{c})/(\overline{c} - \underline{c})]^\alpha$). Then the optimal price function $P(c) = \Phi^{-1}(\Gamma(c))$ would be linear and the distribution of prices would be a power distribution $G_p(p) = G(P^{-1}(p)) = [(p - \underline{p})/(\overline{p} - \underline{p})]^\alpha$ with some $\underline{p}$, $\overline{p}$. Taking the empirical distribution of prices $\hat{G}_p$, we have the relation $\ln \hat{G}_p(p_i) = \alpha \ln(\overline{p} - \underline{p}) - \alpha \ln(p_i - \underline{p}) = \beta_0 + \beta_1 \ln(p_i - \underline{p})$. The slope in the regression β_1 is equal to the negative elasticity of supply $-\alpha$. Note that this assumes that (quality adjusted) prices are observed without error and that one can determine the lowest possible price $\underline{p}$ accurately.

First-Order Effects – Perturbation Analysis We now show that adding dynamics to a static model, by adding an infinitesimal perturbation to the seller's distribution and considering the first-order effect of a change of ϵ evaluated at $\delta = 0$ and $\epsilon = 0$, makes the seller's supply more elastic and decreases fees.

Proposition 6. *Set $\epsilon = 0$ and $\delta = 0$. The first-order partial effects of an increase of ϵ on the seller's distribution G are that the elasticity $\eta_s(c)$ increases and the fee $\omega(p)$ decreases.*

The first-order effect shown in Proposition 6 is important, since it identifies a driving force behind the level of fees. The proposition can be interpreted as saying that dynamics acts as a form of competition: it increases the elasticity of sellers' supply and lowers fees. This is in contrast to a setup with public information about v and c, in which the Diamond paradox occurs and brokers extract the full surplus $v - c$ for any $\epsilon, \delta < 1$.[33]

The effect of dynamics on how well linear fees approximate optimal fees is more subtle than the effect of a power transformation as described in Proposition 2. For $\tilde{F}_0$ uniform on $[0, 1]$, we can derive conditions under which dynamics drives fees to linearity. We denote the inverse hazard rate of the distribution G as $H(c) := G(c)/g(c)$ and, accordingly, the inverse hazard rate of the distribution $\tilde{G}_0(\tilde{c})$ as $\tilde{H}_0(\tilde{c})$. The hazard rate $\tilde{H}_0$ is said to be log-convex if $[\ln \tilde{H}_0(\tilde{c})]'' > 0$. Note that log-convexity of $\tilde{\Gamma}_0$ is equivalent to $\tilde{\Gamma}_0''(\tilde{c}) > -\tilde{H}_0'(\tilde{c})^2/\tilde{H}_0(\tilde{c})$ and is implied by convexity.

Proposition 7. *Assume $\tilde{F}_0(\tilde{v}) = \tilde{v}$ and set $\epsilon = 0$ and $\delta = 0$. The first-order partial effect of an increase of ϵ is that $\Gamma''(c)$ decreases if and only if $\tilde{H}_0(\tilde{c})$ is log-convex.*

Proposition 7 implies that as ϵ increases the absolute curvature $|\Gamma''(c)|$ decreases either if $\tilde{\Gamma}_0$ is both convex and log-convex or if $\tilde{\Gamma}_0$ is both concave and log-concave.

[33] The reason for the Diamond paradox in this setup is that if in an equilibrium, a buyer were to get a positive expected utility U in a mechanism, then her outside option in the current period is δU. A broker will offer her a mechanism that sets her indifferent between accepting and choosing the outside option. Since all brokers do this in equilibrium, for the equilibrium mechanism $U = \delta U$ holds, which implies $U = 0$ for any $\delta < 1$. The same applies to the utility of sellers. Therefore, the broker gets the full surplus $v - c$. We thank Stephan Lauermann for indicating the effects of private versus public information in this setup. See Lauermann (2007) for general conditions under which markets converge to a Walrasian equilibrium as frictions go to zero.

This is equivalent to the condition that $\tilde{\Gamma}_0$ is either convex or sufficiently concave. Note also that if the log-convexity condition in Proposition 7 holds for all c, so that $|\Gamma''(c)|$ decreases for all c, then the distance between the lower convex bound $p - \underline{\Gamma}^{-1}(p)$ and the upper concave bound $p - \overline{\Gamma}^{-1}(p)$ of the fee $\omega(p)$ becomes smaller for all c. This can be interpreted as the fees becoming more linear (as defined after Proposition 2). Unfortunately, it is hard to find a simple economic interpretation of the convexity and log-convexity of the inverse hazard rate. One interpretation that sheds some light on the issue is that the inverse hazard rate is equal to the inverse semi-elasticity $c/\eta_s(c)$. A convex inverse semi-elasticity can be seen as supply becoming more elastic with an accelerating rate as prices decrease.[34]

However, in addition to the first-order partial effects, we are also interested in the second-order effects and equilibrium effects. We will consider these in the numerical calculations below. For the numerical calculations, we will also use a more precise metric for how well linear fees perform: the ratio of the best linear profit to the optimal profit.

Numerical Results for Equilibrium Effects In the numerical calculations, we let the entrant type distributions for both buyers and sellers be given by beta distributions with $\alpha = \beta = 2$, that is $\tilde{f}_0(x) = \tilde{g}_0(x) = 6x(1-x)$.[35] These densities are illustrated by the dotted lines in Figure 2.

We solve the system of differential equations numerically by a combination of a Chebyshev projection and a homotopy continuation method starting from the static setup.[36]

[34] Since the shape of $\eta_s(c)$ for a specific function is not known a priori, this observation was one of the motivations for our empirical analysis in Loertscher and Niedermayer (2011).

[35] To be precise, we take $\tilde{g}_0(x) = \tilde{f}_0(x) = [6x(1-x) + 0.1]/1.1$ to avoid numerical problems with division by zero at $x = 0$ and 1.

[36] The Chebyshev projection method involves approximating the functions to be solved by (sixth degree) Chebyshev polynomials and minimizing the inconsistencies in the equations. The basic idea of a homotopy continuation method is the following. If solving a certain complex problem (say finding the roots of some function $y(x) = 0$) is too difficult, one first solves another, simpler problem (say finding the roots of $z(x) = 0$). Then, one constructs a homotopy that transforms the simple problem into the more complicated one (say a homotopy $H(x,0) = z(x)$ and $H(x,1) = y(x)$, $H(\cdot,\cdot)$ being continuous in the second argument). Then, one begins with the solution to the simple problem and gradually transforms it to the solution to the more complex one. For the given example, this means finding the roots $x(t)$ of $H(x,t)$ as t moves from 0 to 1. See Judd (1998, p. 179) for further details. There is a natural homotopy in our model, since for $\epsilon = \delta = 0$ we have the static model with an analytical solution. Starting from this, one can gradually increase ϵ and δ to the desired level.

The homotopy continuation method has the advantage that we are finding the equilibria on a continuous path from the static setup $\delta = \epsilon = 0$ to the desired level of ϵ and δ. We define the "closeness" of the steady-state distribution (of the dynamic types) to a power distribution as the ratio of profits with the best linear fee to profits with the optimal fee function, $\Pi_{\text{linear}}/\Pi_{\text{optimal}}$, where $\Pi_{\text{optimal}} := E_c[\omega(P(c))(1 - F(P(c)))]$ and $\Pi_{\text{linear}} := \max_{\xi,\zeta} \Pi_{\xi,\zeta}$, where $\Pi_{\xi,\zeta} := E_c\left[(\xi P_{\xi,\zeta}(c) + \zeta)\left(1 - F\left(P_{\xi,\zeta}(c)\right)\right)\right]$ and $P_{\xi,\zeta}(c) := \Phi^{-1}\left(\frac{c+\zeta}{1-\xi}\right)$ is the price set by a seller facing linear fees $\xi p + \zeta$.[37]

Figure 2 depicts the exogenous and endogenous distributions as the rematching frequency (and hence also δ and ϵ) increases. It illustrates graphically that as the rematching frequency increases the steady-state dynamic distribution looks "more similar" to a power distribution. Figure 3 shows that our measure of closeness to a power distribution indeed increases as the rematching frequency increases; that is, for more dynamic markets linear fees do better. One can see that for the static market, linear fees already achieve more than 99.8 per cent of what is obtainable with the optimal mechanism.[38]

The solid lines in Figure 4 depict the dynamic steady-state distributions for sellers and buyers, denoted $\sigma g(c)$ and $\sigma f(v)$ for $\delta = e^{-\bar{\delta}\tau} = 0.54$ and $\epsilon = e^{-\bar{\varepsilon}\tau} = 0.73$. The dashed lines represent the steady-state distributions of the static types of sellers and buyers, denoted as $\sigma\tilde{g}(\tilde{c})$ and $\sigma\tilde{f}(\tilde{v})$ while the dotted lines are the distributions of the static entrant types, which are uniform by assumption and denoted as $\tilde{g}_0(\tilde{c})$ and $\tilde{f}_0(\tilde{v})$.

For the example given in Figures 2 and 3, an increase of ϵ and δ leads to an increase of $\Pi_{\text{linear}}/\Pi_{\text{optimal}}$. If one starts with a power distribution G and a uniform F (which means linear virtual costs, $\Gamma''(c) = 0$), then by Proposition 7 the first-order partial effect will decrease Γ'' and hence increase $|\Gamma''|$. Therefore, one starts with $\Pi_{\text{linear}}/\Pi_{\text{optimal}} = 100\%$ and the first-order effect decreases this ratio. Computing the equilibrium numerically with

[37] The price $P_{\xi,\zeta}(c)$ is given by the first-order condition of the seller's maximization problem $\max_p((1-\xi)p - \zeta)(1 - F(p))$. Intuitively, $(c + \zeta)/(1 - \xi)$ is the seller's "perceived cost" and Φ^{-1} is the optimal pricing function according to Myerson (1981). The broker's profit is the fee $\xi p + \zeta$ times the probability of sale $1 - F(p)$.

[38] That simple mechanisms can achieve a large percentage of the optimal surplus or profit has been shown in a variety of contexts; see McAfee (2002) for assortative matching, Rogerson (2003) for incentives in procurement and Chu, Leslie, and Sorensen (2011) for bundling. In our empirical companion paper (Loertscher and Niedermayer, 2011), we find that using a 6 per cent fee intermediaries achieve between 75 per cent and 88 per cent of the profit under the optimal mechanisms. Using the optimal proportional fee, they achieve between 81 per cent and 88 per cent of the maximum profit.

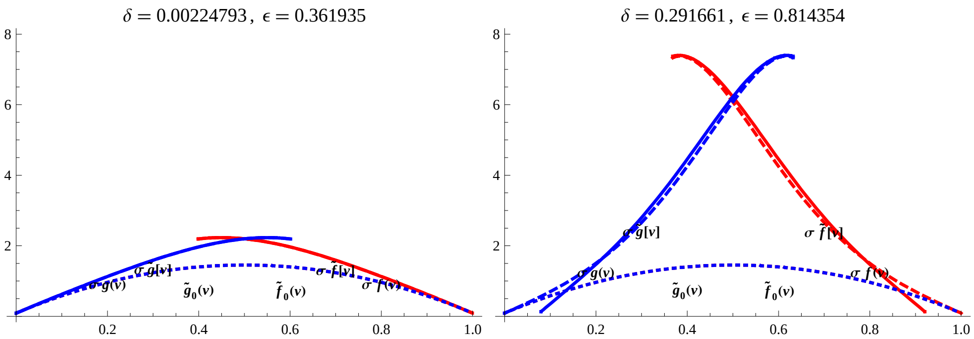

Figure 2: Exogenous (dotted: entrant static) and endogenous (dashed: steady-state static. solid: steady-state dynamic) densities with a low (left) and a high (right) rematching frequency. For the lowest rematching frequency ($\epsilon = \delta = 0$), we are back to the static model and the endogenous distributions coincide with the exogenous ones (dotted lines).

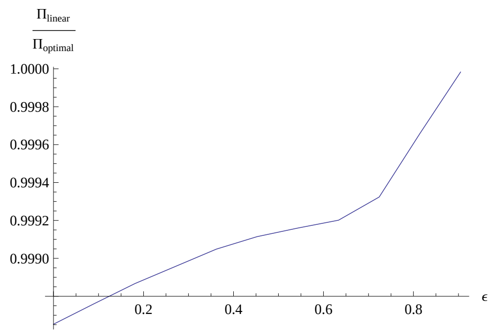

Figure 3: Ratio of best linear to optimal profits as the rematching frequency increases. The horizontal axis is the probability of remaining in the market ϵ with $\epsilon = e^{-0.1\tau}$, the discount factor δ is given by $\delta = e^{-0.2\tau}$.

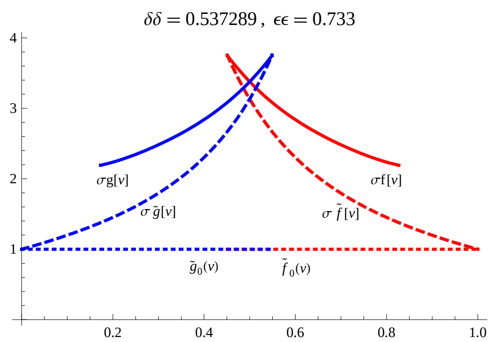

Figure 4: Relation between static entrant (dotted), static steady-state (dashed), and dynamic steady-state (solid) distributions for buyers and sellers. $\delta = e^{-\overline{\delta}\tau} = 0.54$ and $\epsilon = e^{-\overline{\varepsilon}\tau} = 0.73$.

uniform exogenous distributions $\tilde{F}_0$ and $\tilde{G}_0$ (see Figure 4) reveals that the total equilibrium effect goes in the same directions as the partial equilibrium effect: the performance of linear fees starts out at 100 per cent and decreases (slightly).

To sum up, our theoretical results show that in the static model additional outside offers for the seller (power transformations) always lead to lower and "more linear" fees (see Propositions 2 and 3). Dynamics has the partial first-order effect that it decreases fees and, under the conditions of Proposition 7, also makes the fees more linear. The numerical computations of the equilibrium suggest that the full equilibrium effect goes in the same direction. There exist, however, simple counter examples. For example, for the case in which $\tilde{G}_0$ exhibits a constant elasticity of supply (that is, is a power distribution), we know from Proposition 4 that dynamics cannot make the optimal fee more linear and from Proposition 6 that the first-order partial effect is that the optimal fee becomes non-linear because $\tilde{\Gamma}_0'' = 0$ when $\tilde{G}_0$ is a power distribution. Numerical simulations confirm that for $\tilde{G}_0$ uniform the total equilibrium effect is that linear fees indeed become slightly suboptimal. Therefore, what ratio of the optimal profit can be achieved with linear fees in a given market is ultimately an empirical question. We address this question in our companion paper (Loertscher and Niedermayer, 2011) for the real estate brokerage market in Boston in the 1990s using a simple extension of the model developed here. The counterfactual analysis based on the empirical estimates of the parameters suggests that linear fees perform reasonably well. Using a 6 per cent fee intermediaries achieve between 75 per cent and 88 per cent of the profit under the optimal mechanisms. Using the optimal proportional fee, they achieve between 81 per cent and 88 per cent of the maximum profit. This application provides the motivation for the following section, in which we develop additional theoretical results that are of particular relevance in the context of real estate brokerage.

4 Real Estate Brokerage

We now apply our model to real estate brokerage. By deriving additional results and relating them to the large existing literature on real estate brokerage, we next show that our institutionally simple model sheds new light on this important industry.

Of course, our model abstracts away from institutional details that are important in the real estate industry, such as the multiple listings service (also known as MLS), bargaining between buyers and sellers, and empirically observed differences between listing prices and transaction prices.[39] Further, the level of competitiveness between intermediaries has been kept fixed in our model. A prominent way of incorporating different levels of competition is the one pursued by Burdett and Mortensen (1998). In a setup without intermediaries and with public information, they model competitiveness as the probability a currently matched worker receives outside offers from other employers. Probability one of outside offers is the most competitive market, whereas probability zero is the monopolistic market (due to the Diamond paradox). Following Burdett and Mortensen (1998) in spirit, we can model the competitiveness of intermediated markets as the probability ν that a seller is rematched with a new broker in the subsequent period. In this sense, the baseline model considered so far is the most competitive benchmark with $\nu = 1$. Here the broker cannot extract any rents from a trader's option value of future trade, since traders can switch to a different broker in the subsequent period. A lower probability that the seller and the broker are rematched means a lower level of competitiveness of brokers (see Appendix C). Most results are qualitatively the same for $\nu < 1$, provided the broker cannot extract all rents from the option value of future trade, which corresponds to assuming $\nu > 0$.[40]

[39]Though differences between the listing price and the transaction price may occur, there is ample evidence that there is a strong and positive correlation between listing prices and transaction prices. For example, in the data of Genesove and Mayer (2001), which we use for our analysis in Loertscher and Niedermayer (2011), the correlation coefficient between quality adjusted listing price and quality adjusted transaction price is 0.9584. Therefore, fee-setting mechanisms appear to be a sensible approximation to the mechanisms used by real estate brokers. Also note that our results on the optimal fee carry over without modification to a setup in which the seller sets a reserve price and multiple buyers bid in an auction. This is briefly discussed in Section 5 and derived in detail in the working paper version of this article.

[40]Of course, there are many alternative ways of modeling imperfect competition between brokers. While formal results have yet to be derived, it seems plausible that an increase in competition will have the effect of increasing the elasticity of supply, making the inverse elasticity $1/\eta_s(c)$ "more constant", and preventing a broker from fully extracting a seller's option value of future trade.

4.1 Further Results

Quality-Adjusted Price and Time On Market We now relax the assumption maintained thus far that objects (say, houses) are homogeneous. We assume that everyone agrees that the objective value of a certain house is ϑ. The valuation of a buyer of type v for a house of quality ϑ is ϑv and, accordingly, the cost of selling a house of quality ϑ for a seller of type c is ϑc. Empirically observed prices $\hat{p}$ are given by $\hat{P}(c) = \vartheta P(c)$, where $P(c)$ is the quality-adjusted price. Rutherford, Springer, and Yavas (2005) define the degree of overpricing (DOP) as $\text{DOP} = (\hat{P}(c) - \vartheta)/\vartheta$. The quality-adjusted price is thus $P(c) = \text{DOP} + 1$. The price $P(c)$ in our model can be interpreted as a quality-adjusted price,[41] and our model can be interpreted as, say, consisting of several separate submarkets that differ only in their ϑ's, which are observed by the buyer, seller and the broker (but not necessarily by the econometrician). This widely used way of modeling vertical differentiation implies that the slope ξ^* of the optimal linear fee $\xi^* p + \zeta^*(\vartheta)$ does not vary with ϑ, whereas $\zeta^*(\vartheta)$ does.[42]

Having thus defined quality-adjusted prices, we can now derive predicted time on market for houses of the same objective quality ϑ. Interestingly, the baseline model predicts that the average time on market is the same for sold and unsold houses if the objective quality ϑ can be perfectly controlled for. The continuous-time approximation of the discrete-time geometric distribution is described in the first part of Proposition 8.[43] The second part addresses the case in which a data set includes heterogeneous submarkets under the assumption that heterogeneity is not or is only imperfectly controlled for. In this case, the model predicts that unsold houses have a longer time on the market.

[41] Since Rutherford, Springer, and Yavas (2005) estimate DOP as the average listing price a house with certain characteristics has, this means that DOP has mean 0. This corresponds to a quality-adjusted price where the mean is normalized to 1.

[42] To see why this is so, notice first that $\vartheta\Phi(v)$ and $\vartheta\Gamma(c)$ are the virtual valuation and virtual cost associated with ϑv and ϑc, respectively. Given a linear fee $\xi^* p + \zeta$ a seller of type ϑc thus optimally sets the price $\vartheta P_{\xi,\zeta}(c)$, where $P_{\xi,\zeta}(c)$ is as given in Footnote 37. The intermediary optimal linear fee is then given as $\arg\max_{(\xi,\zeta)}(\xi\vartheta P_{\xi,\zeta}(c) + \zeta)(1 - F(P_{\xi,\zeta}(c))) = \arg\max_{(\xi,\zeta)} \vartheta(\xi P_{\xi,\zeta}(c) + \zeta/\vartheta)(1 - F(P_{\xi,\zeta}(c)))$. Thus, letting (ξ^*, ζ^*) be the optimal linear fee when $\vartheta = 1$, the optimal linear fee for an arbitrary ϑ, denoted $(\xi^*(\vartheta), \zeta^*(\vartheta))$, satisfies $(\xi^*(\vartheta), \zeta^*(\vartheta)) = (\xi^*, \vartheta\zeta^*)$.

[43] We provide the continuous-time version of the distribution since it allows for a simpler notation. The discrete-time version is derived in the formal proof in Appendix A. In general, closed-form solutions cannot be obtained for the distributions that arise with imperfect quality adjustment.

Proposition 8. *(a) Homogeneous Market/Perfect Quality Adjustment: For homogeneous houses, the time on market of sold and unsold houses has the same distribution. The continuous-time approximation of this distribution is exponential, with the cumulative distribution function* $1 - \exp(-(\overline{F}(p) + \overline{\varepsilon})t)$ *and mean*

$$T(p) = \frac{1}{\overline{F}(p) + \overline{\varepsilon}},$$

where $\overline{F}(p)$ *is such that* $e^{-\overline{F}(p)\tau} = F(p)$*. For a price* p *the ratio of houses ever sold, denoted* $1 - F_\infty(p)$*, is*

$$1 - F_\infty(p) = \frac{\overline{F}(p)}{\overline{F}(p) + \overline{\varepsilon}}.$$

(b) Heterogeneous Submarkets/Imperfect Quality Adjustment: Let $T^s(p)$ *and* $T^u(p)$ *be, respectively, the mean time on market for sold and unsold houses with price* p*. If heterogeneous submarkets are in the observed sample, the time on market is lower for sold houses than for unsold ones:* $T^s(p) \leq T^u(p)$*.*

The result in part (b) stems from the fact that houses that have a higher probability of selling at the same price have a shorter time on market and are also relatively overrepresented in the set of sold houses.[44] A similar result is achieved when times on market are estimated as averages over all prices rather than for a specific price. According to Proposition 8, differences in sample means of time on market for sold and unsold houses indicate that that the quality adjustment is not perfect. Part (b) of Proposition 8 makes an empirically testable prediction assuming, as seems plausible, that quality can only be imperfectly controlled for. One can show that adding a measurement error (i) to the time on market and (ii) to the quality index of our baseline model leads to (i) the modal value of the distribution of time on market being positive rather than zero and (ii) time on market having a fat tail rather than an exponential distribution.[45] This is what we observe in Loertscher and Niedermayer (2011).

[44]This result is consistent with Larsen and Park (1989)'s observation that failing to include unsold houses may lead to a bias in the estimation of time on market. Our analysis suggests that such a bias stems from heterogeneity.

[45](i) stems from the fact that the sum of two continuously distributed positive random variables has a distribution with a positive mode. The intuition behind (ii) is that houses spending a long time on the market are more likely to be houses that are difficult to sell. As a simple example, if the quality index ϑ is measured with an error such that $\overline{F}(p) = \overline{F}(\hat{p}/\vartheta)$ follows a Gamma distribution with density

Houses For Sale By Owner and Brokers Selling their Own Houses Call a broker who owns the house he is selling and an owner who sells without a broker a *direct seller* while an individual who sells via an intermediary is called an *indirect seller*. From the perspective of our model, houses owned and sold by brokers and houses for sale by owners (FSBO) are the same.[46] Hendel, Nevo, and Ortalo-Magné (2009) compared prices of houses sold via brokers and houses for sale by owners, and found that prices achieved by FSBOs were higher, though not significantly so. Similarly, Rutherford, Springer, and Yavas (2005) and Levitt and Syverson (2008) observed that brokers selling their own houses achieved higher average prices than did independent sellers selling comparable houses. The commonly given explanations for these observations is that there is a moral hazard problem or a selection effect.

We now show briefly how a simple extension of our dynamic model can shed new light on the price-setting behavior of direct and indirect sellers and thereby provide an alternative explanation. The partial equilibrium analysis is based on the assumption that both kinds of sellers face the same (endogenous) distribution of buyers F. This is the case if buyers join the FSBO market and the intermediary market with equal probabilities or if buyers "multi-home", that is, search on both platforms.[47]

In a static setup, the optimal mechanism of a direct seller is posting a price (see Myerson (1981)). This direct seller's profits will exceed the joint profits of an indirect seller and his intermediary. However, his price $\Phi^{-1}(c)$ will be smaller than the price of an indirect seller $\Phi^{-1}(\Gamma(c))$ because of the externality the intermediary's fee imposes, which is analogous to double marginalization. Interestingly, dynamics reverses this result for low cost sellers for any distribution: the direct seller will have a higher dynamic cost because his option value of not selling and staying in the market is higher than that for

proportional to $\overline{F}^{\alpha-1}e^{-\overline{F}/\beta}$ and time on market without quality measurement error is exponentially distributed as in Proposition 8 (a), then, with quality measurement error, the time on market t is distributed with density $e^{-\overline{\varepsilon}t}(t\beta+1)^{-(\alpha+1)}(t\beta\overline{\varepsilon}+\alpha\beta+\overline{\varepsilon})$, which exhibits a fat tail.

[46]This is so because we do not model agency issues and institutional details such as the multiple listings service (MLS), which only brokers can access.

[47]The argument is easiest if one assumes that overall the mass of direct sellers is negligible for then F and G are as given in the main text. The assumption is not crucial, though: if direct sellers have positive measure and buyers are randomly matched to a direct or an indirect seller in every period, then F and G will be different from the expressions in the text, but they will nonetheless determine P^I and P^D as derived in the proof of the following proposition.

an indirect seller with the same static cost $\tilde{c}$.

Proposition 9. *(a) An indirect seller at the lower end of the distribution ($\tilde{c} = \underline{\tilde{c}}$) sets a lower price than a direct seller with the same static cost $\tilde{c}$.*

(b) An indirect seller at the upper end of the distribution ($\tilde{c} = \overline{\tilde{c}}$) sets a higher price than a direct seller with the same static cost $\tilde{c}$.

(c) If G has a monotone hazard rate, then there is a $\hat{\tilde{c}} \in (\underline{\tilde{c}}, \overline{\tilde{c}})$ such that a direct seller sets a higher price than an indirect seller with the same static cost if and only if $\tilde{c} < \hat{\tilde{c}}$.

Since prices are a positive monotone transformation of costs, this proposition has the empirically testable implication, assuming there is no selection effect between direct and indirect sellers, that in the lower (upper) quantiles of the price distributions, direct sellers set higher (lower) prices than do indirect sellers. Whether the average prices of direct and indirect sellers are higher depends on the shape of the distribution. It is easy to find examples, in which direct sellers set higher prices on average than indirect sellers even in the absence of agency problems and selection effects (that is, even if $\tilde{G}_0$ is the same for direct and indirect sellers).[48]

The above reasoning referred to price averages that were taken over the entrant distribution $\tilde{G}_0$. If prices are averaged over the distribution of *sold* houses (as is typically done in empirical studies), there is a selection effect for sold houses, which occurs independently of whether or not there is a selection effect for the entrant distributions. This selection effect stems from the different probabilities of selling for a direct and an indirect seller with the same $\tilde{c}$. Under a monotone hazard rate assumption it can be shown that this selection effect increases the average price of direct sellers and hence makes the impact of dynamics even stronger.[49]

[48]A simple example can be found by solving the problem backward and starting with the endogenous distributions $F(v) = 2v - 1$ for $v \in [1/2, 1]$, $G(c) = 2c$ for $c \in [0, 1/2]$ and $\overline{\delta} = \overline{\varepsilon}$. This uniquely determines the exogenous distributions $\tilde{F}_0$ and $\tilde{G}_0$. Using the exogenous and endogenous distributions, it can be shown that the average transaction price is higher for direct sellers than for indirect sellers for all $\varepsilon > 3/4$.

[49]This follows from Proposition 9 (c) and the fact that the probability of selling decreases with price.

There is an additional effect increasing the average price of direct sellers that occurs even in a static setup. Sellers with relatively high static costs can profitably enter the market if and only if they are direct sellers because the fees levied by intermediaries would induce them to set prices that are never accepted. This "static entry" effect can lead to a higher average price of direct sellers even in a static model:

Proposition 10. *Assume Φ is linear and $\overline{c} = \overline{v}$ in a static model and take into account additional entry by direct sellers. Compared to the average price of indirect sellers, the average price of direct sellers is (i) equal if Γ is linear, (ii) higher if Γ is concave, and (iii) lower if Γ is convex.*

Proposition 10 also indicates that with free entry by sellers (and buyers) intermediaries can, in principle, perform the socially valuable role of keeping inefficient traders out of the market.[50] Though Proposition 10 certainly falls short of describing a model of platform competition, it suggests that competition between fee-setting platforms need not result in fees equal to zero even if platforms have large capacities and set their fees publicly as, all else being equal, the platform with the intermediary optimal fees may in fact offer prices that are lower on average than the prices offered by a platform without any fees.

Inefficient Free Entry by Intermediaries Hsieh and Moretti (2003) analyze free entry into real estate brokerage and find that the number of transactions per intermediary and year decreases during real estate booms and that the number of intermediaries increases in proportion to the overall profits made in real estate brokerage during booms while the mechanism the intermediaries employ, a 6 per cent fee, is independent of the number of active intermediaries. A simple extension of our dynamic model provides a concise explanation for both these findings. Let $\kappa > 0$ be an individual's per period opportunity cost of being a real estate broker. Assuming that there is free entry into the industry, intermediaries will enter until expected per period profits equal κ. Denote the mass of active intermediaries under free entry by ι^*.

[50] As we discuss next, the literature has so far focused on the inefficiency of free entry by intermediaries. Free entry by sellers is another potential source of inefficiency.

Proposition 11. *If, under free entry, there is a sufficient mass of intermediaries to match every seller and every buyer – that is, if $\iota^* > \sigma$ – then the equilibrium mechanism is independent of the opportunity cost of entry κ and, consequently, independent of the number of active intermediaries ι^*.*

The number of active intermediaries only affects the probability of a match for an intermediary, but does not alter the optimal mechanism because it does not affect the probability of a match for a buyer or a seller. Proposition 11 also shows that the comparative statics of our model with respect to the number of intermediaries are similar to those of a model with complete information, such as Diamond (1971), because the equilibrium mechanisms are invariant with respect to this number in both types of models.[51] Heterogeneity among brokers can also be easily accommodated for. For example, heterogenous fixed costs of entry or heterogenous matching probabilities, which depend on functional assumptions, can give rise to star brokers and a middle class doing real estate brokerage (Hsieh and Moretti, 2003, Appendix A).

Per Transaction Costs of Intermediation Let us now briefly consider the case, in which intermediation services also involve a variable cost. Assume that the seller's cost is given by a power distribution $G(c) = [(c - \underline{c})/(\overline{c} - \underline{c})]^\alpha$ with $\alpha > 0$ (or, alternatively, assume that a power distribution is a good approximation for the problem at hand) and recall that a power distribution with $\underline{c} > 0$ implies a negative fee $\zeta = -\underline{c}/(\alpha + 1) < 0$; that is, the fee involves a subsidy from the intermediary to the seller, which may take the form of free of charge services like advertising, showing the house to potential buyers and legal advice intermediaries provide as well. This interpretation is consistent with the observation that in the United Kingdom (U.K.), real estate brokers typically charge a lower fee (2.5 per cent) than do U.S. brokers, but that they also provide considerably less services than do brokers in the U.S. According to our model and assuming that power distributions are good approximations for both markets with the same or similar $\underline{c}$, this would mean that the exponent α is larger in the U.K. than in the U.S, which is

[51] Recall that in a Diamond (1971) style model all sellers charge the monopoly price independent of the number of sellers. Thus, we have one "half" of the Diamond paradox: fees do not change as the number of sellers increases, but they do decrease as rematching frequency increases.

equivalent to the elasticity of supply $\eta_s(c) = \alpha c/(c - \underline{c})$ being larger in the U.K. than in the U.S.

Of course, this explanation begs the question as to why these distributions should so differ. Labor market related mobility is obviously an important factor influencing individuals' decision to buy and sell houses. If these mobility patterns differ across the two countries in such a way that in the U.K. more job-related moves occur within a radius that still allows commuting than in the U.S., then according to our model British buyers and sellers would exhibit a higher probability ϵ of staying in the market than their North-American counterparts. A numerical analysis, that is based on the same assumptions that underlie Figure 2 (or, alternatively, Figure 4) and that is not reproduced here, shows that the optimal proportional fee indeed decreases in ϵ.

4.2 Stylized Facts and Empirical Observations

The following stylized facts are observed in real estate markets. First, real estate brokers charge 6 per cent of the transaction price, a commission rate that shows very little variance over time and across regions. Second, sellers with a higher loan-to-value ratio ask higher prices (Genesove and Mayer, 1997). Third, sellers who had bought their houses when average real estate prices were high, ask for higher prices than those who had bought when prices were low (Genesove and Mayer, 2001). Fourth, quality-adjusted prices and time on market of houses are positively correlated in cross-sectional data and negatively correlated in longitudinal data. Fifth, broker fees are the same irrespective of the number of intermediaries and house prices. Sixth, when industry profits double, the number of brokers doubles as well (Hsieh and Moretti, 2003). Seventh, direct sellers, either houses for sale by owner or brokers selling their own houses, sell at higher prices than indirect sellers selling through a broker (see Rutherford, Springer, and Yavas, 2005; Levitt and Syverson, 2008; Hendel, Nevo, and Ortalo-Magné, 2009).

A counterfactual empirical analysis, based on data from real estate transaction in the 1990s,[52] which is performed in Loertscher and Niedermayer (2011), shows that 6 per cent fees allow intermediaries to achieve 75 per cent or more of the maximum profit, given the

[52] The data is from Genesove and Mayer (2001).

parameter estimates.[53] That higher loan-to-value ratios induce sellers to set higher prices is an immediate implication of our model if higher-loan-to-value ratios are interpreted as higher costs. Similarly, sellers who bought their houses during booms when prices were high are quite naturally interpreted in our model as sellers whose valuations are high. Consequently, they will ask higher prices.[54]

A large empirical literature addresses the relation between the quality-adjusted listing price and the time on market and typically finds that the quality-adjusted listing price and the time on market are positively correlated in cross-sectional data.[55] Our model is consistent with this stylized fact. Further, assuming that during booms houses sell faster and at higher prices than during recessions, our model also generates the negative correlation observed in longitudinal data.[56] That the equilibrium mechanism is independent of the number of active brokers is a direct implication of our matching assumption and of the assumption that enough brokers are active. Assuming free entry, the number of active brokers is proportional to industry profit if all brokers face the same fixed cost of entry. Lastly, sellers with low static costs set higher prices when they sell directly than when they sell indirectly because they have a sufficiently higher continuation payoff that more than compensates for the absence of a price-increasing fee.

Figure 5 is taken from our companion paper (Loertscher and Niedermayer, 2011) and is based on the maximum likelihood estimates of the parameters of the parameterized model for the year 1993. For the current paper, it serves as an example and a graphical illustration of the data our model can generate. In Loertscher and Niedermayer (2011),

[53] Our numerical analysis suggest that the explanation of why simple mechanism like percentage fees or linear fees fare so well may reside in the dynamics of the model.

[54] A natural objection is that this argument relies on the assumption that the valuations of buyers and sellers are differently distributed despite the fact that in real estate markets most buyers are also sellers, and vice versa. However, our argument only requires the *dynamic steady-state* distributions to be different for buyers and sellers (that is, the solid lines in Figure 4). The fundamentals – that is, the *entrant static type* distributions (which are given by the dotted lines) – can be the same for buyers and sellers. The transformation from the dotted to the solid line breaks the original symmetry for two reasons. First, low valuation buyers remain longer in the market, whereas for sellers it is the ones with high costs. Second, the option value of future trade has to be subtracted for a buyer, but added for a seller.

[55] See Kang and Gardner (1989), Larsen and Park (1989), Yavas and Yang (1995), Genesove and Mayer (1997, 2001), Rutherford, Springer, and Yavas (2005) and Hendel, Nevo, and Ortalo-Magné (2009).

[56] In our model this means that F and G change in booms such that both the average price $\int_{\underline{c}}^{P^{-1}(\bar{v})} P(c)dG(c)$ increases and the overall average time on market $\int_{\underline{c}}^{P^{-1}(\bar{v})} T(P(c))dG(c)$ decreases.

we use the model of the current paper with three simple modifications: (i) a simple form of heterogeneity, a buyer and a seller are a potential fit with some probability λ, (ii) quality-adjusted prices are observed with an error, and (iii) time on market is observed with an error. Panel (a) displays the estimated endogenous densities $f(v)$ (dashed) and $g(c)$ (solid). Panels (c), (d) and (f) show, respectively, the relationships between time on market and quality-adjusted price, density of quality-adjusted prices, and the probability of selling. In each case, the empirically observed relationship is displayed as a dashed line while the relationship that our model implies when evaluated at the estimated parameter values is shown as a solid line. The densities of time on the market for sold and unsold houses are displayed, respectively, with solid and dashed lines, in panel (b) as implied by our model and the parameter estimates and in panel (e) for what is observed in the data. Panel (g) shows the optimal fee $\omega(p)$ as a solid curve, the optimal linear fee $\omega_{\text{linear}}(p)$ as a dotted line and the optimal proportional fee $\omega_{\text{prop}}(p)$ as a dashed line as implied by our model evaluated at the estimated parameter values. The empirically observed 6 per cent fee is displayed as a dashed line. Figure 5 suggests that our model has the flexibility to generate predictions similar to empirical data. In Loertscher and Niedermayer (2011) we argue further that our theory also gives a plausible explanation for simple percentage fees used in practice.

5 Alternative Applications and Mechanisms

We now briefly discuss two other applications of our model, intermediaries in auctions and stockbrokers, and relate fee setting to two alternative mechanisms that are widely used by intermediaries: price posting and slotting allowances. See the working paper version of this article for a more detailed and formal discussion of these results.

Auction Houses and Auction Sites Auction houses like Sotheby's and Christie's and auction internet sites like eBay set percentage fees[57] and the seller then sets a reservation price rather than a take-it-or-leave-it price. Our results regarding the optimal

[57]Before they were convicted of collusion, this fee was 20 per cent for Sotheby's and Christie's. eBay used to charge a linear fee of 5 per cent before it adopted a regressive fee structure.

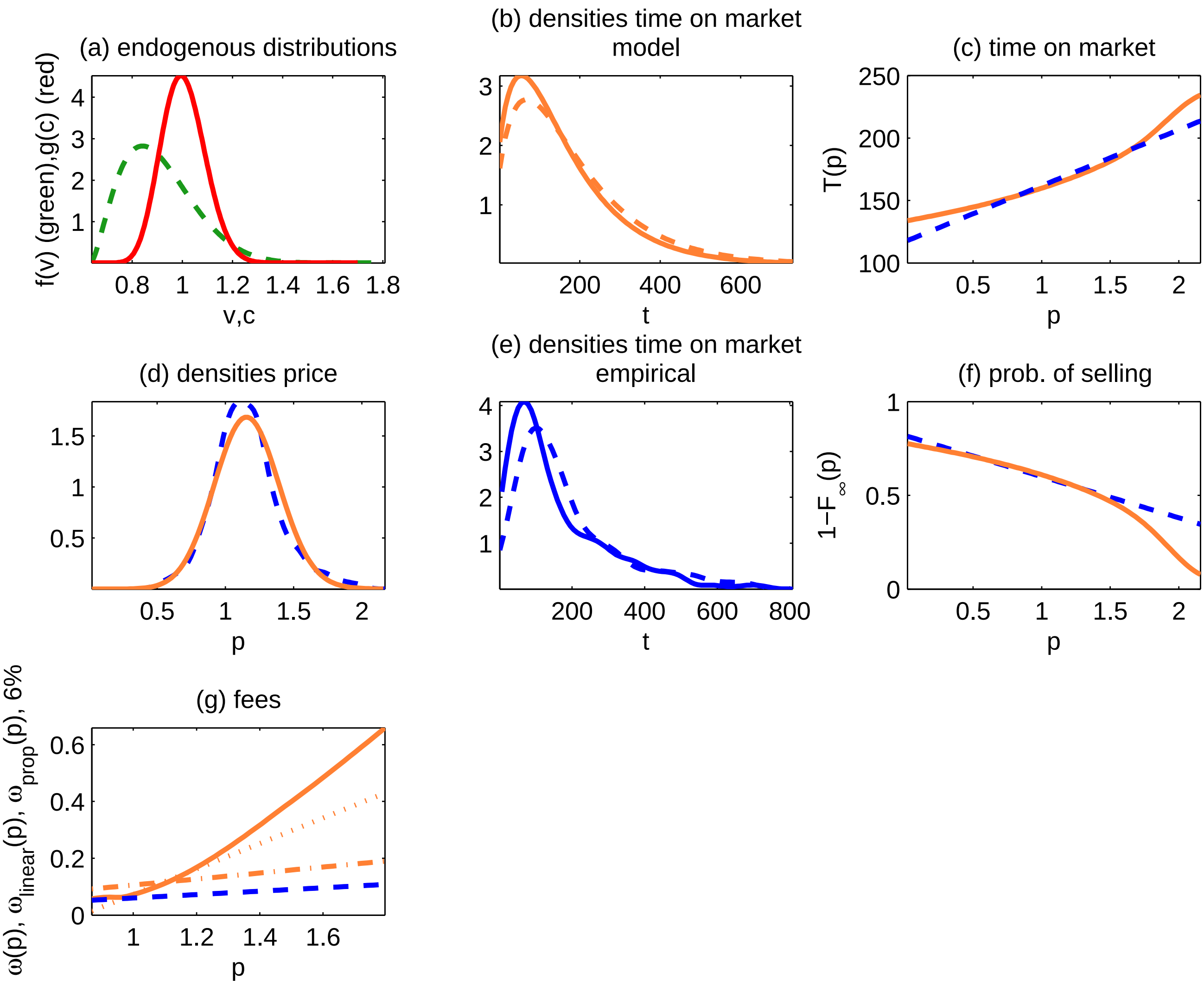

Figure 5: Example of distributions and fees that our model can generate, shown solid in panels (c), (d), (f), solid and dashed in panel (b), and solid, dash-dotted, and dotted in panel (g). The dashed lines in panels (c) to (g) and the solid line in Panel (e) show empirical observations (ordinary least squares and kernel density estimators). Figure taken from Loertscher and Niedermayer (2011).

fee directly translate to an auction environment with the following modification. The seller sets a reservation price $\Phi^{-1}(\Gamma(c))$ in a standard auction (for example, a first-price auction or a second-price auction), buyers bid, the highest bidding buyer gets the good at gross price p (which is her own bid in a first-price auction or the second highest in a second-price auction), the intermediary levies the fee $\omega(p) = p - E[\Gamma^{-1}(\Phi(v))|v \geq p]$ on the gross price, and the seller receives the net price $p - \omega(p)$. Note that the fee $\omega(p)$ is the same as with one buyer, except that it is levied on the auction price rather than the price set by the seller. This fee-setting mechanism is optimal in the general class of mechanisms.

Stockbrokers Linear fees are widely used in stock brokerage (see, for example, Stoughton, Wu, and Zechner, 2011). The empirical finance literature describes two kinds of fees used: flat fees and percentage fees (see Conrad, Johnson, and Wahal, 2001). In stock markets, a broker willing to sell looks at one buyer's broker at a time. The short moment during which two brokers look at each other can be interpreted as the period τ between subsequent rematching (without modeling the buyer's and the seller's broker).[58]

An important reason why stockbrokers may prefer percentage fees to (bid and ask) price posting (see below) may be the highly volatile nature of the value of the goods (that is, stocks) in the exchange of which they are engaged. This feature is arguably well captured by our model, in which the buyer's and seller's valuations v and c are stochastic. Consequently, the market price exhibits substantial variability and price posting is not optimal for the intermediary – as discussed below and shown formally in the working paper version – whereas percentage fee setting is. Interestingly, the literature on market microstructure and intermediation mainly focuses on price posting by brokers. For example, Gehrig (1993) studies a static model with a monopolistic broker who quotes bid and ask prices. Similarly, Duffie, Garleau, and Peddersen (2005) analyze market making by price-setting intermediaries in a dynamic setup, in which the intermediaries' search intensity is determined endogenously and price posting is the exogenously given mechanism.

[58] We thank Mark Satterthwaite for providing us with this interpretation.

Price Posting Consider price posting, which is widely used, for example, by used car dealers and currency exchange offices: the intermediary posts an ask (or buyer) price p^B and a bid (or seller) price p^S. The quantity traded is the minimum of the number of buyers and sellers who are willing to trade at these prices, and the intermediary earns the bid-ask spread $p^B - p^S$ on each buyer-seller pair that trades.

It can be shown that price posting cannot be optimal for one buyer and one seller. However, as the number of buyers and sellers goes to infinity, price posting becomes optimal. If the broker can store the good between rematchings, it is as if all buyers and sellers arrived at once. Hence, with storability, price posting is optimal as well. It is remarkable that price posting is used in those markets, in which liquidity is high (a large number of buyers and sellers), or in which the broker stores the good.

Slotting Allowances Retailers often require upfront payments by sellers to allocate scarce shelf space and then charge a fee on the revenue generated by the seller. In a static model with one buyer, several sellers with independently and identically distributed costs, and one slot on the shelf being available, it can be shown that it is optimal for the intermediary to first auction off the shelf space, then let the highest bidding seller set the sales price and finally charge the fee given in Proposition 1 on the price set by the seller.

6 Conclusions

We use a dynamic random-matching model that builds on previous theoretical work (see Satterthwaite and Shneyerov, 2007, 2008) to study intermediation from an applied Bayesian mechanism design perspective, assuming that the intermediaries have all the temporary bargaining power. We show that fee-setting mechanisms, which are widely used in practice, but so far little understood in economics, are optimal mechanisms for intermediaries and that increasing dynamics has the tendency of making the optimal fees smaller and closer to linear. Adding institutional details from specific industries of interest to the present setup seems a particulary fruitful avenue for further research.

Appendix

A Proofs

Since we will refer to the properties of intermediary optimal mechanisms often, we summarize Myerson and Satterthwaite (1983)'s Theorems 3 and 4 on intermediation in the following lemma.

Lemma 1 (Myerson-Satterthwaite)**.** *An incentive compatible, interim individually rational mechanism is intermediary optimal if and only if it is such that (i) the good is transferred if and only if* $\Phi(v) \geq \Gamma(c)$ *and (ii) the seller with the highest cost* $\bar{c}$ *and the buyer with the lowest valuation* $\underline{v}$ *both have zero expected utility.*

We also state the following result that is helpful in several proofs.

Lemma 2. *The following holds for the virtual cost and virtual valuation functions: (i)* $E_c[\Gamma(c)|c \leq p] = p$ *and (ii)* $E_v[\Phi(v)|v \geq p] = p$*.*

Proof. We only provide the proof for (i) since (ii) is analogous. Note that (i) holds for $p = \underline{c}$ since $E[\Gamma(c)|c \leq \underline{c}] = \Gamma(\underline{c}) = \underline{c}$. Further, (i) can be rewritten as $\int_{\underline{c}}^{p} \Gamma(c)g(c)dc = pG(p)$. The derivative of the right-hand-side is $[pG(p)]' = \Gamma(p)g(p)$, which is equal to the derivative of the left-hand-side. This completes the proof. □

Proposition 1: Optimal Fee.

Proof of Proposition 1. Even though a shorter proof can be obtained by first considering the dominant strategy implementation, we will derive our results through the incentive compatibility constraint because other proofs rely on the intermediate steps of this proof.

If through an appropriately chosen fee function $\omega(p)$ the seller with cost c can be induced to set the price $p = P(c)$, we know that we have an an intermediary optimal mechanism because the buyer will accept the offer if and only if $v \geq P(c) \Leftrightarrow \Phi(v) \geq \Gamma(c)$. We first show that such a function $\omega(p)$ exists and is unique. Denote the expected payoff to the seller of type c who sets the "prescribed" price $P(c)$ as $U(c) := (P(c) - \omega(P(c)) - c)(1 - F(P(c)))$. By Revenue Equivalence (see, for example, the proof of Theorem 1 in

Myerson and Satterthwaite (1983) or Chapter 5 in Krishna (2002)), for $p = P(c)$ to be an equilibrium strategy for the seller, $U(c)$ has to satisfy

$$U(c) = U(\bar{c}) + \int_c^{\bar{c}} [1 - F(P(t))]dt = \int_c^{\bar{c}} [1 - F(P(t))]dt, \tag{8}$$

where the second equality follows because $U(\bar{c}) = 0$. Moreover, since $1 - F(P(c)) = 0$ for all c such that $P(c) \geq \bar{v} \Leftrightarrow c \geq P^{-1}(\bar{v}) := \Gamma^{-1}(\Phi(\bar{v}))$, the upper limit of the integral can be written as $P^{-1}(\bar{v})$. Inserting the definition of $U(c)$ into (8) and rearranging yields

$$\omega(P(c)) = P(c) - c - \int_c^{P^{-1}(\bar{v})} \frac{1 - F(P(t))}{1 - F(P(c))} dt. \tag{9}$$

Substituting $p = P(c)$ into (9) and integrating using this substitution yields

$$\omega(p) = p - P^{-1}(p) - \frac{\int_p^{\overline{v}} (1 - F(v))[P^{-1}(v)]' dv}{1 - F(p)} = p - E_v[P^{-1}(v) \mid v \geq p], \tag{10}$$

where the second equality follows after integrating by parts ($E_v[P^{-1}(v) \mid v \geq p] = (\int_p^{\bar{v}} f(v)P^{-1}(v)dv)/(1 - F(p))$ being the expectation of $P^{-1}(v)$ taken with respect to the distribution F conditional on v being larger than p). For given F and G the function $\omega(p)$ is unique. Since in equilibrium prices $p > \overline{v}$ never induce trade, $\omega(p)$ for $p > \overline{v}$ can be chosen arbitrarily.

Next we show that given $\omega(p)$ the Subgame Perfect Nash equilibrium is essentially unique. (The equilibrium is not unique merely because a seller of type $c > P^{-1}(\overline{v})$ can set any $p > \overline{v}$.) Since the buyer's unique best response is to accept whenever $p \leq v$, the problem of the seller with cost c, given $\omega(p) = p - E_v[P^{-1}(v)|v \geq p]$, is to choose p to maximize $(E_v[P^{-1}(v)|v \geq p] - c)(1 - F(p))$. By construction of $\omega(p)$, the first-order condition is satisfied at $p = P(c)$. Since at the first-order condition, the second-order condition $0 > -f(p)[P^{-1}(p)]'$ is also satisfied, uniqueness follows. □

Proposition 2: Upper and Lower Bounds.

Proof of Proposition 2. (i) Note that $\underline{\Gamma}$ convex and below Γ implies $\underline{\Gamma}^{-1}$ concave and above Γ^{-1}. For the lower bound we have the chain of inequalities $\omega(p) = p - E[\Gamma^{-1}(\Phi(v))|v \geq p] \geq p - E[\underline{\Gamma}^{-1}(\Phi(v))|v \geq p] \geq p - \underline{\Gamma}^{-1}(E[\Phi(v)|v \geq p]) = p - \underline{\Gamma}^{-1}(p)$. The first inequality stems from $\underline{\Gamma}^{-1} \geq \Gamma^{-1}$, the second from the concavity of $\underline{\Gamma}^{-1}$ and Jensen's inequality, and the last (equality) from Lemma 2. Therefore, the last term in the chain is a lower bound of ω. The result for the upper bound can be derived by the same reasoning.

(ii) Note that for the transformation $G_\alpha = G^\alpha$, the virtual cost function becomes $\Gamma_\alpha(c) = (1/\alpha)\Gamma(c) + (1 - 1/\alpha)\, c$. The convex hull of the transformed Γ is the transformation of the convex hull: $\underline{\Gamma}_\alpha(c) = (1/\alpha)\underline{\Gamma}(c) + (1 - 1/\alpha)\, c$, the same applies for the concave hull. An increase of α brings the convex and the concave hull closer to the linear function c. This can be used to prove the statements in the proposition. $|\Gamma''_\alpha(c)| < |\Gamma''(c)|$ follows from $\Gamma''_\alpha(c) = (1/\alpha)\Gamma''(c)$.

□

Proposition 3: Supply Side.

Proof of Proposition 2. (i) ω_α decreasing with α follows almost directly from the fact that Γ_α decreases with α and $\omega_\alpha(p) = p - E[\Gamma_\alpha^{-1}(\Phi(v))|v \geq p]$. Convergence of the fee to zero can be shown in the following way. Since $\lim_{\alpha\to\infty} \Gamma_\alpha(c) = c$, $\lim_{\alpha\to\infty} \omega_\alpha(p) = p - E[\Phi(v)|v \geq p] = p - p = 0$, where the second equality stems from Lemma 2.

(ii) Since $\Gamma(c) = c(1 + 1/\eta_s(c))$, $\eta_s(c) < \hat{\eta}_s(c)$ for all c implies $\Gamma(c) > \hat{\Gamma}(c)$, which in turn implies $-\Gamma^{-1}(p) > -\hat{\Gamma}^{-1}(p)$ for all p. This, in combination with Proposition 1 implies the statement. □

Proposition 4: Linear Fee.

Proof of Proposition 4. By the same standard arguments leading to (8) we also get $U'(c) = -q(c)$ almost everywhere because of incentive compatibility. Equating this with the derivative obtained from the definition $U'(c) = [(P(c) - \omega(P(c)) - c)q(c)]'$ and rearranging yields

$$\Phi(P(c)) = P(c) - \frac{P(c) - \omega(P(c)) - c}{1 - \omega'(P(c))}. \tag{11}$$

(i) implies (ii). Take $\omega(p) = \xi p + \zeta$. Then the right hand side of (11) becomes $(c+\zeta)/(1-\xi)$. Equating this with $\Gamma(c)$ to achieve optimality according to Lemma 1 (i) gives the differential equation $g(c) = G(c)(1-\xi)/(\xi c + \zeta)$. With the condition $G(\underline{c}) = 0$

one obtains the expression in part (ii) of the proposition with $\alpha = (1-\xi)/\xi$ and $\underline{c} = -\zeta/\xi$. The upper bound of the support $\bar{c}$ remains arbitrary.

(ii) implies (i) Observe that with the distribution G specified in part (ii) one has $\Gamma^{-1}(p) = (1-\hat{\xi})p - \hat{\zeta}$ with $\hat{\xi} := 1/(\alpha+1)$ and $\hat{\zeta} := -\underline{c}/(\alpha+1)$. Therefore, $P^{-1}(p) = \Gamma^{-1}(\Phi(p)) = (1-\hat{\xi})\Phi(p) - \hat{\zeta}$. Take (11) and replace $P(c)$ with p, c with $P^{-1}(p)$, and Φ with its definition. Rearranging leads to

$$(1-F(p))(1-\omega'(p)-(1-\hat{\xi})) - f(p)(p-\omega(p)-((1-\hat{\xi})p-\hat{\zeta})) = 0. \tag{12}$$

Defining $l(p) := p - \omega(p) - ((1-\hat{\xi})p - \hat{\zeta})$ equation (12) leads to $[l(p)(1-F(p))]' = 0$. From part (ii) of Proposition 4 follows that $p - \omega(p)$ is not singular at $p = \bar{v}$ (actually $\omega(\bar{v}) = \bar{v} - P^{-1}(\bar{v})$). Since $1 - F(\bar{v}) = 0$ it follows that $l(p) \equiv 0$; that is, $\omega(p) = \xi p + \zeta$ as in part (i) Proposition 4 is satisfied with $\xi = \hat{\xi}$ and $\zeta = \hat{\zeta}$. □

Proposition 5: Invariance and Linearity of Fees.

Proof of Proposition 5. The optimality condition (i) of Lemma 1 implies $\Phi(P(c)) = \Gamma(c)$. If we want optimality to hold for arbitrary distributions F, and hence for arbitrary functions $P(c)$, equating the right hand side of (11) and $\Gamma(c)$ yields $\Gamma(c) = p - (p - \omega(p) - c)/(1-\omega'(p))$ for arbitrary p. This differential equation in ω has the solution

$$\omega(p) = p - (1-\xi)(p - \Gamma(c)) - c \tag{13}$$

defined up to a constant $1-\xi$. If we want this to hold for any c we need $c-(1-\xi)\Gamma(c) = -\zeta$ for some constant ζ. Therefore, $\Gamma(c) = (c+\zeta)/(1-\xi)$. Substituting this back to (13) results in $\omega(p) = \xi p + \zeta$, that is, a linear fee. This also implies a generalized power distribution G by Proposition 4. □

Proposition 6: First-Order Effects.

Proof of Proposition 6. In the following, we will keep $\delta = 0$ and change ϵ infinitesimally. Since $\delta = 0$ implies no option value of future trade, $c = \tilde{c}$, $G = \tilde{G}$, $F = \tilde{F}$. We will simplify notation in this proof by dropping the tilde (hence $\tilde{G}_0$ will be G_0, $\tilde{\Gamma}_0 =$

$c + \tilde{G}_0/\tilde{g}_0$ will be Γ_0, and so on); by dropping the argument c; and, most of the time, dropping the second-order effect $O(\epsilon^2)$. Now recall that the steady-state condition is $\sigma(1-(1-\rho_S(c))\epsilon)g(c) = g_0(c)$, where $\rho_S(c) = 1 - F_0(\Phi_0^{-1}(\Gamma_0(c)))$ and σ is a constant such that the density function g adds up to one. In the following, we want to have a function γ that infinitesimally perturbs g

$$g(c) = (1+\epsilon\gamma(c))g_0(c)$$

and ensures that g adds up to one, that is, $\int \gamma g_0 = 0$.

Since we are only concerned with first-order effects, $g(c) = (1+\epsilon\gamma(c))g_0(c)$ can be rewritten with the Taylor expansion of $y_c(\epsilon) = 1/(1+\epsilon\gamma(c))$ at $\epsilon = 0$; that is, $y_c(0) + y_c'(0)\epsilon + y_c''(0)\epsilon^2/2 + \sum_i y^{(n)}(0)\epsilon^n/n!$, truncated after the first-order effect:

$$g_0 = \frac{1}{1+\epsilon\gamma} g = (1 - \epsilon\gamma + O(\epsilon^2))g, \tag{14}$$

where $O(\epsilon^2)$ stands for the second-order effect. Taking a constant α with $1 + \alpha\epsilon = \sigma$, this has to be equal to

$$(1+\alpha\epsilon)(1-\epsilon(1-\rho_S(c)))g(c) = (1-\epsilon[(1-\rho_S(c)) - \alpha] + O(\epsilon^2))g(c). \tag{15}$$

α has to be chosen as $\alpha = \int(1-\rho_S)g$ so that the density functions add up to one. Therefore, equating the right hand sides of (14) and (15) results in

$$\gamma(c) = F_0(\Phi_0^{-1}(\Gamma_0(c))) - \int_{\underline{c}}^{\overline{c}} F_0(\Phi_0^{-1}(\Gamma_0(t)))g_0(t)dt. \tag{16}$$

We know that γ is increasing, γ and g_0 are orthogonal ($\int \gamma g_0 = 0$), $\gamma(\underline{c}) < 0$, and $\gamma(\overline{c}) > 0$.

Note that one can write $g(c)/G(c)$ as $(\ln G(c))'$. We will first show that $(\ln G(c))'$ increases if ϵ increases, that is, $\frac{\partial^2}{\partial\epsilon\partial c}\ln G > 0$. Using $\frac{\partial}{\partial\epsilon}G = \frac{\partial}{\partial\epsilon}\int(1+\epsilon\gamma)g_0 = \int\gamma g_0$, we get $\frac{\partial}{\partial\epsilon}\ln G\Big|_{\epsilon=0} = \frac{1}{G_0}\int_{\underline{c}}^{c} g_0(c')\gamma(c')dc'$. Taking the derivative with respect to c yields

$$\frac{\partial}{\partial c}\frac{\partial}{\partial\epsilon}\ln G\Big|_{\epsilon=0} = -\frac{g_0}{G_0^2}\int_{\underline{c}}^{c} g_0(c')\gamma(c')dc' + \frac{1}{G_0}g_0(c)\gamma(c),$$

the sign of which is to be determined. Multiplying by the positive expression G_0^2/g_0 we get, for the right hand side,

$$G_0(c)\gamma(c) - \int_{\underline{c}}^{c} g_0(c')\gamma(c')dc' = \int_{\underline{c}}^{c} g_0(c')[\gamma(c) - \gamma(c')]dc' > 0$$

since $\gamma'(c) > 0$ and $c > c'$. Therefore, the whole expression is positive, which proves the statement $\frac{\partial^2}{\partial\epsilon\partial c}\ln G > 0$, or, equivalently, $\partial[g(c)/G(c)]/\partial\epsilon > 0$. Now $g(c)/G(c)$ increasing in ϵ obviously implies $cg(c)/G(c)$ increasing in ϵ. The decrease of $\omega(p)$ can be shown by a similar logic as in Proposition 3 (ii). □

Proposition 7: Convergence to Linearity.

Convergence to Linearity. We simplify the notation as we did in the proof of Proposition 6. The following analysis can be simplified by defining a further function ψ, such that

$$G(c) = (1 - \epsilon\psi(c))G_0(c). \tag{17}$$

The relation between ψ and γ (defined in the proof of Proposition 6) is the following $g\gamma = -(G_0\psi)'$ or $\psi = -\frac{1}{G_0}\int g_0\gamma$, which is equal to $-\partial\ln G/\partial\epsilon$. Therefore, $\psi' < 0$ by the arguments in the proof of Proposition 6. Since $G(\overline{c}) = G_0(\overline{c})$, we know that $\psi(\overline{c}) = 0$. Together with $\psi' < 0$ this implies $\psi \geq 0$.

The derivative of $G(c)$ with respect to c is $g = g_0 - \epsilon(g_0\psi + G_0\psi')$.

By definition

$$\Gamma = c + \frac{G}{g} = c + \frac{[1-\epsilon\psi]G_0}{[(1-\epsilon\psi) - \epsilon(G_0/g_0)\psi']g_0} = c + \frac{G_0}{g_0}\frac{[1-\epsilon\psi]}{g_0 - \epsilon(\psi + (G_0/g_0)\psi')}. \tag{18}$$

The Taylor expansion is

$$c + \frac{G_0}{g_0}[1-\epsilon\psi]\left[1 + \epsilon(\psi + \frac{G_0}{g_0}\psi')\right] + O(\epsilon^2) = c + \frac{G_0}{g_0}\left[1 + \epsilon\frac{G_0}{g_0}\psi'\right] + O(\epsilon^2) \tag{19}$$

Using the definition of Γ this gives us

$$\Gamma = \Gamma_0 + \epsilon\left(\frac{G_0}{g_0}\right)^2\psi' + O(\epsilon^2). \tag{20}$$

Using the definitions of ψ we get $\psi' = [-(\int_{\underline{c}}^{c} g_0\gamma]' = -g_0A/G_0^2$, where $A := \int_{\underline{c}}^{c}(\gamma(c) - \gamma(t))g_0(t)dt$. Plugging this into (20) we get $\Gamma = \Gamma_0 - \epsilon A/g_0$. Using the fact that $\int_{\underline{c}}^{c}\Gamma(t)g(t)dt = cG(c)$ and the assumption $F_0(v) = v$, the expression for γ in (16) simplifies to $\gamma(c) = (\Gamma_0(c) - \overline{c})/2$. This in turn simplifies A to $A = G_0^2/(2g_0)$. Plugging this back to the expression for Γ, we get $\Gamma = \Gamma_0 - \frac{1}{2}\epsilon H_0^2$. This implies $\Gamma'' = \Gamma_0'' - \epsilon(H_0'^2 + H_0H_0'')$ and the statement in the proposition, since $\Gamma_0'' = H_0''$. □

Proposition 8: Time on Market.

Proof of Proposition 8. (a) *Discrete Time.* Consider first a cohort of sellers, who entered the market at some point t, normalized to $t = 0$, and offered their houses for some price p. Label the number of rematchings since $t = 0$ with $k := t/\tau$ and the expected number of sellers in the cohort staying in the market at the beginning with N_0, and in subsequent periods with N_k. The probability that a seller stays in the market until the next rematching is the probability that he cannot sell multiplied by the probability that he does not drop out for exogenous reasons, that is, $\epsilon F(p)$ with $\epsilon := e^{-\bar{\varepsilon}\tau}$. Therefore, the number of sellers in period k is $N_k = (\epsilon F(p))^k N_0$. Time on market for the total population of both sold and unsold houses follows a geometric distribution, with the cumulative distribution function $1 - (\epsilon F(p))^{t/\tau}$ and mean $T(p) = \tau/(1 - \epsilon F(p))$. Denote the number of sellers who leave the market in period k because they sell as N_k^s and those who leave with unsold houses as N_k^u. Clearly, $N_k^s = (1 - F(p))N_k$ and $N_k^u = (1 - \epsilon)F(p)N_k$. Therefore, the ratio of sellers able to sell is $(1 - F(p))/(1 - \epsilon F(p))$. Now consider only the subsample of sellers who managed to sell their houses. Since N_k^s is a constant factor smaller than N_k, the distribution of time on market of this subsample is the same as for the total population. Therefore, the cumulative distribution function is also $1 - (\epsilon F(p))^k$ and the mean time on market for sold houses is $T^s(p) = \tau/(1 - \epsilon F(p))$. The same reasoning applies for sellers who did not sell their houses, so that $T^u(p) = \tau/(1 - \epsilon F(p))$ is the mean time on market for unsold houses. Since we are looking at a market in a stationary equilibrium, in every period the same number of N_0 sellers enters and the previous argument carries over to a setup, in which cohorts of sellers enter every period rather than only one cohort entering at $t = 0$.

Continuous Time. The same logic applies to the continuous-time approximation of the distribution. Denote the mass of sellers in the cohort at period $t = 0$ as $N(0)$. The number of sellers remaining in the market in period t is $N(t) = N(0)e^{-(\bar{F}+\bar{\varepsilon})t}$, dropping the argument p in $\overline{F}(p)$. In each period $dN^s(t) = N(t)\overline{F}dt$ houses are sold and $dN^u(t) = N(t)\bar{\varepsilon}dt$ drop out unsold. Cumulatively, we have $N^s(t) = \int_0^t dN^s(t') = (\overline{F}/(\overline{F} + \bar{\varepsilon}))\,[N(0) - N(t)]$ and $N^u(t) = \int_0^t dN^u(t') = (\bar{\varepsilon}/(\overline{F} + \bar{\varepsilon}))\,[N(0) - N(t)]$. After an infinite number of periods, fraction $1 - F_\infty := N^s(\infty)/N(0) = \overline{F}/(\overline{F} + \overline{\varepsilon})$ of houses have been sold. The average time on market for sold houses is

$$T^s = \frac{\int_0^\infty t dN^s(t)}{\int_0^\infty dN^s(t)} = -\frac{\partial}{\partial \overline{F}} \ln \int_0^\infty e^{-(\overline{F}+\overline{\varepsilon})t} dt = -\frac{\partial}{\partial \overline{F}} \ln \frac{1}{\overline{F} + \overline{\varepsilon}} = \frac{1}{\overline{F} + \overline{\varepsilon}}.$$

By the same logic, the average time on market of unsold houses is $T^u = 1/(\overline{F} + \overline{\varepsilon})$.

(b) Consider multiple submarkets, indexed by i, with different probabilities of sale $\overline{F}_i(p)$. Houses of each submarket are represented with weight w_i in the total sample. Taking averages over submarkets, the mean time on market for sold $T^s(p)$ and unsold $T^u(p)$ houses is $T^s = \left(\sum_i w_i \frac{\overline{F}_i}{\overline{F}_i+\overline{\varepsilon}} \frac{1}{\overline{F}_i+\overline{\varepsilon}}\right) \left(\sum_i w_i \frac{\overline{F}_i}{\overline{F}_i+\overline{\varepsilon}}\right)^{-1}$ and $T^u = \left(\sum_i w_i \frac{\overline{\varepsilon}}{\overline{F}_i+\overline{\varepsilon}} \frac{1}{\overline{F}_i+\overline{\varepsilon}}\right) \left(\sum_i w_i \frac{\overline{\varepsilon}}{\overline{F}_i+\overline{\varepsilon}}\right)^{-1}$, where the parameter p has been dropped. The ratio of the two means is

$$\frac{T^u}{T^s} = \frac{\sum_i w_i \frac{\overline{\varepsilon}}{(\overline{F}_i+\overline{\varepsilon})^2} \sum_j w_j \frac{\overline{F}_j}{\overline{F}_j+\overline{\varepsilon}}}{\sum_i w_i \frac{\overline{F}_i}{(\overline{F}_i+\overline{\varepsilon})^2} \sum_j w_j \frac{\overline{\varepsilon}}{\overline{F}_j+\overline{\varepsilon}}} =: \frac{N}{D}.$$

The difference between the numerator N and the denominator D is

$$N - D = \overline{\varepsilon} \sum_{ij} w_i w_j \frac{\overline{F}_j - \overline{F}_i}{(\overline{F}_i + \overline{\varepsilon})^2 (\overline{F}_j + \overline{\varepsilon})} = -\overline{\varepsilon} \sum_{ij} w_i w_j \frac{\overline{F}_j - \overline{F}_i}{(\overline{F}_i + \overline{\varepsilon})(\overline{F}_j + \overline{\varepsilon})^2},$$

where the second equation comes from interchanging the summation variables. Adding the two expressions for $N - D$ one gets

$$2(N - D) = \overline{\varepsilon} \sum_{ij} w_i w_j \frac{(\overline{F}_j - \overline{F}_i)^2}{(\overline{F}_i + \overline{\varepsilon})^2 (\overline{F}_j + \overline{\varepsilon})^2} \geq 0.$$

Therefore, $T^u \geq T^s$. The inequality is strict for heterogeneous submarkets.

□

Proposition 9: Direct Sellers.

Proof of Proposition 9. Denote, respectively, by $W_S(\tilde{c})$ and $W_S^D(\tilde{c})$ the expected continuation payoff of staying in the market (that is, when not selling) of an indirect seller with static type $\tilde{c}$ who sells via an intermediary and of a direct seller of the same type. Their dynamic types are $S(\tilde{c}) = \tilde{c} + \delta W_S(\tilde{c})$ and $S^D(\tilde{c}) := \tilde{c} + \delta W_S^D(\tilde{c})$. Now $W_S^D(\tilde{c}) > W_S(\tilde{c})$ holds for all $\tilde{c} \in [\underline{\tilde{c}}, \overline{\tilde{c}}]$. This is because the direct seller always has the option of setting the same price as the indirect seller, in which case they both sell with the same probability. Because the indirect seller does not have to pay a fee, it follows (via a revealed preference argument if their prices differ) that in every period the direct seller's expected payoff exceeds the expected payoff of an indirect seller. Consequently, $W_S^D(\tilde{c}) \geq W_S(\tilde{c})$. One can show that the inequality is strict at the lower end of the distribution.

(a) The indirect seller sets the price $\tilde{P}(\tilde{c}) := P(S(\tilde{c})) = \Phi^{-1}(\Gamma(\tilde{c} + \delta W_S(\tilde{c})))$ and the direct seller sets the price $\tilde{P}^D(\tilde{c}) := \Phi^{-1}(\tilde{c} + \delta W_S^D(\tilde{c}))$, where Φ and Γ are derived from the distributions of the dynamic types of buyers and sellers F and G that are endogenous to the model. Since $\Gamma(\underline{c}) = \underline{c}$, $\Gamma(\underline{\tilde{c}} + \delta W_S(\underline{\tilde{c}})) = \underline{\tilde{c}} + \delta W_S(\underline{\tilde{c}}) < \underline{\tilde{c}} + \delta W_S^D(\underline{\tilde{c}})$. Since Φ^{-1} is increasing, this implies $\tilde{P}^D(\underline{\tilde{c}}) > \tilde{P}(\underline{\tilde{c}})$.

(b) In equilibrium, the mechanism is such that an indirect seller at the upper end of the distribution $\overline{\tilde{c}}$ has utility zero and sells with probability zero. Hence, $W_S(\overline{\tilde{c}}) = 0$. Therefore, a direct seller with cost $\overline{\tilde{c}}$ will sell with probability zero if he sets a price $\tilde{P}^D(\overline{\tilde{c}}) \geq \tilde{P}(\overline{\tilde{c}})$, in which case his option value $W_S^D(\overline{\tilde{c}})$ would be equal to 0. However, his price would then have to be lower than that of an indirect seller, since $\Phi^{-1}(\overline{\tilde{c}} + W_S^D(\overline{\tilde{c}})) = \Phi^{-1}(\overline{\tilde{c}}) < \tilde{P}(\overline{\tilde{c}})$, which is a contradiction. Hence $\tilde{P}^D(\overline{\tilde{c}}) < \tilde{P}(\overline{\tilde{c}})$ follows.

(c) Parts (a) and (b) together with continuity imply that there will be at least one point of intersection between $\tilde{P}^D(\tilde{c})$ and $\tilde{P}(\tilde{c})$. Let $\hat{\tilde{c}}_i$ be a point in the intersection. We now show that $\Gamma' > 1$ implies that $\tilde{P}'(\hat{\tilde{c}}_i) > \tilde{P}^{D'}(\hat{\tilde{c}}_i)$ at every point of intersection $\hat{\tilde{c}}_i$. This in turn implies that $\tilde{P}(\tilde{c}) \geq \tilde{P}^D(\tilde{c})$ for all $\tilde{c}$ above the first (that is, smallest) point of intersection.

Notice first that $\tilde{P}'(\hat{\tilde{c}}_i) = \Phi^{-1'}(\Gamma(\hat{\tilde{c}}_i + \delta W_S(\hat{\tilde{c}}_i)))\Gamma'(\hat{\tilde{c}}_i + \delta W_S(\hat{\tilde{c}}_i))(1 + \delta W_S'(\hat{\tilde{c}}_i))$ and $\tilde{P}^{D'}(\hat{\tilde{c}}_i) = \Phi^{-1'}(\hat{\tilde{c}}_i + \delta W_S^D(\hat{\tilde{c}}_i))(1 + \delta W_S^{D'}(\hat{\tilde{c}}_i))$. Since the prices in both expressions are the same, the arguments of $\Phi^{-1'}$, and hence $\Phi^{-1'}$, must be the same in both expressions. Given the assumption $\Gamma' > 1$, the result thus follows if we can show that $1 + \delta W_S'(\hat{\tilde{c}}_i) \geq 1 + \delta W_S^{D'}(\hat{\tilde{c}}_i)$. Recall that $W_S'(\hat{\tilde{c}}_i) = -\rho_S(\hat{\tilde{c}}_i)/(1 - (1 - \rho_S(\hat{\tilde{c}}_i))\delta)$, where $\rho_S(\hat{\tilde{c}}_i) = 1 - F(\tilde{P}(\hat{\tilde{c}}_i))$. By analogous arguments, $W_S^{D'}(\hat{\tilde{c}}_i) = -\rho_S^D(\hat{\tilde{c}}_i)/(1 - (1 - \rho_S^D(\hat{\tilde{c}}_i))\delta)$, where $\rho_S^D(\hat{\tilde{c}}_i) = 1 - F(\tilde{P}^D(\hat{\tilde{c}}_i))$. Since $\tilde{P}(\hat{\tilde{c}}_i) = \tilde{P}^D(\hat{\tilde{c}}_i)$ at every point of intersection, $\rho_S(\hat{\tilde{c}}_i) = \rho_S^D(\hat{\tilde{c}}_i)$ and $\delta W_S'(\hat{\tilde{c}}_i) = \delta W_S^{D'}(\hat{\tilde{c}}_i)$ follows. This completes the proof that $\tilde{P}(\tilde{c}) \geq \tilde{P}^D(\tilde{c})$ for all $\tilde{c}$ above the first (that is, smallest) point of intersection.

□

Proposition 10: Direct Sellers and Additional Entry in a Static Model.

Proof of Proposition 10. Direct sellers set the price $\Phi^{-1}(c)$ (see Myerson (1981)) and enter if $c \leq \overline{v} = \overline{c}$. Indirect sellers set the price $\Phi^{-1}(\Gamma(c))$ (see Proposition 1) and enter if $c \leq \Gamma^{-1}(\Phi(\overline{v})) = \Gamma^{-1}(\overline{c})$. Therefore, average prices of direct and indirect sellers are $P_A^D = E[\Phi^{-1}(c)]$ and $P_A^I = E[\Phi^{-1}(\Gamma(c))|c \leq \Gamma^{-1}(\overline{c})]$. Using the linearity of Φ and Lemma 2 (i), prices can be rewritten as $P_A^D = \Phi^{-1}(E[c])$ and $P_A^I = \Phi^{-1}(\Gamma^{-1}(\overline{c}))$. Since Φ and Γ are increasing, $P_A^D \geq P_A^I$ is equivalent to $\Gamma(E[c]) \geq \overline{c}$. This implies the proposition because of $E[\Gamma(c)] = \overline{c}$ by Lemma 2 and Jensen's inequality. □

Proposition 11: Invariant Mechanisms.

Proof of Proposition 11. Denote an intermediary's expected profit in a given period if matched to a buyer and a seller by $\Pi := \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \max\{0, \Phi(v) - \Gamma(c)\} f(v) g(c) dc dv > 0$. (If unmatched, then his per period profit is 0). Provided $\iota^* > \sigma$, each active intermediary's expected per period probability of being matched with a buyer and a seller is σ/ι^* and his per period profit is $(\sigma/\iota^*)\Pi$. The equilibrium number of intermediaries in the market is given by $(\sigma/\iota^*)\Pi = \kappa$ under free entry. Note that changes in ι^* only adversely affect the volume of transactions per intermediary and thereby expected equilibrium profit per intermediary. Since every buyer and seller is matched with probability 1 in every period regardless of the number of intermediaries, this leaves every intermediary's mechanism design problem in every period unaffected. Therefore, the equilibrium mechanisms intermediaries employ do not vary as ι^* changes, provided only ι^* is not smaller than σ.[59] Therefore, business stealing is the only effect of entry, so that free entry leads to excessive entry. □

[59] Note that if $\iota^* < \sigma$ were the case, then the equilibrium mechanisms would vary with ι^* since buyers and sellers are now matched with probability $\iota^*/\sigma < 1$. The equilibrium number of intermediaries is then given by $(\sigma/\iota^*)\hat{\Pi}(\iota^*) = \kappa$ where $\hat{\Pi}(\iota)$ is some decreasing function that determines the equilibrium per transaction profits of an intermediary.

B The General Mechanism Design Problem (not for publication)

Preliminaries. We first define the ironed virtual valuation and virtual cost, denoted respectively Φ_{iron} and Γ_{iron}, following the procedure in Myerson (1981, p.68). For the buyer define

$$J_b(q) := \text{conv} \int_0^q \Phi(F^{-1}(r))dr,$$

where conv stands for the convex hull of a function. The ironed virtual valuation function of the buyer can now be defined as $\Phi_{iron}(v) = J_b'(F(v))$. Wherever Φ_{iron} is strictly increasing, $\Phi_{iron}(v) = \Phi(v)$.

For the seller, construct the ironed virtual cost function in a similar way by defining

$$J_s(q) := \text{conv} \int_0^q \Gamma(G^{-1}(r))dr,$$

and the ironed virtual cost function as $\Gamma_{iron}(c) = J_s'(G(c))$.

Lemma 3. *In the general setup (in which Myerson's regularity assumption may not hold), it is optimal for the intermediary to allow trade if and only if* $\Phi_{iron}(v) \geq \Gamma_{iron}(c)$.

Proof. By the same logic as Myerson's (1981) theorem on p. 69. □

Optimal Fee-Setting Mechanism. The easiest way to derive the optimal fee is by the following brief detour through the dominant strategy implementation of the optimal allocation rule. Similar to Myerson and Satterthwaite (1983, p.280), we can implement the allocation rule with dominant strategies in the general case by letting the buyer pay $P_{gen}(c) := \Phi_{iron}^{-1}(\Gamma_{iron}(c))$ and the seller receive $P_{gen}^{-1}(v) := \Gamma_{iron}^{-1}(\Phi_{iron}(v))$ in the case of trade. Here, the subscripts stand for "general" and the inverses are redefined as

$$\Phi_{iron}^{-1}(x) := \min\{v | v \geq \underline{v} \text{ and } \Phi_{iron}(v) \geq x\}, \tag{21}$$

$$\Gamma_{iron}^{-1}(x) := \max\{c | c \leq \overline{c} \text{ and } \Gamma_{iron}(c) \leq x\}. \tag{22}$$

Without trade, both get 0. We can now use the same logic as in the main text to derive the optimal fee-setting mechanism. The dominant strategy implementation already gives us the right price for the buyer, $P_{gen}(c)$. To ensure that the seller's payment only depends on c, we have to consider the expectation of $P_{gen}^{-1}(v)$, which he gets in the dominant strategy implementation. The net price the seller gets is $E_v[P_{gen}^{-1}(v)|v \geq P_{gen}(c)]$, since we have to take expectations conditional on trade taking place. The optimal fee is the gross minus the net price, in the indirect mechanism representation.Using p rather than $P_{gen}(c)$, this is

$$\omega(p) = p - E_v[P_{gen}^{-1}(v)|v \geq p] \tag{23}$$

for $p \leq \overline{v}$ and any $\omega(p) \geq \overline{v} - p$ for $p > \overline{v}$. This is the same as in the main text, except that we are using ironed virtual valuations. If $\Phi_{iron}(\overline{v}) > \Gamma_{iron}(\overline{c})$ (violating Condition 1), the fee becomes

$$\omega(p) = p - E_v[P_{gen}^{-1}(v)|v \geq p] + \left\{ \frac{1 - F(\overline{P}(\overline{c}))}{1 - F(\overline{P}(c))} E[P_{gen}^{-1}(v)|v \geq \overline{P}(\overline{c})] - \overline{c} \right\}, \tag{24}$$

where the expression in brackets ensure that sellers of type $\overline{c}$ have no informational rents (see Lemma 1 (ii)).

The price paid by the buyer, $P_{gen}(c)$ is a left continuous function (the discontinuities, if any, being determined by the discontinuities of Φ_{iron}^{-1}) with horizontal intervals where Γ_{iron} is horizontal. The inverse $P_{gen}^{-1}(v)$ is a right continuous function (the discontinuities, if any, arising due to Γ_{iron}^{-1}) with horizontal intervals where Φ_{iron} is horizontal.

C Sketch of Model with Alternative Matching Assumptions (not for publication)

We now provide a brief sketch of how the model and the optimal stationary fee-setting mechanism work under the alternative matching assumption mentioned at the beginning of Section 4. All assumptions are maintained here unless stated otherwise.

We let $\nu \in (0, 1)$ be the probability that a seller does not stay with the same broker from one period to the next, conditional on not selling and on staying in the market, and we confine attention to stationary fee-setting mechanisms. These mechanisms have the following properties. The broker announces a fee function $\omega(p)$ to the seller, which will be used for all transactions of this seller through this broker. The seller chooses a price p

for which he will offer his good while selling through this particular broker. Notice that we only restrict the intermediary to choosing a stationary mechanism, the seller's choice of a stationary price is optimal given the stationarity of his problem. In every period, a new buyer arrives and accepts the offer if her dynamic valuation v, which is drawn from the distribution F, is above the price p. Thus, the probability of trade at price p in any given period is $1-F(p)$. Interpreting the discount factor as the probability that the world does not end, the probability that a seller who sets the price p will ever sell through the broker he is currently matched to is thus $(1-F(p))(1+(1-\nu)\delta F(p)+((1-\nu)\delta F(p))^2+..)$. Taking this geometric sum until infinity, we get the "ultimate discounted probability of trade through the current broker"

$$\frac{1-F(p)}{1-(1-\nu)\delta F(p)} =: 1-F_{\nu\delta}(p).$$

Notice that $F_{\nu\delta}(p)$ is increasing, 0 at $\underline{v}$, 1 at $\overline{v}$, and can therefore be seen as a cumulative distribution function with support $[\underline{v},\overline{v}]$. As before denote by c the seller's dynamic type, which is given as the sum of his static type $\tilde{c}$ and the expected discounted value of his option value of trade outside the broker he is currently matched to $W_S^{\text{outside}}(c,\tilde{c})$, which will be determined shortly. Note that because we restrict ourselves to stationary fee-setting mechanisms, the broker's profit maximization problem when choosing the optimal fee function $\omega(p)$ is the same as if there were only one buyer whose valuation is drawn from a distribution $F_{\nu\delta}(v)$. Consequently, our results on the optimal fee-setting mechanism carry over and we get, for the optimal fee $\omega_{\nu\delta}(p)$ set by the broker and the optimal price $P_{\nu\delta}(c)$ set by the seller with dynamic type c,

$$\omega_{\nu\delta}(p) = p - E_{v\sim F_{\nu\delta}}[\Gamma^{-1}(\Phi_{\nu\delta}(v))|v \geq p] \quad \text{and} \quad P_{\nu\delta}(c) = \Phi_{\nu\delta}^{-1}(\Gamma(c)),$$

where $\Phi_{\nu\delta}$ is the virtual valuation function associated with $F_{\nu\delta}$. Letting $D_S(c) = P_{\nu\delta}(c) - \omega_{\nu\delta}(P_{\nu\delta}(c))$ and $\rho_S(c) = 1-F(P_{\nu\delta}(c))$, the total option value of future trade $W_S(c,\tilde{c})$, which includes trade through the current broker and trade outside the current broker, is

$$W_S(c,\tilde{c}) = (D_S(c)-\tilde{c})\rho_S(c) + (1-\rho_S(c))\delta W_S(c,\tilde{c}). \tag{25}$$

Similarly, the option value of inside trade $W_S^{\text{inside}}(c,\tilde{c})$ – that is, the value of future trade through the current broker – is

$$W_S^{\text{inside}}(c,\tilde{c}) = (D_S(c) - \tilde{c})\rho_S(c) + (1 - \rho_S(c))\delta(1-\nu)W_S^{\text{inside}}(c,\tilde{c}), \tag{26}$$

and the option value of outside trade is $W_S^{\text{outside}}(c,\tilde{c}) = W_S(c,\tilde{c}) - (1-\nu)W_S^{\text{inside}}(c,\tilde{c})$. Using the above two equations, this can be shown to be the same as

$$W_S^{\text{outside}}(c,\tilde{c}) = \nu\frac{(D_S(c) - \tilde{c})\rho_S(c)(1-\rho_S(c))\delta}{(1-(1-\rho_S(c))\delta)(1-(1-\rho_S(c))(-\nu)\delta)}. \tag{27}$$

Observe that $W_S^{\text{outside}}(c,\tilde{c}) = W_S(c,\tilde{c})$ if $\nu = 1$ and $W_S^{\text{outside}}(c,\tilde{c}) = 0$ if $\nu = 0$. Since $c = \tilde{c} + \delta W_S^{\text{outside}}(c,\tilde{c})$, this implies that the dynamic type coincides with the static type when $\nu = 0$. Further, at $\nu = 1$ the dynamic type in this model is identical to the dynamic type in the main model.

The analogue to equation (1) in this variant of the model, which relates the entrant static type distribution $\tilde{g}_0(\tilde{c})$ to the steady-state static type distribution $\tilde{g}(\tilde{c})$, is

$$\sigma(1-(1-\rho_S(S(\tilde{c})))\epsilon)\tilde{g}(\tilde{c}) = \tilde{g}_0(\tilde{c}). \tag{28}$$

The distribution of primary importance for any given broker is the distribution of seller types he faces, which are newly matched to him. We denote the density of the distribution of these static types by $\tilde{g}_1(\tilde{c})$, which is given by $\sigma\tilde{g}_1(\tilde{c}) = (\tilde{g}_0(\tilde{c}) + \nu(\sigma\tilde{g}(\tilde{c}) - \tilde{g}_0(\tilde{c})))$, because all new entrants and fraction ν of sellers who did not exit in the previous period are available for a new match. Notice that as ν becomes smaller, $\tilde{g}_1(\tilde{c})$ puts more weight on the entrant distribution $\tilde{g}_0(\tilde{c})$. The distribution of dynamic types that is of concern for the brokers can now readily be derived based on $\tilde{g}_1(\tilde{c})$ and the relationship between static and dynamic types derived above.

Both the cumulation effect and the way the seller's option value of future trade enters the broker's mechanism design problem depends on ν. ν can be interpreted in the following way. For $\nu = 1$, the competitive threat is the greatest. For $\nu = 0$, there is no competitive threat, since the seller has to stay with the same broker forever. Note that our results carry over qualitatively to a setup, in which there is at least some degree of competitive threat, that is, $\nu > 0$.

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