Market Structure and the Competitive Effects of Vertical Integration*

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Abstract

We analyze the competitive effects of backward vertical integration when firms exert market power upstream and compete à la Cournot downstream. Contrasting with previous literature, a small degree of vertical integration is always procompetitive because efficiency gains dominate foreclosure effects, and vertical integration even to full foreclosure can be procompetitive. Surprisingly, vertical integration is more likely to be procompetitive if the industry is otherwise more concentrated. Extensions analyze incentives to integrate and differentiated Bertrand competition downstream. Our analysis suggests that antitrust authorities should be wary of vertical integration when the integrating firm faces many competitors and should be permissive otherwise.

Keywords: Vertical Integration, Market Structure, Downstream Oligopsony, Competition Policy.

JEL-Classification: D43, L41, L42

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1 Introduction

The effects of vertical integration on consumer surplus and overall welfare are subjects of ongoing debate amongst economists, antitrust lawyers, and policy makers. Over the last two decades substantial progress has been made in identifying pro- and anticompetitive effects of vertical integration. Productivity increases due to cost synergies have been advanced as a major source of efficiency gains from vertical integration while the ability of integrating parties to raise their rivals' costs has been recognized as a factor fostering foreclosure.[1] Yet, an open theoretical question of substantial practical relevance is how these effects depend on the underlying market structure. In particular, is vertical integration more likely to harm consumers when the industry consists of many competitors, or should antitrust authorities be more vigilant when the integrating firm's competitors are small in number and exert substantial market power?

To shed light on these questions we present a model that permits us to study the competitive effects of vertical integration as a function of the underlying market structure and of the degree of vertical integration, taking into account both productivity gains and incentives to raise rivals' costs. The following is a sketch of our basic model, which builds on Riordan (1998). There are a number of non-integrated firms and one partly vertically integrated firm. All firms exert oligopolistic market power downstream, where they compete à la Cournot, and oligopsonistic market power upstream. To produce the final good, firms need a fixed input, termed capacity, that is competitively offered on an upward sloping supply curve. The more capacity a firm purchases on the market, the lower is its marginal cost of producing the final good. The vertically integrated firm produces some capacity internally. This is referred to as its degree of vertical integration. It can be as low as zero or so large that the integrated firm completely forecloses its rivals, or anything in between. The degree of vertical integration plays an important strategic role: The internally produced units are protected from the capacity price increase when the integrated firm purchases additional capacity. So an increase in the degree of vertical integration induces the integrated firm to behave more aggressively on the input market. Therefore, increases in the degree of vertical integration lead to increases in the market price of capacity, which raises the costs of the integrated firm's rivals and thus leads to (partial) foreclosure. At the same time, as the degree of vertical integration increases, the marginal

[1] For recent surveys on the effects of vertical integration, see Church (2008), Rey and Tirole (2007) and Riordan (2008).

production cost of the integrated firm falls, due to the increased purchase of capacity. Hence, the firm produces more output. Thus, our model explicitly allows for productivity gains from vertical integration.[2] In what follows, we use the term "procompetitive" ("anticompetitive") effects as a shorthand expression for saying that consumer surplus increases (falls) following increases in the degree of vertical integration.[3]

Within this setup, we obtain the following results. First, vertical integration is more likely to be procompetitive (i) the *more* concentrated is the industry, i.e., the fewer are the non-integrated rivals, and (ii) the smaller is the degree of integration.[4] While result (ii) is arguably as one would expect, result (i) is probably surprising.[5] However, a clear intuition for this result based on our model exists and will be provided below. It implies that antitrust authorities should be more vigilant vis-à-vis vertical mergers when there is a *larger* number of rival firms in the industry. We also demonstrate that the effects from vertical integration on consumer surplus can be substantial even if the number of firms is large. Second, vertical integration is procompetitive under a fairly wide array of circumstances. In the extreme, even complete foreclosure of the non-integrated firms can enhance consumer surplus because the integrated firm expands its quantity by a large extent after integrating.[6] Third, we show that, as the number of competitors becomes large, vertical integration is anticompetitive irrespective of the degree of vertical integration. In the limit, our model thus yields Riordan's (1998) powerful result that vertical integration by a dominant firm who faces a competitive fringe is always anticompetitive.[7] Fourth, even if it is procompetitive, vertical integration is not necessarily welfare increasing. Procompetitive but welfare reducing mergers are possible because vertical integration changes the cost structure in the industry. Fifth, endogenizing the degree of vertical integration in an extension, we find that the privately optimal degree of integration is smaller than the ones that maximize social welfare and consumer surplus when the number

[2]Such productivity gains are not only commonly advanced by merging parties as motivation for their desire to vertically integrate but they are also well documented empirically (see e.g., Hortaçsu and Syverson, 2007). Church (2008) argues that one of the reasons why vertical mergers are complicated to evaluate is that the incentives to integrate often arise because of non-price efficiencies and are usually not attributable to market power effects.

[3]Similarly, we use the term "efficiency" effects when referring to the effects on total welfare.

[4]When saying that something is more (less) likely we mean that it occurs for a larger (smaller) set in the parameter space.

[5]For example, Lafontaine and Slade (2007) note that most empirical studies on vertical integration are conducted for highly concentrated markets because evidence for foreclosure is thought most likely to be found there.

[6]This point is related to but distinct from Quirmbach's (1986) observation that consumer prices fall *after* vertical integration to monopoly is complete. Our result is that consumer prices can fall along all the way towards complete foreclosure.

[7]This means also that the dominant firm model provides a good approximation to nearby market structures.

of non-integrated rivals is small but above those thresholds when the number is large. Therefore, explicit consideration of the incentives to integrate reinforces the message that antitrust authorities should be the more vigilant vis-à-vis vertical mergers the *larger* is the number of rivals. Moreover, this same message remains valid when we consider differentiated Bertrand competition rather than Cournot competition in the downstream market, in which case vertical integration is even more likely to be procompetitive.

Let us now develop the basic intuition for these results, starting with a few preliminaries. If a firm purchases more capacity, it faces lower production costs and produces a larger quantity. This implies that firms with higher capacities incur larger inframarginal losses from a price decrease. Therefore, a firm with a larger capacity utilizes its capacity less intensively. Conversely, firms with little market power have marginal costs that are approximately equal to price — much like fringe firms in the dominant firm model — and utilize their capacity intensively. By increasing the degree of vertical integration, the integrated firm purchases more capacity because its internally produced units are protected from the capacity price increase. This increases both the aggregate capacity level and the market clearing price for capacity. Therefore, rival firms purchase less capacity. That is, capacities are strategic substitutes. Vertical integration has the strongest negative effect on consumer surplus if rival firms have little to no market power. Operating already close to the point where marginal costs equal price, their only way to adapt is to decrease their output. In contrast, if rival firms exert market power themselves, the anticompetitive effect of reducing the capacity available to them will be partly offset because smaller capacities induce them to use capacity more intensively. In other words, market power of rival firms mitigates the anticompetitive foreclosure effect of vertical integration.

Based on these preliminary observations, rather intuitive explanations for our main results are now at hand. In an industry with a large number of competitors, the market power of each non-integrated firm is low. As noted above, the non-integrated firms therefore operate relatively efficiently. Vertical integration shifts production from the non-integrated firms to the integrated firm, which utilizes its capacity less intensively. As a consequence, vertical integration is more likely to be anticompetitive when the industry consists of a large number of firms. This is also the reason why, in the limit as the number of firms grows large, our model encompasses the case with a dominant firm who faces a competitive fringe, in which vertical integration is always anticompetitive.

When the degree of vertical integration becomes larger, the output expansion of the integrated firm becomes smaller due to the lower capacity utilization. At the same time, the output reduction of its non-integrated rivals increases. Therefore, vertical integration is more likely to be anticompetitive the larger is the integrated firm's degree of vertical integration.

As vertical integration leads to larger capacity purchases of the integrating firm, aggregate capacity employed in the industry rises. This effect countervails the effect that the integrated firm utilizes its capacity less intensively. If all firms have considerable market power, the effect of less intensive capacity utilization is small. Therefore, the dominating effect is that aggregate capacity increases, implying that even vertical integration to full foreclosure can be procompetitive.

In determining the effects of vertical integration on social welfare rather than on consumer surplus, one needs additionally to account for the costs of production. As vertical integration shifts capacity to the integrated firm that utilizes it less intensively, aggregate costs of production increase with vertical integration. This may render vertical integration welfare reducing even when it is consumer surplus enhancing.

Finally, with differentiated Bertrand competition downstream, an increase in vertical integration leads to a decrease in the price of the integrated firm because it reduces its cost of production. In contrast, the prices of its non-integrated rivals rise because their costs of production increase. Consumers then optimally substitute away from the more expensive products of the rivals. This mitigates the reduction in surplus they incur because of the price increases of the non-integrated firms while the benefits due to the decreased price of the integrated firm becomes larger, which tends to make vertical integration procompetitive.

Our paper is most closely related to Riordan (1998), whose setup includes a dominant, partly integrated firm facing a competitive fringe. We extend this by allowing the integrated firm's rivals to exert market power as well. Riordan's model is a notable exception within the theoretical literature on vertical integration because it incorporates exercise of market power by a single firm in both markets whereas most of this literature is concerned with the trade-off between avoidance of double marginalization, that is, the exercise of market power by different firms, and foreclosure. For example, Hart and Tirole (1990), Ordover, Saloner, and Salop (1990) and Chen and Riordan (2007) are only concerned with foreclosure motives. In Salinger (1988), Choi and Yi (2000), Chen (2001) and Inderst and Valletti (2011a), the downstream market is comprised of an oligopoly and both effects are present but downstream firms have no market power in the intermediate goods market.[8]

A different approach to vertical integration is developed by De Fontenay and Gans (2005),[9] in which there is efficient bilateral bargaining between pairs of upstream and downstream firms.[10] As Gans (2007) notes, the bargaining approach fits relatively well to an industry with few upstream and downstream firms, while in our model, the input is supplied competitively, which corresponds to general mass markets for inputs.

A paper that, like ours, considers a competitive upstream industry is Esö, Nocke, and White (2010). They study a model in which competing downstream firms bid for scarce upstream capacity and show that if this capacity is sufficiently large, the asymmetric downstream market structure analyzed here and in Riordan (1998) emerges endogenously.[11]

As in most of the literature, we consider the case of one-shot interaction between firms. An important exception is the paper by Nocke and White (2007),[12] who consider a dynamic model and show that vertical integration facilitates upstream collusion because it reduces the number of buyers for rival firms, which decreases their incentives to deviate from a collusive agreement.[13]

Our model is also broadly consistent with recent evidence. In a comprehensive review of empirical studies on the effects of vertical integration for several highly concentrated industries, Lafontaine and Slade (2007) find that the efficiency effect dominates the foreclosure effect in almost all studies, and that, therefore, vertical integration has led to a fall in the final good price in almost all cases. In a similar vein, Hortaçsu and Syverson (2007) find that vertical integration in the cement and ready-mixed concrete industries has led to output increases and price decreases and show that these effects can be attributed to productivity increases that

[8]Hendricks and McAfee (2010) present a model with both effects, where upstream and downstream firms exert market power in the intermediate goods market. However, when analyzing vertical mergers, they keep the downstream price fixed and suppose that the market structure consists of no vertical integration at the outset in order to keep the model tractable. Under these assumptions they show that output increases with vertical mergers. In contrast, in our model the downstream price is flexible and, as argued above, we show that a crucial variable to determine the competitive effects of vertical integration is the degree to which there is already integration.

[9]This approach is used by Gans (2007) to derive concentration measures for vertical and horizontal mergers in an oligopolistic vertical market structure.

[10]For a related analysis of vertical integration with multilateral bargaining, see Bolton and Whinston (1993).

[11]Inderst and Valletti (2011b) consider a model with take-it-or-leave-it offers of an upstream firm but without vertical integration. They allow for one buyer to be larger than the others and show that this buyer obtains a favorable deal because its outside option is higher. However, this leads to higher wholesale prices for rival buyers to the detriment of consumers.

[12]For a similar analysis but with a different upstream pricing regime, see Normann (2009).

[13]We also concentrate on the case of a single (or marginal) vertical merger. Recently, Nocke and Whinston (2010) considered the case in which multiple horizontal mergers might arise over time and showed under which conditions the optimal policy for an antitrust authority is myopic.

arise from firm size.

The remainder of the paper is organized as follows. Section 2 lays out the model and Section 3 presents the equilibrium. In Section 4 we derive the competitive effects of vertical integration and show how these effects change with the number of firms in the industry. Section 5 analyzes the effects of vertical integration on social welfare. In Section 6 we study the incentives to acquire capacity. Section 7 considers price competition with differentiated goods and Section 8 concludes. All proofs are in the appendix.

2 The Model

There are two types of firms, one (partially) vertically integrated firm, which we index by I and $N \geq 1$ non-integrated firms. A typical non-integrated firm is indexed by j. All firms produce a homogenous good and compete à la Cournot on the downstream market. The inverse demand function is $P(Q)$, where $P(Q)$ is the market clearing price for the aggregate quantity $Q \equiv q_I + \sum_{j=1}^{N} q_j$ satisfying $P'(Q) < 0$. To produce the final good firms require a fixed input, referred to as capacity. The cost function of firm $i = I, 1, ..., N$ for production of q_i units is given by

$$c(q_i, k_i) = k_i C\left(\frac{q_i}{k_i}\right),$$

where k_i is firm i's capacity and $C'(q_i/k_i) \geq 0$ and $C''(q_i/k_i) > 0$. This type of cost function, introduced by Perry (1978) and used e.g., by Perry and Porter (1985), Riordan (1998) and Hendricks and McAfee (2010), presumes that capacity is combined with variable inputs to produce the final good.[14] This cost function is homogeneous of degree 1 in the vector (q_i, k_i), which is an implication of a production technology that exhibits constant returns to scale.

Capacity is supplied competitively with an inverse supply function of $R(K)$, with $R'(K) > 0$ and $K \equiv k_I + \sum_{j=1}^{N} k_j$, i.e., K is the aggregate purchase of capacity. Firm I is partially vertically integrated, that is, it produces $\underline{k} \geq 0$ units of capacity internally. We refer to $\underline{k}$ as its degree of vertical integration, which is taken as given.[15]

The timing of the game is as follows: In the first stage, the capacity stage, all firms i simultaneously choose their level of capacity k_i. The degree of vertical integration $\underline{k}$ is common

[14] We focus on the case in which the integrated firm and the non-integrated firms have the same cost function. Riordan (1998) allows firm I to have a fixed cost advantage per unit of output in addition to being vertically integrated. We provide the analysis of this extension on Appendix B [not intended for publication] and show that our results carry over to this case.

[15] Section 6 provides the analysis of an augmented model in which $\underline{k}$ is determined endogenously.

knowledge. Firm I purchases $k_I - \underline{k}$ units of capacity at the market price $R(K)$. Thus, the profit function of firm I is given by

$$\Pi_I(q_I, k_I) = P(Q)q_I - k_I C\left(\frac{q_I}{k_I}\right) - (k_I - \underline{k})R(K), \tag{1}$$

and the one of a non-integrated firm j is $\Pi_j(q_j, k_j) = P(Q)q_j - k_j C(q_j/k_j) - k_j R(K)$. In the second stage, the quantity stage, all firms simultaneously choose their quantities after having observed all capacity levels $\mathbf{k} = (k_I, k_1, .., k_N)$. The aggregate quantity Q determines the market clearing price $P(Q)$, and payoffs are realized.

Equation (1) implies that firm I has the opportunity to supply undesired capacity to an outside market, which occurs if $k_I < \underline{k}$, that is, $\underline{k}$ is not firm-specific. An increase in the degree of vertical integration reduces the number of inframarginal units of capacity $k_I - \underline{k}$ on which the integrated firm bears the market clearing price $R(K)$ when purchasing additional units of capacity. Because the firm supplies $\underline{k}$ units internally, it captures internally the effect of a higher capacity price on the first $\underline{k}$ units. As we will see shortly, this reduction in the number of inframarginal units induces the integrated firm to purchase capacity more aggressively because the effective cost of capacity is lower.

We focus on symmetric subgame perfect equilibria, where symmetry means that the non-integrated firms play the same strategies. To ensure interior solutions and a unique equilibrium, we make some shape assumptions on the demand, supply and cost function. We suppose that $\lim_{Q\to\infty} P(Q) = 0$, that $P''(Q)$ is not too positive and that $P'''(Q)$, $C'''(q_i/k_i)$ and $R''(K)$ are not too negative. These assumptions are relatively mild and guarantee a unique equilibrium. A special case that satisfies these assumptions is the linear-quadratic model, in which $P(Q) = \alpha - \beta Q$ for $Q \in [0, \alpha/\beta]$, $R(K) = \delta K$, $C(q_i/k_i) = \frac{c}{2}\left(\frac{q_i}{k_i}\right)^2$, where α, β, c and δ are positive constants.

3 Equilibrium

We solve the game by backward induction.

3.1 The Quantity Stage (Stage 2)

At the quantity stage, $\mathbf{k}$ is already determined. As $\underline{k}$ has a direct effect only on k_I but not on q_I, the first-order condition for a profit maximum for each firm does not depend directly on $\underline{k}$. Consequently, the first-order condition of any firm $i \in \{I, 1, .., N\}$ in the subgame of the quantity stage is given by[16]

$$P + P'q_i = C_i'. \tag{2}$$

It is easy to see that the second-order conditions are satisfied given that P'' is not too positive, which we assumed above. Our assumptions also imply that firm i's reaction function has a negative slope greater than -1. Therefore, every quantity-stage subgame has a unique equilibrium. We denote by $q_i^*(\mathbf{k})$ the equilibrium quantity of firm i, given any vector of capacities $\mathbf{k}$. From the first-order conditions we get the following intuitive lemma.

Lemma 1

$$\frac{dq_i^*(\mathbf{k})}{dk_i} > 0 \quad and \quad \frac{dq_i^*(\mathbf{k})}{dk_j} < 0 \quad for\ all \quad i \neq j,\ i,j = I, 1, ..., N.$$

Therefore, all own effects are positive and all cross effects are negative. That is, a firm's optimal quantity increases in its own capacity and falls in the capacity of its rivals, independently of the type of the firm. Next we obtain the following result:

Lemma 2 $\frac{q_i^*(\mathbf{k})}{k_i}$ *decreases in* k_i $\forall i \in \{I, 1, ..., N\}$.

The same result is obtained by Riordan (1998). As observed above, a firm with a larger capacity produces a larger quantity, but because it produces more inframarginal units, it suffers more from a fall in the final output price. As a consequence, it utilizes its capacity less intensively than firms with lower capacity. This means that q_i^*/k_i is smaller.

3.2 The Capacity Stage (Stage 1)

We now move on to the first stage of the game, the capacity choice game. Using the envelope theorem and dropping all arguments, the first-order condition of a non-integrated firm j in the capacity stage is given by

$$\frac{\partial \Pi_j}{\partial k_j} = P' \frac{dQ_{-j}^*}{dk_j} q_j^* - C_j + C_j' \frac{q_j^*}{k_j} - R - k_j R' = 0, \tag{3}$$

where Q_{-j}^* is the equilibrium quantity of all firms but firm j. The first-order condition of the integrated firm I is given by

$$\frac{\partial \Pi_I}{\partial k_I} = P' \frac{dQ_{-I}^*}{dk_I} q_I^* - C_I + C_I' \frac{q_I^*}{k_I} - R - (k_I - \underline{k}) R' = 0. \tag{4}$$

[16]To simplify notation, in the following we abbreviate $P(Q)$ by P, $C(q_i/k_i)$ by C_i and $R(K)$ by R. We do so also for all derivatives.

Showing that an equilibrium exists and, if it does, is unique is more involved in the capacity stage than in the quantity stage. The reason is that now a change in firm i's capacity has an effect on the equilibrium quantity of each firm in the second stage. Thus, the expression for the reaction function is more complicated than in a standard single stage game.[17] Nevertheless, the next lemma establishes that an equilibrium exists and is indeed unique.

Lemma 3 *There exists a unique symmetric equilibrium in the capacity stage. In this equilibrium, k_I^* and k_j^*, $j = 1, ..., N$, are determined by* (3) *and* (4)*.*

From the two first-order conditions we can now derive the following lemma:

Lemma 4 *Capacities are strategic substitutes, that is,*

$$\frac{dk_i}{dk_j} < 0 \quad \textit{for all} \quad i \neq j,\ i, j = I, 1, ..., N.$$

The lemma states that capacity choices between any two firms are strategic substitutes. Two economic forces are the key to the intuition for this result: First, in stage 2 quantity choices of the firms are strategic substitutes, and this translates to strategic substitutability of the capacity choices. As shown in Lemma 1, if a firm purchases more capacity in stage 2, it will increase its quantity, thereby inducing the rivals to lower their quantities. When producing a lower quantity, an additional unit of capacity becomes then less valuable for each rival firm, implying that each rival optimally lowers its capacity. Second, if a firms purchases more capacity, the capacity price R rises. Therefore, buying capacity becomes more expensive for the rivals, inducing each of them to lower its optimal capacity.

Drawing on the previous lemma, the next result then states how equilibrium capacity choices are affected by a change in $\underline{k}$:

Lemma 5

$$\frac{dk_I^*}{d\underline{k}} > 0 \quad \textit{and} \quad \frac{dk_j^*}{d\underline{k}} < 0, \quad j = 1, ..., N.$$

That k_I^* increases and k_j^* decreases in $\underline{k}$, is intuitive. If $\underline{k}$ increases, firm I produces more capacity units internally. Thus, firm I's marginal opportunity costs of purchasing additional units of capacity are lower, because the number of inframarginal units for which it has to

[17] Moreover, the game is not an aggregative game. The reaction of a non-integrated firm is different if firm I changes its capacity than if a non-integrated firm changes its capacity because this has different effects on the overall quantity produced in the second stage.

pay the capacity price R on the upstream market decreases. As a consequence, firm I finds it optimal to increase its overall amount of capacity. While $\underline{k}$ does not directly influence the optimal capacity of the non-integrated firms, we know from Lemma 4 that capacities are strategic substitutes. Therefore, each non-integrated firm optimally acquires less capacity as $\underline{k}$ rises. Hence, non-integrated firms become (partially) foreclosed as $\underline{k}$ increases.

It follows immediately from Lemma 5 that $k_I^* > k_j^*$ for $\underline{k} > 0$, i.e., $\underline{k} > 0$ commits firm I to a more aggressive reaction in stage 1. Thus, if firm I is vertically integrated to some extent, its equilibrium capacity is larger than the one of the non-integrated firms. From Lemma 2 we know that this implies that its capacity utilization q_I^*/k_I^* is lower than for the non-integrated firms.

4 Competitive Effects of Vertical Integration

We now turn to the analysis of the effects of vertical integration on consumer surplus. As competition authorities both in the U.S. and in Europe base their decisions mainly on the effects on consumer surplus, this analysis is most relevant for competition policy.

4.1 Competitive Threshold

We first analyze under which conditions vertical integration is pro - or anticompetitive, i.e., whether a marginal change in $\underline{k}$ increases or decreases the aggregate equilibrium quantity supplied in the downstream market. From above it follows that an increase in $\underline{k}$ has a direct positive effect on k_I and an indirect negative effect on all k_j.[18] This in turn leads to an increase in q_I and to a decrease in all q_j. Thus, vertical integration is procompetitive at the margin if and only if

$$\frac{dQ}{d\underline{k}} = \left(\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}\right)\frac{dk_I}{d\underline{k}} + N\left(\frac{dq_I}{dk_j} + \frac{dq_j}{dk_j} + (N-1)\frac{dq_i}{dk_j}\right)\frac{dk_j}{d\underline{k}} > 0, \quad i \neq j,$$

or equivalently

$$\frac{\left(\frac{dk_j}{d\underline{k}}\right)}{\left(\frac{dk_I}{d\underline{k}}\right)} > -\frac{\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}}{N\left(\frac{dq_I}{dk_j} + \frac{dq_j}{dk_j} + (N-1)\frac{dq_i}{dk_j}\right)}, \quad i \neq j. \tag{5}$$

The left-hand side of (5) expresses the relative change of a non-integrated firm's capacity with $\underline{k}$ to the change in the integrated firm's capacity at the equilibrium. Because capacity choices are strategic substitutes, this relative change is negative. The right-hand side gives a benchmark

[18] To simplify notation here and in what follows we omit the superscript $*$ on equilibrium quantities and capacities.

against which to compare this term. The inequality says that if the relative change is small enough in absolute terms, then vertical integration is procompetitive. Intuitively, if k_j does not fall by much after firm I becomes more integrated, the positive effect resulting from the increase in q_I dominates the negative effect that stems from the decrease in q_j of all non-integrated firms.

Inserting the respective derivatives (derived in the proof of Lemma 1) into the right-hand side of (5) and simplifying yields

$$\left(\frac{dk_j}{d\underline{k}}\right) / \left(\frac{dk_I}{d\underline{k}}\right) > -\frac{C_I'' \frac{q_I}{k_I}(C_j'' - k_j P')}{N C_j'' \frac{q_j}{k_j}(C_I'' - k_I P')}. \tag{6}$$

To gain some intuition for this formula suppose that $\underline{k}$ is zero. In this case all $N+1$ firms are the same and we have $q_I = q_j$, $k_I = k_j$ and thus $C_I'' = C_j''$. As a consequence, the right-hand side of (6) simplifies to $-1/N$. Thus, to keep overall output constant, the aggregate capacity reduction of the non-integrated firms must be the same as the increase in the capacity of firm I. Because all N non-integrated firms are symmetric, each of them must lower its capacity by $1/N$ of the increase in the integrated firm's capacity.

Suppose now that $\underline{k} > 0$. From the above lemmas we know that in this case $k_I > k_j$, $q_I/k_I < q_j/k_j$ and thus $C_I'' < C_j''$. Then, the right-hand side of (6) is in absolute value smaller than $1/N$. The reason is that the integrated firm utilizes its capacity less intensively than a non-integrated firm. As a consequence, if all non-integrated firms reduced their capacity in sum by the same amount as the capacity increase of the integrated firm, overall output would fall because capacity is shifted to the less efficient firm. Thus, to keep output constant the reduction in capacity by non-integrated firms has to be smaller and overall capacity must rise.

To characterize how vertical integration changes overall output, we begin with the case where $\underline{k}$ is small.

Proposition 1 *For any finite N, there exists a competitive threshold $\underline{k}^* > 0$, such that for all $\underline{k} < \underline{k}^*$, vertical integration is procompetitive at the margin.*

Intuitively, if $\underline{k}$ is small, firm I utilizes its capacity only slightly less intensively than its non-integrated rivals. However, the aggregate reaction of the rivals to an increase in $\underline{k}$ is always smaller than the increase in k_I^*. Thus, the aggregate equilibrium capacity increases and overall output rises.

Next assume that $\underline{k}$ is so large that the equilibrium value of k_I^* is large enough to induce $k_j^* = 0$ for all $j \neq I$ and define $\bar{\underline{k}}$ as the degree of vertical integration at which $k_j^* = 0$. Observe that this implies $q_j^* = 0$. In words, at $\underline{k} = \bar{\underline{k}}$, only the integrated firm is active and its non-integrated rivals are fully foreclosed.[19] Accordingly, we refer to the case where $\underline{k}$ approaches $\bar{\underline{k}}$ as vertical integration to full foreclosure.

Proposition 2 *For any finite N, vertical integration to full foreclosure can be procompetitive at the margin.*

Thus, even marginal vertical integration that leads to a complete foreclosure of rival firms is not necessarily detrimental to consumer surplus. In addition, as we will show below, vertical integration to monopoly may not only be locally procompetitive, i.e., when $\underline{k}$ is close to $\bar{\underline{k}}$, but also globally, i.e., for any $\underline{k} \in [0, \bar{\underline{k}})$. This implies that starting from any $\underline{k} \in [0, \bar{\underline{k}})$ vertical integration to $\bar{\underline{k}}$ may maximize consumer surplus. This can occur because our model explicitly takes into account efficiency gains in production beyond pure avoidance of double marginalization. If a firm acquires such a large amount of capacity that its competitors stop producing, its production costs may become so low that it produces a larger quantity than the oligopoly quantity without the capacity increase.

Even though according to Proposition 2 vertical integration to full foreclosure can be procompetitive, it need not necessarily be so. The reason is that firm I utilizes its capacity less intensively as $\underline{k}$ rises. Thus, for vertical integration to be procompetitive, the decrease in k_j (relative to the increase in k_I) as a reaction to the rise in $\underline{k}$ must become smaller as $\underline{k}$ rises.

We now turn to the analysis of intermediate values of $\underline{k}$, that is, values of $\underline{k} \in (\underline{k}^*, \bar{\underline{k}})$. It is of particular interest to explore if there is a unique threshold of $\underline{k}$ below which vertical integration is procompetitive and above which vertical integration is anticompetitive. Moreover, if no such threshold exists, is vertical integration procompetitive over the whole range from 0 to $\bar{\underline{k}}$? The expressions that are involved in the calculations are too complicated to allow us to answer this question in general. Nonetheless, we are able to show that the threshold, provided it exists, is indeed unique for two important subclasses of the general specification. The first class consists of models where the supply function $R(K)$ is very inelastic.[20] The second class is the widely

[19] Such a $\bar{\underline{k}}$ necessarily exists because from Lemma 5 we know that $dk_I/d\underline{k} > 0$ and $dk_j/d\underline{k} < 0$. In addition, variable production costs $c(q_j, k_j)$ are decreasing in k_j because $C''(q_j/k_j) > 0$. Thus, both production and capacity costs are increasing in $\underline{k}$ for a non-integrated firm j, while revenue is decreasing because q_j is decreasing and q_I is increasing. So if $\underline{k}$ and therewith k_I is large enough, j's costs are too high relative to $P(Q)$, and so it is optimal for firm j to stop producing.

[20] A steeply increasing supply curve can be observed in many high technological industries. For example, dedicated fiber-optic cables or several semiconductor devices like customized integrated circuits that are produced in specialized plants exhibit large production costs that are steeply increasing once a plant produces close to its capacity limit. In many cases, a firm, which also produces downstream products, already owns some of these

used linear-quadratic specification introduced above.

Proposition 3 *Suppose either that (i) the supply function $R(K)$ is sufficiently inelastic, or that (ii) the model is linear-quadratic. Then, for any finite N vertical integration is always procompetitive or there exists a unique $\underline{k}^* \in (0, \bar{\underline{k}})$, such that vertical integration is procompetitive at the margin for all $\underline{k} < \underline{k}^*$ and anticompetitive at the margin for all $\underline{k} > \underline{k}^*$.*

The intuition for case (i) of the proposition is that if $R(K)$ is sufficiently inelastic, the capacity reaction of a non-integrated firm to a change in $\underline{k}$ is independent of the value of $\underline{k}$. Therefore, $(dk_j/d\underline{k})/(dk_I/d\underline{k})$ stays constant as $\underline{k}$ varies. However, the right-hand side of (6) is strictly increasing because firm I utilizes its capacity less intensively with further integration. Thus, there is a unique intersection point between the left-hand and the right-hand-side of (6). Case (ii) of the proposition is important because it shows that the threshold is unique (given that it exists) in the general linear-quadratic specification used in many industrial organization models. In addition, this indicates that the threshold is unique also for specifications that are close to the linear-quadratic one and suggests that the threshold may be unique even more generally.[21]

Comparison to the Dominant Firm Model Our result that the efficiency gains of vertical integration are often larger than the foreclosure effects contrasts with the findings obtained in the model where a dominant firm faces a competitive fringe. In both models, vertical integration increases rivals' costs and thereby leads to (partial) foreclosure. As fringe firms have no market power, their marginal cost is equal to the final consumer price. As a consequence of foreclosure, a positive mass of fringe firms exits the market, which has highly detrimental effects on the aggregate output because fringe firms utilize their capacity intensively. In contrast, oligopolistic non-integrated firms also exert market power, restricting their output to keep the final goods price high. Following an increase in vertical integration and the associated foreclosure effect, each oligopolistic rival uses its capacity more intensively (because it buys a smaller amount). Therefore, the detrimental effects of foreclosure through vertical integration are partly offset by rivals' more efficient production, whose marginal costs become closer to consumer prices as

plants while other firms need to purchase the specialized inputs from scratch. In our terminology, the firm that owns some plants would be considered vertically integrated.

[21] It may also be worth mentioning that, although we tried several different specifications, we have not found any counterexamples, i.e., cases where the left-hand and right-hand side of (6) are the same for different values of $\underline{k}$.

vertical integration increases. As a result, in the dominant firm model the output contraction of fringe firms after foreclosure is larger than the reaction of rival firms under oligopoly.

Relation to Horizontal Mergers It is also instructive to contrast the results of our model with those arising in models of horizontal mergers such as Perry and Porter (1985). For that purpose, consider a game that only consists of stage 2 of our model, where each of the $N+1$ firms compete à la Cournot on a product market, with each firm being endowed with some units of capacity. Following Perry and Porter (1985), we can then analyze the effects of a merger between two firms, where the newly merged firm uses the aggregate capacity of both stand-alone firms.

There are three substantial differences between such horizontal mergers and our analysis of vertical integration. First, the explicit modeling of an upstream market for capacity makes firms' capacities endogenous. This allows us to focus on equilibrium values of capacities rather than given capacities, which is the working assumption in the analysis of horizontal mergers.[22] Second, in stage 2 of our model a horizontal merger is always anticompetitive because it decreases the number of firms and because larger firms utilize their capacity less intensively (Lemma 2). In contrast, vertical integration even to full foreclosure can be procompetitive. Therefore, despite the similarity of the economic forces at work, the models' predictions are almost reversed, and so are the models' implications for antitrust policy. Third, an important lesson from the analysis of Perry and Porter (1985) is that the profitability of horizontal mergers hinges critically on the (exogenous) endowment of capacities. For example, if all firms are symmetric, it is easily shown that a horizontal merger is not profitable for more than three firms. In contrast, marginal increases in $\underline{k}$ at $\underline{k} = 0$ – that is, when all firms are ex ante symmetric – are always profitable in our model, as we will show in Section 6 below. Therefore, the models' predictions are, again, almost reversed.

4.2 Anticompetitive Integration with Many Rival Firms

We now consider the effect of a change in the number of firms on the competitive effects of vertical integration. Understanding this relationship is particularly relevant for antitrust policy. We start by looking at the case in which the number of firms becomes large. This case is also of theoretical interest as the limit corresponds to the model with a dominant firm facing a competitive fringe.

[22] For similar arguments, see also Riordan (1998).

Proposition 4 *If $N \to \infty$, then vertical integration is anticompetitive for all $\underline{k} \in [0, \bar{\underline{k}}]$.*

Hence, as the number of firms grows large, vertical integration is always anticompetitive. Intuitively, the aggregate reaction of the non-integrated firms to an increase in $\underline{k}$ is larger, the more firms are in the market. Therefore, the aggregate capacity reduction and, hence, the quantity reduction of the non-integrated firms increases in their number. As N goes to infinity, this effect dominates any cost advantage of the integrated firm. Thus in the limit, as the market power of the non-integrated firms vanishes, we obtain the result of Riordan (1998). As the integrated firm has no first-mover advantage in our model, but has one in Riordan's, Proposition 4 also shows that his strong result stems genuinely from the dominant firm's market power rather than from the first-mover advantage.[23]

We now turn to the case of N being finite, and analyze how $\underline{k}^*$ changes with N. From the previous subsection we know that a unique threshold $\underline{k}^*$ exists in the linear-quadratic specification or if the supply function is sufficiently inelastic. For tractability reasons we therefore restrict ourselves to these cases. In the following, we denote the threshold as $\underline{k}^*(N)$ to explicitly account for its dependence on N. We start with the case of a sufficiently inelastic supply function. Here we obtain the following result:

Proposition 5 *Suppose that $R(K)$ is sufficiently inelastic. If $C(\cdot)$ is quadratic, then $\underline{k}^*(N)$ is strictly decreasing in N.*

Although Proposition 5 is restricted by the assumption that $C''(\cdot)$ is a constant, the basic insight does not seem to be confined to this condition. It is easy to demonstrate numerically that the result also holds for $C(q_i/k_i) = (q_i/k_i)^g$ with $g > 1$.

Proposition 5 shows that, with quadratic costs and a sufficiently inelastic supply of capacity, the competitive threshold $\underline{k}^*(N)$ decreases in the number of non-integrated rivals the integrated firm faces. While the result may come as a surprise at first glance, the intuition behind it is relatively simple. As the number of non-integrated firms increase, each of them becomes smaller and thus utilizes its capacity more intensively. Because the non-integrated firms are foreclosed through integration, overall capacity utilization in the industry falls. This effect is more likely to dominate the countervailing force that integration leads to an increase in the overall capacity, if the number of non-integrated rivals is larger.

[23] That Riordan's result is obtained as the limit of an imperfectly competitive model is not trivial and has been an open question hitherto.

For the linear-quadratic specification, numerical computations also demonstrate that the threshold $\underline{k}^*(N)$ decreases in N.[24] This is displayed in Figure 2 in Section 6, where $\underline{k}^*(N)$ is depicted by $\underline{k}^*$. As the figure shows, there is a flat segment at the beginning. This is because for $N = 1$ and $N = 2$ vertical integration is procompetitive for all $\underline{k}$, i.e., with few non-integrated rivals vertical integration to full foreclosure is procompetitive. For these values the curve does not depict $\underline{k}^*$ but $\bar{\underline{k}}$.

We note that this result, like all our previous results, also holds if all rival firms are vertically integrated to the same extent. That is, none of our results depend on only one firm being integrated.[25] Interestingly, when all rival firms are integrated, the competitive threshold increases in the degree of integration of the rivals because more integrated rivals are less responsive to increases in the input price, which reduces the foreclosure effect. This insight puts additional emphasis on the notion that antitrust authorities should be less wary of vertical integration the more market power the integrating firm's rivals have.

4.3 Quantifying the Effects of Vertical Integration

So far we have been focusing on the direction or sign of output changes upon vertical integration. This leaves open the question how important these effects are quantitatively.

To shed light on this question, we use numerical computations for the linear-quadratic model. Figure 1a displays the percentage change in consumer surplus $CS(\underline{k})$ as a function of N when the integrated firm's degree of vertical integration increases marginally while its downstream market share is kept fixed at 50%.[26,27] The results displayed in Figure 1a show that the marginal effect of vertical integration is positive and large when N is small and negative yet still sizeable in absolute terms when N is large. Additionally, vertical integration is often not a continuous process but involves acquiring a non-negligible fraction of the intermediate good market. Thus, the computation shows that even in industries with a large number of rivals, the absolute effect of a discrete vertical merger is sizeable.

Another important feature of our model is that vertical integration up to full foreclosure of the non-integrated firms can enhance consumer surplus. Figure 1b illustrates the order of

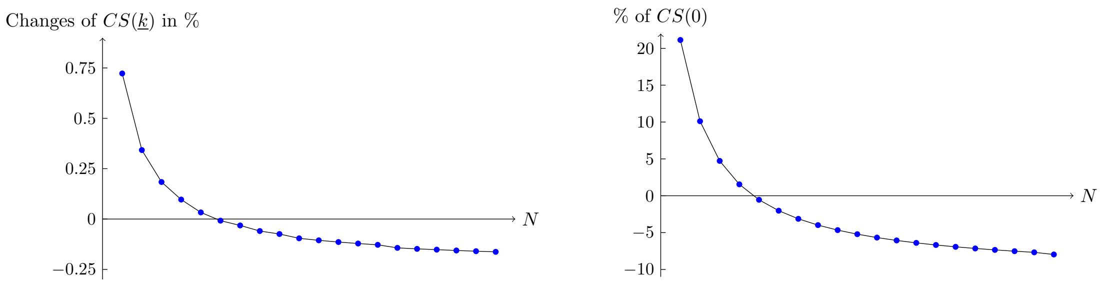
[24] All computations were done in Python and are available upon request. The numerical computations are based on the parameterization $\alpha = \beta = c = \delta = 1$.

[25] The arguments underlying this statement are provided in Appendix B [not for publication].

[26] Put formally, Figure 1a displays $100(CS(\underline{k} + 0.01) - CS(\underline{k}))/CS(\underline{k})$ at the point where $\underline{k}$ is such that $q_I^*/Q^* = 1/2$. Here, 0.01 is the smallest increment for changes in $\underline{k}$ that we used in our simulations.

[27] This exercise is also insightful as it captures the way in which many antitrust authorities may think about evaluating the competitive effects of vertical integration.

magnitude of these effects in the linear-quadratic model. It depicts the difference in consumer surplus between vertical integration to monopoly and no vertical integration, i.e., $CS(\bar{k}) - CS(0)$, as percentage of $CS(0)$ as a function of N. If the only objective were to maximize consumer surplus and if the degree of vertical integration were 0, then vertical integration that would lead to full foreclosure should be permitted when the number of competitors is small (absent vertical integration to full foreclosure) but not when it is large.



(a) Changes in $CS(\underline{k})$ when $\underline{k}$ increases marginally in % of $CS(\underline{k})$ when I's downstream market share is kept at 50%.

(b) The difference in consumer surplus between vertical integration to full foreclosure and no vertical integration, i.e., $CS(\bar{k}) - CS(0)$, in % of $CS(0)$.

Figure 1: Quantifying the effects of vertical integration of consumer surplus.

At first glance, it seems like a very intuitive proposition that vertical integration is most harmful to consumers when there are few non-integrated rivals but has little effect when their number is large.[28] This leads to the policy recommendation of prohibiting vertical integration when N is small but not when it is large. Our numerical results show that such intuitive, but in our model ultimately misguided policy recommendations, can lead to mistakes with substantial costs to consumers at both ends of the spectrum of market structures.

5 Welfare Effects

Our focus thus far has been on the competitive effects of vertical integration. Yet, it is also important to analyze the implications of vertical integration on social welfare, which can be expressed as

$$W = \int_0^Q P(x)dx - k_I C\left(\frac{q_I}{k_I}\right) - N k_j C\left(\frac{q_j}{k_j}\right) - \int_0^K R(y)dy.$$

[28]For example, the guidelines of the U.S. Department of Justice for non-horizontal mergers express the view that "Adverse competitive effects are likely only if overall concentration, or the largest firm's market share, is high." and state that the Department is unlikely to challenge a merger "unless overall concentration of the acquired firm's market is above 1800 HHI," with a lower concentration sufficing under certain conditions; see `http://www.justice.gov/atr/public/guidelines/2614.pdf`.

The first term is gross consumer surplus, the second term is the variable cost of the integrated firm, while the third term represents the variable cost of all non-integrated firms. The last term is the opportunity cost of capacity. Differentiating this expression with respect to $\underline{k}$ (and dropping arguments), we obtain that welfare is increasing in $\underline{k}$ if and only if

$$P\frac{dQ}{d\underline{k}} - NC_j\frac{dk_j}{d\underline{k}} - Nk_jC_j'\left(\frac{1}{k_j}\frac{dq_j}{d\underline{k}} - \frac{q_j}{k_j^2}\frac{dk_j}{d\underline{k}}\right) - C_I\frac{dk_I}{d\underline{k}} - k_IC_I'\left(\frac{1}{k_I}\frac{dq_I}{d\underline{k}} - \frac{q_I}{k_I^2}\frac{dk_I}{d\underline{k}}\right) - R\frac{dK}{d\underline{k}} > 0. \quad (7)$$

We can now solve the first-order conditions of the quantity stage, given by (2), for C_j' and C_I', and insert them into (7). Similarly, inserting C_j' and C_I' from (2) into the first-order conditions from the capacity stage, (3) and (4), and solving them for C_j and C_I, we can replace C_j and C_I in (7). After rearranging we obtain

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} > -\frac{-P'\left(q_I\frac{dQ}{dk_I} + q_j\frac{dQ_{-I}}{dk_I}\right) + R'(k_I - \underline{k})}{N\left[-P'\left(q_j\frac{dQ}{dk_j} + (N-1)q_j\frac{dq_i}{dk_j} + q_I\frac{dq_I}{dk_j}\right) + R'k_j\right]}. \quad (8)$$

This inequality has a similar structure as (5). The left-hand side is again the equilibrium ratio of the response of k_j to a change in $\underline{k}$ over the response of k_I. The right-hand side is now different because when considering social welfare we have to take into account that the cost structure and therefore the absolute value of overall costs changes as $\underline{k}$ varies. Nevertheless, one can show that for any finite N there exists a $\underline{k}_W^* > 0$ such that for all $\underline{k} < \underline{k}_W^*$ vertical integration is welfare increasing at the margin. It is also possible that vertical integration to full foreclosure increases overall welfare.[29]

The intuition is similar to the one for Propositions 1 and 2. If the degree of vertical integration is low, further vertical integration mainly increases final output. Therefore, it is welfare increasing. On the other hand, if $\underline{k}$ is already large, the overall quantity may decrease and, in addition, the less efficient firm produces more, which raises production costs even for a given quantity.

A result that is akin to Proposition 3 can also be shown: If $R(K)$ is sufficiently inelastic or if the model is linear-quadratic, then for any finite N there either exists a unique $\underline{k}_W^* \in (0, \bar{\underline{k}})$ so that vertical integration is welfare enhancing at the margin for all $\underline{k} < \underline{k}_W^*$ and welfare reducing at the margin for all $\underline{k} > \underline{k}_W^*$, or vertical integration is always welfare enhancing.[30]

[29] A formal statement and a sketch of the proof are in Appendix B [not intended for publication].

[30] The sketch of the proof is in Appendix B [not intended for publication].

The analysis so far resembles the one of the previous section. However, the threshold value of $\underline{k}$ obtained in the welfare analysis is different from the one obtained for consumer surplus because, as mentioned, the variable costs of production and the opportunity costs of capacity change with an increase in $\underline{k}$. The next proposition shows that for the linear-quadratic specification, a comparison of these thresholds delivers a clear-cut result.

Proposition 6 *In the linear-quadratic case,* $\underline{k}_W^* < \underline{k}^*$.

The proposition states that marginal vertical integration is less likely to be welfare enhancing than consumer surplus enhancing. The intuition for this is two-fold: First, an increase in the degree of vertical integration leads to an increase in aggregate capacity K. This is because the rise in k_I following an increase in $\underline{k}$ is larger than the fall in the capacity of non-integrated rivals. Therefore, capacity costs increase. Second, firm I utilizes its capacity less intensively than a non-integrated firm. This implies that vertical integration increases overall production costs at $\underline{k}^*$ for constant aggregate quantity. Thus, even if aggregate quantity increases slightly, the effect of increased production costs dominates and welfare falls.[31] The result is interesting because it seems natural to conjecture that procompetitive vertical integration also improves welfare because firms' profits should rise as the industry becomes more integrated. However, what is missing in this reasoning is that vertical integration shifts production costs between firms. Proposition 6 shows that this effect can be so large that procompetitive but welfare reducing mergers are possible.[32]

Another important issue for practical application is to derive conclusions about the welfare effects of vertical integration that are based on observable market conditions.[33] For the linear-quadratic specification, one can numerically compute the critical input or output market shares of the integrated firm, given the thresholds $\underline{k}^*$ and $\underline{k}_W^*$, beyond which further vertical integration reduces consumer surplus or social welfare. In line with our previous results, these critical input and output market shares fall in the number of rivals. In addition, these critical market shares are almost identical in the input and the output market, suggesting that it may be sufficient for antitrust authorities to look at either of the two markets.

[31] The comparison between the two thresholds is illustrated in Figure 2 in the next section.

[32] If the integrated firm has an additional cost advantage that is sufficiently strong, the result can be reversed. In this case, anticompetitive but welfare enhancing mergers can occur. We show this result in Appendix B [not intended for publication].

[33] A particularly nice feature of Riordan's (1998) dominant firm model is that it establishes an indicator about the welfare effects of vertical integration that is based on the ratio of input to output market shares.

6 Incentives for Vertical Integration

Our model, like Riordan's (1998), assumes that one firm – firm I – owns some exogenous amount of capacity $\underline{k}$ at the outset, thereby taking vertical integration as given. Therefore, an important question is whether firm I has the incentive to acquire capacity $\underline{k}$ in the first place, and if so, what its optimal ownership stake of capacity would be.[34] The answers to these questions are far from obvious because, as noted by Rey and Tirole (2007), a rise in $\underline{k}$ leads to a higher wholesale price, which increases firm I's expenses to buy the ownership stake. This higher wholesale price could dissuade firm I from buying the ownership stake in the first place.[35] To address these questions, we now extend our model by adding an ex ante stage, in which firm I can make an offer to purchase $\underline{k}$ units of capacity from upstream suppliers. We show analytically that firm I always has an incentive to acquire at least a small stake of the competitively supplied capacity input, and we use numerical computations for the linear-quadratic model to determine firm I's profit maximizing level of integration.

Setup To fix ideas, we assume that the upstream market consists of a continuum of suppliers with heterogeneous costs of producing one unit of capacity, which yields the perfectly competitive inverse supply function $R(K)$ for aggregate capacity K, with $R'(K) > 0$.

We now consider a three-stage game, whose first stage – called stage 0 or ex ante stage – precedes the game analyzed thus far. In stage 0, firm I makes a public offer to buy $\underline{k}$ units of capacity at price r.[36,37] Stages 1 and 2 are then exactly the same as stage 1 and 2 in the previous analysis, i.e., conditional on the acquired capacity of firm I, all $N + 1$ firms choose their capacities in stage 1 and then compete in quantities in stage 2.

Analysis We start with the optimal decision of an upstream supplier. Because each supplier has measure 0, firm I's level of vertical integration will be $\underline{k}$ if $\underline{k}$ other suppliers sell their unit of capacity at price r. Any supplier who does not sell his capacity at price r in the ex ante stage can therefore sell it at price $R(K(\underline{k}))$ on the capacity spot market in stage 1, where $K(\underline{k})$

[34] At the end of Section 7, we show that the answers to these questions are not sensitive to whether firms compete in quantities or prices downstream.

[35] Notice also that these have been open questions even for the analytically simpler model of a dominant firm facing a competitive fringe.

[36] With some abuse of notation we denote the maximum quantity firm I is willing to buy and the quantity it buys by $\underline{k}$. As shown shortly, these two quantities will be the same in equilibrium.

[37] The assumption that only firm I can make an offer to the upstream suppliers in the ex ante stage is in line with the assumption of a single integrated firm that underlies our main analysis.

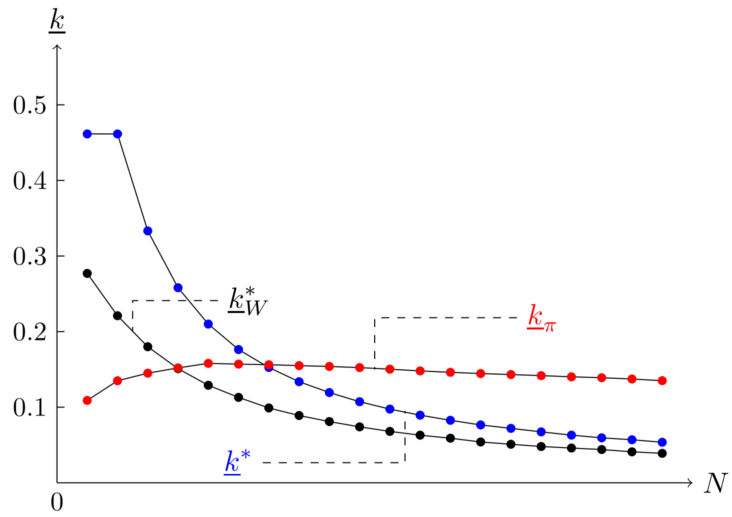


Figure 2: The competitive threshold $\underline{k}^*$, the welfare threshold $\underline{k}_W^*$, and the profitability threshold $\underline{k}_\pi$ in the linear-quadratic model.

is the aggregate equilibrium capacity given that I is vertically integrated to the degree $\underline{k}$. It thus follows that firm I can acquire $\underline{k}$ units ex ante if and only if $r \geq R(K(\underline{k}))$. As there is no point for I to leave money on the table, it will set $r = R(K(\underline{k}))$.[38] Therefore, if firm I wants to acquire $\underline{k}$ units of capacity in the ex ante stage, its overall cost of doing so is $\underline{k}R(K(\underline{k}))$.

For a given degree of vertical integration $\underline{k}$, the profit accruing to firm I in stages 1 and 2 is

$$\Pi_I(\underline{k}) = P(Q(\underline{k}))q_I(\underline{k}) - k_I(\underline{k})C\left(\frac{q_I(\underline{k})}{k_I(\underline{k})}\right) - (k_I(\underline{k}) - \underline{k})R(K(\underline{k})), \tag{9}$$

where $q_I(\underline{k})$, $k_I(\underline{k})$, and $Q(\underline{k})$ are, respectively, the equilibrium values of I's quantity, I's capacity and aggregate equilibrium quantity for a given amount of vertical integration $\underline{k}$. Differentiating this with respect to $\underline{k}$ and using the envelope theorem yields

$$\frac{\partial \Pi_I(\underline{k})}{\partial \underline{k}} = N\left[P'(Q(\underline{k}))q_I(\underline{k})\frac{\partial q_j(\underline{k})}{\partial \underline{k}} - (k_I(\underline{k}) - \underline{k})R'(K(\underline{k}))\frac{\partial k_j(\underline{k})}{\partial \underline{k}}\right] + R(K(\underline{k})) > 0,$$

where the inequality follows because, by Lemmas 1 and 5, $\partial q_j/\partial \underline{k} < 0$ and $\partial k_j/\partial \underline{k} < 0$.

We now evaluate the incentives to acquire a small (i.e., marginal) stake of upstream capacity for firm I at $\underline{k} = 0$. To do so, we compare the marginal change in profit in stages 1 and 2

[38] Given the offer to buy $\underline{k}$ at price $r = R(K(\underline{k}))$, $\underline{k}$ is the unique capacity sold to I in equilibrium. To see this, recall that K is increasing in $\underline{k}$ and R increasing in K. Therefore, if suppliers offered less (more) than $\underline{k}$, the price offer by I would beat (be worse than) the spot market price. Notice also that the fact $R(K(\underline{k}))$ is increasing in $\underline{k}$ reflects the well-known free riding phenomenon in the takeover literature (see e.g., Burkart, Gromb, and Panunzi (1998)): The larger the ownership stake a firm wants to acquire, the higher the price it has to pay for it. Joskow and Tirole (2000) and Gilbert, Neuhoff, and Newbery (2004) observe a similar problem in the trading of transmission contracts in electricity markets.

evaluated at $\underline{k}$, $(\partial \Pi_I(\underline{k})/\partial \underline{k})\,|_{\underline{k}=0}$, with the marginal change in the cost of procurement, which is the derivative of $\underline{k}R(K(\underline{k}))$ with respect to $\underline{k}$. Evaluated at $\underline{k}=0$, this derivative is simply $R(K(0))$, yielding the following result

$$\begin{aligned}\frac{\partial \Pi_I(\underline{k})}{\partial \underline{k}}|_{\underline{k}=0} = N\left[P'(Q(0))q_I(0)\frac{\partial q_j(0)}{\partial \underline{k}} - k_I(0)R'(K(0))\frac{\partial k_j(0)}{\partial \underline{k}}\right] \quad &+ \quad R(K(0)) \\ > R(K(0)) \quad &= \quad \frac{\partial [\underline{k}R(K(\underline{k}))]}{\partial \underline{k}}|_{\underline{k}=0}.\end{aligned}$$

Notice that this inequality holds for any positive N. This implies that some vertical integration is always profitable for firm I. Summarizing, we have established the following:

Proposition 7 *For any N, the acquisition of a marginal ownership stake at $\underline{k}=0$ is always profitable for firm I.*

Interestingly, a small degree of vertical integration is always profitable, independent of the shape of the demand, supply, and cost functions. This shows that even if firm I needs to pay for its acquisition, it finds it profitable to buy at least a small share. The result is of importance in relation to Proposition 1, according to which marginal vertical integration is procompetitive at $\underline{k}=0$. Proposition 7 shows that, for the fairly natural capacity acquisition game analyzed here, such marginal vertical integration will indeed take place. Consequently, the market will implement these gains in consumer surplus.

To derive the profit maximizing level of vertical integration, we need to compare the benefits and costs from acquisition for an arbitrary level of $\underline{k}$. When acquiring $\underline{k}$, firm I's profit is given by (9), while the costs of procurement are $\underline{k}R(K(\underline{k}))$. Subtracting the latter from the former gives

$$\Pi_I(\underline{k}) = P(Q(\underline{k}))q_I(\underline{k}) - k_I(\underline{k})C\left(q_I(\underline{k})/k_I(\underline{k})\right) - k_I(\underline{k})R(K(\underline{k})). \tag{10}$$

The optimal level of integration is the maximizer of $\Pi_I(\underline{k})$ in (10) with respect to $\underline{k}$, which we denote by $\underline{k}_\pi$ and evaluate numerically for the linear-quadratic model. Figure 2 shows $\underline{k}_\pi$ together with $\underline{k}^*$ and $\underline{k}^W$.

It is evident that $\underline{k}_\pi$ is lower than $\underline{k}^*$ and $\underline{k}^W$ for small values of N but above $\underline{k}^*$ and $\underline{k}^W$ for high values of N. This implies that for small values of N, there is too little integration in equilibrium, while for larger values, the incentives to vertically integrate are excessive. As N becomes large, any degree of vertical integration is anticompetitive (and reduces welfare because $\underline{k}^*_W < \underline{k}^*$ by Proposition 6) but some vertical integration will occur in equilibrium. Therefore, what emerges from this analysis of equilibrium incentives to integrate is in line with the general theme that emerges from our paper. More vigilance vis-à-vis vertical integration is called for in markets with a large number of non-integrated rivals. Figure 2 shows that as the number of rival firms increases, the incentives to vertically integrate are larger than the socially desirable degree of integration, independent of whether the objective is consumer surplus or welfare.

7 Differentiated Bertrand Competition Downstream

In this section, we show that our main results do not rely on the property that downstream choice variables are strategic substitutes, as is the case with Cournot competition. To this end, we assume that competition downstream is differentiated Bertrand rather than Cournot.

Setup To simplify the analysis, we focus on the linear-quadratic model as does much of the literature on competition with differentiated products (see e.g., Deneckere and Davidson (1985), Raith (1996), or Nocke and White (2007)). In our setup, quadratic costs means $C_i(q_i/k_i) = c\,(q_i/k_i)^2/2$ and we also assume that $R(K) = \delta K$. As before, there is one integrated firm denoted by I and N non-integrated firms indexed by j. All firms now produce symmetrically differentiated products. There is a unit mass of identical consumers with utility function

$$CS(\mathbf{q},\mathbf{p}) = \alpha Q - \frac{\beta}{2}\sum_{j=1}^{N} q_j^2 - \frac{\beta}{2}q_I^2 - \sigma q_I(Q - q_I) - \sigma \sum_{j=1}^{N}\sum_{k=1}^{j-1} q_j q_j + m,$$

where $Q = q_I + \sum_{j=1}^{N} q_j$, and m represents the utility from consuming the numeraire good "money", and $\mathbf{q} = (q_I, q_1, .., q_N)$ and $\mathbf{p} = (p_I, p_1, .., p_N)$ are vectors of quantities and prices. The parameter $\sigma \in [0, \beta)$ measures the degree of substitutability between the products. The products become perfect substitutes as $\sigma \to \beta$ and independent for $\sigma = 0$. Each consumer maximizes this utility function over $\mathbf{q} = (q_I, q_1, .., q_N)$ subject to the constraint $m + \mathbf{p}\cdot\mathbf{q} \leq Y$, where Y is income and $\mathbf{p}\cdot\mathbf{q}$ denotes the inner product. Maximizing this utility function with respect to $\mathbf{q}$ yields the system of inverse demand functions

$$P_i(\mathbf{q}) = \alpha - \beta q_i - \sigma(Q - q_i) \quad i = I, 1, ..., N. \tag{11}$$

As in Hackner (2000), one can invert the system of equations given in (11) to obtain the demand function of firm i, with $i = I, 1, ..., N$, as

$$q_i(\mathbf{p}) = \frac{\alpha}{\beta + \sigma N} - \frac{\beta + \sigma(N-1)}{(\beta - \sigma)(\beta + \sigma N)}p_i + \frac{\sigma}{(\beta - \sigma)(\beta + \sigma N)}(P - p_i), \tag{12}$$

where $P = p_I + \sum_{j=1}^{N} p_j$ is the sum of prices.

The profit function of the integrated firm can then be written as

$$\Pi_I = p_I q_I(\mathbf{p}) - k_I c \left(q_I/k_I\right)^2/2 - (k_I - \underline{k})\delta K$$

and the profit function of a non-integrated firm j is

$$\Pi_j = p_j q_j(\mathbf{p}) - k_j c \left(q_j/k_j\right)^2/2 - k_j \delta K,$$

with $q_I(\mathbf{p})$ and $q_j(\mathbf{p})$ given by (12) with $i = I$ and $i = j$, respectively.

Analysis The analysis proceeds by backwards integration in close analogy to Section 3. Due to our functional form assumptions, there is a unique symmetric equilibrium in the quantity and the capacity stage. Letting $p_i^\star$ and $k_i^\star$ denote equilibrium values of prices and capacities, we can succinctly summarize the equilibrium behavior as follows:

Lemma 6 *With differentiated Bertrand competition downstream, we have that:*

(a) Increases in capacity decrease all equilibrium prices, i.e., $\frac{dp_i^\star(\mathbf{k})}{dk_i} < 0$ *and* $\frac{dp_i^\star(\mathbf{k})}{dk_j} < 0$ *for all* $i \neq j, i, j = I, 1, ..., N$.

(b) Firm i's capacity utilization decreases in i's capacity, i.e., $\frac{q_i(\mathbf{p}^\star(\mathbf{k}))}{k_i}$ *falls in* k_i *for all* i.

(c) Capacities are strategic substitutes, i.e., $\frac{dk_i}{dk_j} < 0$ *for all* $i \neq j, i, j = I, 1, ..., N$.

(d) The equilibrium capacity of the integrated firm increases and the equilibrium capacity of a non-integrated firm decreases in $\underline{k}$*, i.e.,* $\frac{dk_I^\star}{d\underline{k}} > 0$ *and* $\frac{dk_j^\star}{d\underline{k}} < 0$, $j = 1, ..., N$,

where, in part (c), k_i denotes the best response capacity of firm i.

The key difference from the Cournot model, in which the firms' choice variables in the downstream market are strategic substitutes, is the second result in part (a), that is, $dp_i^\star(\mathbf{k})/dk_j < 0$. This reflects the fact that choice variables are strategic complements with Bertrand competition downstream. If a firm purchases more capacity, i.e., has lower production costs, it finds it optimal to set a lower price, inducing its rivals to lower their prices as well. A more surprising result is part (c), which states that just like with Cournot competition downstream, capacities are strategic substitutes with Bertrand competition downstream. If firm i expands its capacity, the market price for capacity rises and any rival finds it optimal to purchase less capacity. Part (c) of Lemma 6 shows that, for the linear-quadratic specification we consider, this raising rivals' costs effect dominates any counter-weighing effect based on strategic complementarity, such as $dp_i^\star(\mathbf{k})/dk_j < 0$.

Competitive Effects of Vertical Integration Vertical integration is procompetitive if equilibrium consumer surplus $CS(\mathbf{q}^*(\mathbf{p}^*(\underline{k})), \mathbf{p}^*(\underline{k}))$ increases with $\underline{k}$, where $\mathbf{p}^*(\underline{k})$ are the equilibrium prices given $\underline{k}$ and $\mathbf{q}^*(\mathbf{p}^*(\underline{k}))$ are the utility maximizing quantities given these prices. Using symmetry of all non-integrated firms and dropping arguments, we can write the equilibrium consumer surplus as

$$\begin{aligned} CS(\mathbf{q}^*(\mathbf{p}^*(\underline{k})), \mathbf{p}^*(\underline{k})) &= \\ \alpha\left(Nq_j^\star + q_I^\star\right) - \frac{\beta}{2}N(q_j^\star)^2 &- \frac{\beta}{2}(q_I^\star)^2 - \sigma N q_I^\star q_j^\star - \sigma\frac{N(N-1)}{2}(q_j^\star)^2 - Np_j^\star q_j^\star - p_I^\star q_I^\star + m. \end{aligned}$$

Differentiating $CS(\mathbf{q}^*(\mathbf{p}^*(\underline{k})), \mathbf{p}^*(\underline{k}))$ with respect to $\underline{k}$, dropping arguments and using the Envelope Theorem, we obtain

$$\frac{dCS}{d\underline{k}} = -\left(Nq_j^\star \frac{dp_j^\star}{d\underline{k}} + q_I^\star \frac{dp_I^\star}{d\underline{k}}\right). \tag{13}$$

We start with the case of $\underline{k}$ small.

Proposition 8 *For any finite N, there exists a threshold $\underline{k}'$, such that for all $\underline{k} < \underline{k}'$, vertical integration is procompetitive at the margin.*

The result resembles the one with quantity competition, where vertical integration is also procompetitive at the margin for $\underline{k}$ small. Intuitively, with $\underline{k}$ small, firm I utilizes its capacity almost as intensively as the non-integrated firms do. Therefore, the effect pointed out in part (b) of Lemma 6 is small. Hence, the effect that the overall capacity $K^\star = k_I^\star + nk_j^\star$ used rises with $\underline{k}$ is the dominating effect. This leads to a reduction in overall production cost, implying that average prices fall. Although the prices of all non-integrated firms rise, the price of the integrated firm falls by a larger amount.[39] Because consumers spend roughly the same amount of money on all goods,[40] this leads a lower expenditure for consumers and thereby to an increase in consumer surplus.

Now we turn to the opposite case in which $\underline{k}$ is so large that additional integration leads to complete foreclosure.

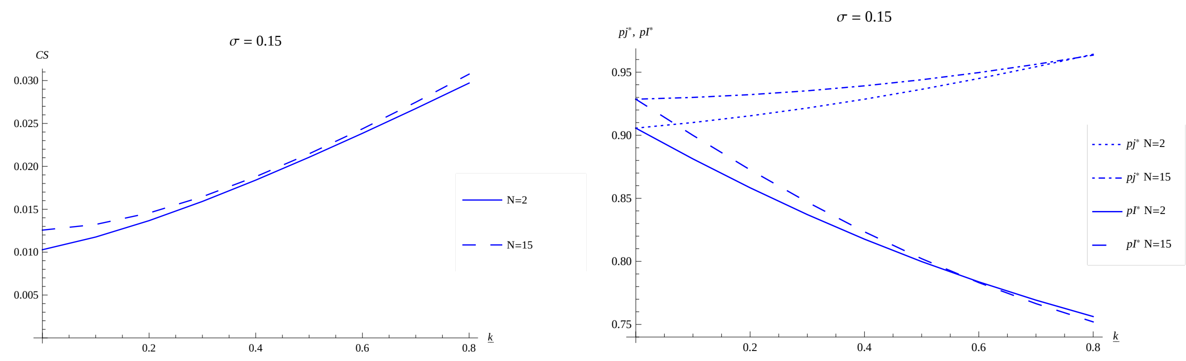
Proposition 9 *For any finite N, vertical integration to complete foreclosure is procompetitive at the margin.*

[39] The proof that the prices of all non-integrated firms increase in $\underline{k}$ at $\underline{k} = 0$ is in Appendix B [not considered for publication].

[40] This is because at $\underline{k}$ close to zero, all firms have almost the same production cost and therefore charge almost the same prices. As products are symmetrically differentiated, consumers buy almost the same quantity of each product.

This result differs from the one of quantity competition, where marginal vertical integration to complete foreclosure can either be procompetitive or anticompetitive. The countervailing effects identified under quantity competition are still present with differentiated Bertrand competition. However, in addition, consumers now have the choice of which product to buy. Thus, they can, and optimally will, substitute away from more expensive products. When $\underline{k}$ is so large that there is almost complete foreclosure, the prices of the non-integrated firms' products are already large, which implies that consumers buy only a small amount of these products. Therefore, as their prices increase further, consumer surplus decreases only by a little.[41] Consequently, the procompetitive effect of firm I's price decrease dominates, and we obtain the rather striking result that with differentiated products vertical integration to complete foreclosure is always procompetitive at the margin.

Propositions 8 and 9 suggest that with differentiated Bertrand competition, quadratic costs and linear supply and demand functions, vertical integration may always be procompetitive. Though an analytical result proves elusive, numerical computations demonstrate that this is indeed the case. Figure 3 shows consumer surplus and equilibrium prices as a function of $\underline{k}$ for different values of N.[42] It is evident from the left-hand panel that consumer surplus is increasing for any level of vertical integration. The right-hand panel shows that the price of the integrated firm falls with vertical integration while the one of its non-integrated rivals rises. Therefore, the effects at work in this case are similar as in the case of quantity competition.



(a) Consumer Surplus (b) Prices

Figure 3: Consumer Surplus (on the left-hand panel) and equilibrium prices p_I^* and p_j^* (on the right-hand panel) as a function of $\underline{k}$ from 0 to $\overline{k}$ for $N \in \{2, 5, 10, 15\}$ and $\sigma = 0.15$.

[41]It should be noted that this result hinges on the assumption of a constant marginal utility that gives rise to linear demand functions. If utility satisfied, say, Inada conditions, the results may differ.

[42]For the computations underlying the figure, we set $\beta = \delta = c = 1$.

Welfare Effects and Incentives to Integrate We can also analyze the welfare effects of vertical integration with differentiated Bertrand competition downstream. As with quantity competition, vertical integration shifts production from the non-integrated firms to the integrated one, which utilizes this capacity less intensively. Therefore, although vertical integration is always procompetitive, it is not necessarily welfare enhancing. This is depicted in Figure 4 by the curve $\underline{k}_W^B$, which provides the threshold beyond which vertical integration becomes welfare reducing.[43]

We now address the incentives to integrate, proceeding along the lines of Section 6. Here we obtain the same result as in Proposition 7, namely that a small amount of vertical integration is always profitable, formally

$$\frac{\partial \Pi_I}{\partial \underline{k}}|_{\underline{k}=0} = N\left[p_I \frac{\partial q_I}{\partial p_j}\frac{\partial p_j}{\partial \underline{k}} - (k_I - \underline{k})\delta \frac{\partial k_j}{\partial \underline{k}}\right] + \delta K > \delta K = \frac{\partial[\underline{k}R(K(\underline{k}))]}{\partial \underline{k}}|_{\underline{k}=0}.$$

Because $\partial q_I/\partial p_j > 0$ and, by part (d) of Lemma 6, $\partial k_j/\partial \underline{k} < 0$, the inequality follows once $(\partial p_j/\partial \underline{k})\,|_{\underline{k}=0} > 0$ is established, which we do in Appendix B [not intended for publication].

To determine the privately optimal level of integration, denoted by $\underline{k}_\pi^B$, we focus again on numerical results. Figure 4 plots $\underline{k}_\pi^B$ alongside $\underline{k}_W^B$. The mere fact that this maximizer is below the level of full foreclosure shows that from the perspective of maximizing consumers surplus the private incentives for vertical integration are insufficient. By contrast, this is not case from a social welfare perspective. As is evident from Figure 4, if the number of firms is large, the private incentives to integrate are excessive. The numerical results therefore reinforce a main theme of the present paper, which is that vertical integration is mostly a concern in industries with intensive competition amongst the rivals of the integrating firm.

8 Conclusions

We have analyzed a model in which the effects of vertical integration on consumer and overall welfare depend on the underlying market structure. We have shown that, surprisingly, vertical integration is more likely to be procompetitive exactly when the market structure consists of a small number of non-integrated rivals. More generally, in our model vertical integration is procompetitive under fairly wide circumstances because efficiency effects tend to dominate foreclosure effects. Because of this, even vertical integration that leads to full foreclosure of the rivals can be procompetitive. However, vertical integration can also increase consumer

[43] As the result and the intuition is similar to the quantity competition case, we focus on the numerical result here.

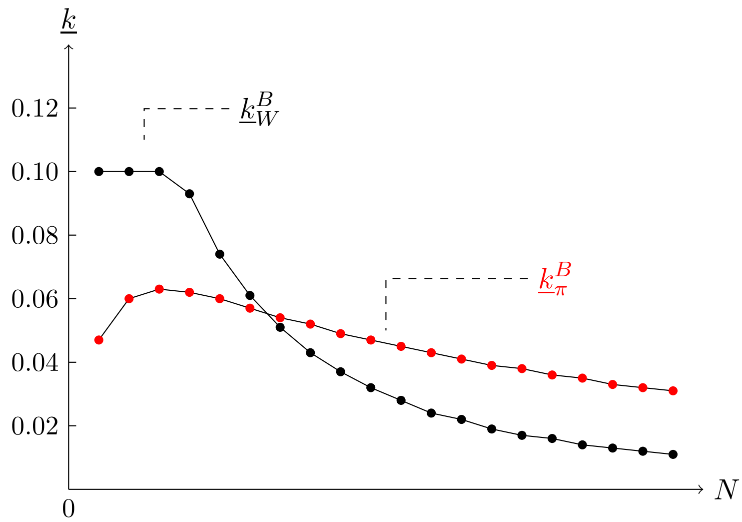


Figure 4: The welfare threshold $\underline{k}_W^B$, and the profitability threshold $\underline{k}_\pi^B$ in the linear-quadratic model with differentiated Bertrand competition.

surplus and decrease total welfare because final output may be produced at higher costs after integration. These results hold for Cournot and differentiated Bertrand competition. Because consumers can substitute away from more expensive goods, the procompetitive effects of vertical integration are amplified when products are differentiated. With regards to the incentives to vertically integrate, we find that a small amount of integration is always profitable despite the free-riding problem the integrating firm faces. In addition, the private incentives to integrate tend to be too weak when the number of rivals is small and excessive when it is large. Our numerical results also indicate that —within the confines of our model— the effects of seemingly intuitive but ultimately misguided policy recommendations can be sizeable.

Our model considers the case in which upstream suppliers are perfectly competitive while downstream firms have full oligopsony power, leaving the open question for future research of what happens when input suppliers exert market power as well. We expect that the main effects will still be at work if input suppliers have some limited bargaining power. If a downstream firm integrates with an upstream supplier, the newly integrated firm owns more capacity units and therefore will purchase more capacity from other upstream suppliers on the input market. At the same time, it will produce more inframarginal units of output and thus utilize its capacity less intensively than non-integrated downstream firms.

In contrast, the effects we identified will not be at work if downstream firms have no market power on the wholesale price as is the case, for example, in the well-known model of Salinger (1988), in which only upstream firms can influence the market price. Our model can therefore be seen as analyzing the other end of the spectrum, in which market power in the input market is purely oligopsonistic instead of oligopolistic, so that the two models are complementary to each other.

Appendix

A Proofs

A.1 Proof of Lemma 1

Let $j \neq h, j \neq I$ and $h \neq I$. Totally differentiating (2) with respect to k_j yields[44]

$$P'\frac{dQ}{dk_j} + P'\frac{dq_j}{dk_j} + P''q_j\frac{dQ}{dk_j} = -C_j''\frac{q_j}{k_j^2} + C_j''\frac{1}{k_j}\frac{dq_j}{dk_j}. \tag{14}$$

We can write dQ/dk_j as $dQ/dk_j = dq_I/dk_j + \sum_{h \neq j} dq_h/dk_j + dq_j/dk_j$, which under the symmetry assumption that $k_h = k_j$ for all $h, j \in \{1, ..., N\}$, becomes

$$\frac{dQ}{dk_j} = \frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j} + \frac{dq_j}{dk_j}.$$

Therefore, (14) can be written as an equation that depends on the three variables dq_h/dk_j, dq_j/dk_j and dq_I/dk_j, which we wish to determine.

Totally differentiating the first-order condition of firm h, which is analogous to (2), with respect to k_j yields

$$P'\frac{dQ}{dk_j} + P'\frac{dq_h}{dk_j} + P''q_h\frac{dQ}{dk_j} = C_h''\frac{1}{k_h}\frac{dq_h}{dk_j}, \tag{15}$$

and differentiating (2) for $i = I$ with respect to k_j yields

$$P'\frac{dQ}{dk_j} + P'\frac{dq_I}{dk_j} + P''q_I\frac{dQ}{dk_j} = C_I''\frac{1}{k_I}\frac{dq_I}{dk_j}. \tag{16}$$

The system of the three equations (14), (15) and (16) is linear in the three unknowns dq_h/dk_j, dq_j/dk_j and dq_I/dk_j. Its unique solution, after imposing symmetry, i.e. $q_h = q_j$, $k_h = k_j$ and $C_h'' = C_j''$, is

$$\frac{dq_I}{dk_j} = \frac{C_j''q_jk_I(P' + P''q_I)}{\eta k_j} < 0 \quad \text{for} \quad j \neq I, \tag{17}$$

$$\frac{dq_h}{dk_j} = \frac{C_j''q_j(C_I'' - P'k_I)(P' + P''q_j)}{\eta(C_j'' - P'k_j)} < 0 \quad \text{for} \quad j \neq h \tag{18}$$

and

$$\begin{aligned} \frac{dq_j}{dk_j} &= \frac{C_j''q_j[(P')^2k_jk_I(N+1) + P'(P''k_jk_I(q_I + (N-1)q_j) - 2C_j''k_I - C_I''k_jN)]}{\eta k_j(C_j'' - P'k_j)} \\ &+ \frac{C_j''q_j[C_j''C_I'' - P''(C_j''k_Iq_I + (N-1)C_I''k_jq_j)]}{\eta k_j(C_j'' - P'k_j)} > 0, \end{aligned} \tag{19}$$

[44]To simplify notation, we omit the superscript $*$ on equilibrium quantities and equilibrium capacities throughout this appendix.

where $\eta \equiv (P')^2(N+2)k_Ik_j + P'[P''k_jk_I(q_I+Nq_j) - C_I'k_j(N+1) - 2k_IC_j''] + C_I''C_j'' - P''(C_j''q_Ik_I + C_I''q_jk_jN) > 0$. The inequality sign follows from P'' being negative or not too positive.

Totally differentiating the first-order conditions of firm I and j with respect to k_I yields

$$P'\frac{dQ}{dk_I} + P'\frac{dq_I}{dk_I} + P''q_I\frac{dQ}{dk_I} = -C_I''\frac{q_I}{k_I^2} + C_I''\frac{1}{k_I}\frac{dq_I}{dk_I} \tag{20}$$

and

$$P'\frac{dQ}{dk_I} + P'\frac{dq_j}{dk_I} + P''q_j\frac{dQ}{dk_I} = C_j''\frac{1}{k_j}\frac{dq_j}{dk_I}, \tag{21}$$

respectively, where under symmetry $dQ/dk_I = dq_I/dk_I + Ndq_j/dk_I$. Using the last equation to replace dQ/dk_I in (20) and (21) yields a system of two linear equations in the two unknowns dq_I/dk_I and dq_j/dk_I. The solution is

$$\frac{dq_j}{dk_I} = \frac{C_I''q_Ik_j(P''q_j + P')}{k_I\eta} < 0 \quad \text{and} \quad \frac{dq_I}{dk_I} = -\frac{C_I''q_I[k_j(P'(N+1) + P''Nq_j) - C_j'']}{k_I\eta} > 0. \tag{22}$$

Again, the inequality sign follows from P'' not being too positive. ■

A.2 Proof of Lemma 2

From Lemma 1 we know that $q_i(\hat{k}_i, \mathbf{k}_{-i}) > q_i(k_i, \mathbf{k}_{-i}) \Leftrightarrow \hat{k}_i > k_i$. Now suppose to the contrary of the claim in the lemma that $q_i(\hat{k}_i, \mathbf{k}_{-i})/\hat{k}_i \geq q_i(k_i, \mathbf{k}_{-i})/k_i$. Because $C_i'' > 0$, this is equivalent to the right-hand side of (2) being weakly greater for $\hat{k}_i$ than for k_i.

Now we can turn to the left-hand side of (2). From (1) we can calculate dQ/dk_j and dQ/dk_I to get

$$\frac{dQ}{dk_j} = \frac{q_jC_j''(C_I'' - k_IP')}{k_j\eta} > 0 \quad \text{and} \quad \frac{dQ}{dk_I} = \frac{q_IC_I''(C_j'' - k_jP')}{k_I\eta} > 0.$$

Because $P' < 0$, the first term of the left-hand side of (2) is smaller for $\hat{k}_i$ than for k_i. Also, because $q_i(\hat{k}_i, \mathbf{k}_{-i}) > q_i(k_i, \mathbf{k}_{-i})$, $P' < 0$ and P'' is negative or not too positive, the second term on the left-hand side of (2) is either smaller for $\hat{k}_i$ than for k_i or only slightly bigger. Therefore, the left-hand side of (2) is strictly smaller for $\hat{k}_i$ than k_i, which is the desired contradiction. ■

A.3 Proof of Lemma 3

Differentiating (3) with respect to k_j and (4) with respect to k_I yields the second-order conditions

$$\frac{\partial^2\Pi_j}{\partial k_j^2} = P'\frac{dq_j}{dk_j}\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right] + P'q_j\left[\frac{d^2q_I}{dk_j^2} + (N-1)\frac{d^2q_h}{dk_j^2}\right] + \tag{23}$$

$$+P''q_j\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right]\left[\frac{dq_j}{dk_j} + \frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right] + C_j''\frac{q_j}{k_j^2}\left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j}\right) - 2R' - k_jR'' < 0$$

and

$$\frac{\partial^2 \Pi_I}{\partial k_I^2} = P' \frac{dq_I}{dk_I} N \frac{dq_j}{dk_I} + P' q_I N \frac{d^2 q_j}{dk_I^2} + \tag{24}$$

$$+P'' q_I N \frac{dq_j}{dk_I}\left[\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}\right] + C_I'' \frac{q_I}{k_I^2}\left(\frac{dq_I}{dk_I} - \frac{q_I}{k_I}\right) - 2R' - (k_I - \underline{k})R'' < 0,$$

with $h \neq j,\ h, j = 1, ..., N$. In the following we show that (23) is indeed fulfilled when the first-order conditions are satisfied. The second-order condition for the integrated firm can then be shown to be fulfilled in exactly the same way.

In the proof of Lemma 1 we determined the equilibrium expressions for dq_i/dk_j, $i = I, 1, ..., N$, that appear in (23). To determine the sign of $\partial^2\Pi_j/\partial k_j^2$ we still have to determine d^2q_I/dk_j^2 and d^2q_h/dk_j^2. To that end we now state the expressions for dq_I/dk_j and dq_h/dk_j without imposing symmetry, i.e., explicitly distinguishing between non-integrated firm h and j, that is between q_h and q_j, k_h and k_j and C_h'' and C_j''. This gives us

$$\frac{dq_I}{dk_j} = \frac{C_j'' q_j k_I (P' + q_I P'')(C_h'' - P'k_h)}{k_j \nu} \quad \text{and} \quad \frac{C_I'' q_j k_h (P' + q_h P'')(C_I'' - P'k_I)}{k_j \nu}, \tag{25}$$

with

$$\nu = -k_I k_j k_h (N+2)(P')^3 + (3C_h'' k_j k_I + k_I k_j (N+1) C_j'' + k_h k_j (N+1) C_I'' - P'' k_I k_h k_j ((N-1) q_h + q_I + q_j))(P')^2 +$$

$$((C_j'' k_h k_I (q_I + (N-1) q_h) + C_h'' k_h k_j (q_j + (N-1) q_h) + C_h'' k_I k_j (q_j + q_I))P'' - N k_h C_I'' C_j'' - 2 k_I C_j'' C_h'' - 2 k_j C_I'' C_h'')P'$$

$$-((N-1) C_I'' C_j'' q_h k_h + C_h''(q_j k_j C_j'' + q_I k_I C_j''))P'' + C_h'' C_I'' C_j''.$$ [45]

Differentiating both equations of (25) with respect to k_j, using dq_h/dk_j, dq_j/dk_j and dq_I/dk_j from the proof of Lemma 1, and inserting the resulting expressions into the second-order condition yields

$$\frac{\partial^2 \Pi_j}{\partial k_j^2} = -\frac{q_j^2 \left(\sum_{s=1}^{9} (P')^s (\sum_{h=1}^{3} \kappa_{sh} (P'')^h + \kappa_{s4} P''' + \kappa_{s5} C_j''' + \kappa_{s6} C_I''' + \kappa_{s7})\right)}{k_j^2 (C_j'' - k_j P')^3 \eta^3} - 2R' - k_j R'', \tag{26}$$

where we have used that in equilibrium $q_h = q_j$, $k_h = k_j$ and $C_h'' = C_j''$. In equation (26) $\kappa_{sh} = \kappa_{sh}(q_j, k_j, q_I, k_I, C_j'', C_I'', P', P'', N)$, $s \in \{1, ..., 9\}$ and $h \in \{1, ..., 7\}$. We do not specify the exact expressions for κ_{sh} here because they stand for rather complex expressions consisting of several terms. Yet, in each case the sign of these expressions is easy to determine and this is the only point of relevance for our purpose. These signs are the following: For $h = \{1, 2, 3\}$

[45] One can easily check that if $q_h = q_j$, $k_h = k_j$ and, therefore, $C_h'' = C_j''$ (which is the case in equilibrium), these formulas yield the expressions in (22).

$\kappa_{sh} \geq 0$, if both s and h are either even or odd and $\kappa_{sh} \leq 0$ if one is even and the other one is odd. κ_{s4}, κ_{s5}, $\kappa_{s6} \geq 0$ for s even and κ_{s4}, κ_{s5}, $\kappa_{s6} \leq 0$ for s odd. $\kappa_{s7} > 0$ for s even and $\kappa_{s7} < 0$ for s odd. Thus, the numerator in the fraction is positive because P'' is not too positive and P''' and C''' are not too negative. Because $\eta > 0$, the denominator is positive as well. Therefore, the first term in (26) is negative. Because R'' is not too negative as well, we get that $\partial^2\Pi_j/\partial k_j^2 < 0$. In exactly the same way we can show that the second-order condition for firm I is satisfied. Thus, the profit function of each firm is quasiconcave in its own capacity and we have an interior equilibrium.

We now turn to the question of uniqueness. From Kolstad and Mathiesen (1987) and Vives (1999) we know that the equilibrium is unique if and only if the Jacobian determinant of minus the marginal profits is positive. In our case this determinant is given by

$$|J| = \begin{vmatrix} -\frac{\partial^2\Pi_j}{\partial k_j^2} & -\frac{\partial^2\Pi_j}{\partial k_j\partial k_h} & \cdots & -\frac{\partial^2\Pi_j}{\partial k_j\partial k_I} \\ -\frac{\partial^2\Pi_h}{\partial k_h\partial k_j} & -\frac{\partial^2\Pi_h}{\partial k_h^2} & \cdots & -\frac{\partial^2\Pi_h}{\partial k_h\partial k_I} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\partial^2\Pi_I}{\partial k_I\partial k_j} & -\frac{\partial^2\Pi_I}{\partial k_I\partial k_h} & \cdots & -\frac{\partial^2\Pi_I}{\partial k_I^2} \end{vmatrix}, \tag{27}$$

with $h \neq j$, $h, j = 1, ..., N$. The terms that are relevant for this determinant are given by the second-order conditions, (23) and (24), and the terms $\partial^2\Pi_j/(\partial k_j\partial k_I)$, $\partial^2\Pi_j/(\partial k_j\partial k_h)$, $\partial^2\Pi_h/(\partial k_h\partial k_j)$, $\partial^2\Pi_I/(\partial k_I\partial k_j)$ and $\partial^2\Pi_I/(\partial k_I\partial k_h)$. Because of symmetry we know that in equilibrium $\partial^2\Pi_h/(\partial k_h\partial k_j) = \partial^2\Pi_j/(\partial k_j\partial k_h)$ and $\partial^2\Pi_I/(\partial k_I\partial k_h) = \partial^2\Pi_I/(\partial k_I\partial k_j)$. The remaining terms can be written as

$$\begin{aligned} \frac{\partial^2\Pi_j}{\partial k_j\partial k_I} &= P'\frac{dq_j}{dk_I}\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right] + P'q_j\left[\frac{d^2q_I}{dk_jdk_I} + (N-1)\frac{d^2q_h}{dk_jdk_I}\right] \qquad (28) \\ &+ P''q_j\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right]\left[\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}\right] + C_j''\frac{q_j}{k_j^2}\frac{dq_j}{dk_I} - R' - k_jR'', \\ \frac{\partial^2\Pi_j}{\partial k_j\partial k_h} &= P'\frac{dq_j}{dk_h}\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right] + P'q_j\left[\frac{d^2q_I}{dk_jdk_h} + (N-2)\frac{d^2q_k}{dk_jdk_h} + \frac{d^2q_h}{dk_jdk_h}\right] \qquad (29) \\ &+ P''q_j\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right]\left[\frac{dq_h}{dk_h} + \frac{dq_I}{dk_h} + (N-1)\frac{dq_j}{dk_h}\right] + C_j''\frac{q_j}{k_j^2}\frac{dq_j}{dk_h} - R' - k_jR'', \\ \frac{\partial^2\Pi_I}{\partial k_I\partial k_j} &= P'\frac{dq_I}{dk_j}N\frac{dq_j}{dk_I} + P'q_I\left[\frac{d^2q_j}{dk_jdk_I} + (N-1)\frac{d^2q_h}{dk_jdk_I}\right] \qquad (30) \\ &+ P''q_IN\frac{dq_I}{dk_j}\left[\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}\right] + C_I''\frac{q_I}{k_I^2}\frac{dq_I}{dk_I} - R' - (k_I - \underline{k})R'', \end{aligned}$$

The second derivatives that appear in these expressions can be derived in the same way as above where we checked that the second-order conditions are satisfied.

Proceeding in a similar way as Kolstad and Mathiesen (1987), i.e., subtracting the first column in (27) from the other columns, and then dividing the i-th row by $\partial^2\Pi_i/\partial k_i \partial k_j - \partial^2\Pi_i/\partial k_i^2$, with $i = I, 1, ..., N$, yields

$$|J| = \begin{vmatrix} -\frac{\frac{\partial^2\Pi_j}{\partial k_j^2}}{\frac{\partial^2\Pi_j}{\partial k_j \partial k_h} - \frac{\partial^2\Pi_j}{\partial k_j^2}} & -1 & -1 & \dots & -\frac{-\frac{\partial^2\Pi_j}{\partial k_j \partial k_I} + \frac{\partial^2\Pi_j}{\partial k_j^2}}{\frac{\partial^2\Pi_j}{\partial k_j \partial k_h} - \frac{\partial^2\Pi_j}{\partial k_j^2}} \\ -\frac{\frac{\partial^2\Pi_h}{\partial k_h \partial k_j}}{\frac{\partial^2\Pi_h}{\partial k_h \partial k_j} - \frac{\partial^2\Pi_h}{\partial k_h^2}} & 1 & 0 & 0 \quad \dots & -\frac{-\frac{\partial^2\Pi_h}{\partial k_h \partial k_I} + \frac{\partial^2\Pi_h}{\partial k_h \partial k_j}}{\frac{\partial^2\Pi_h}{\partial k_h \partial k_j} - \frac{\partial^2\Pi_h}{\partial k_h^2}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{\frac{\partial^2\Pi_I}{\partial k_I \partial k_j}}{\frac{\partial^2\Pi_I}{\partial k_I \partial k_j} - \frac{\partial^2\Pi_I}{\partial k_I^2}} & 0 & 0 & \dots & 1 \end{vmatrix}.$$

We can then calculate the determinant in a relatively straightforward way. Cumbersome but otherwise routine manipulations show that this determinant is unambiguously positive and, therefore, that the equilibrium of the capacity stage is unique. ■

A.4 Proof of Lemma 4

As the second-order conditions are satisfied, we can use the Implicit Function Theorem to show that the sign of dk_j/dk_I equals the sign of $\partial^2\Pi_j/\partial k_j \partial k_I$, given by (28). Similarly, the sign of dk_j/dk_h equals the sign of $\partial^2\Pi_j/\partial k_j \partial k_h$ given by (29) and the sign of dk_I/dk_j equals the sign of $\partial^2\Pi_I/\partial k_I \partial k_j$ given by (30). In what follows, we will determine the sign of the representative term $\partial^2\Pi_j/\partial k_j \partial k_I$. Determining the sign of the other terms works in exactly the same way.

Proceeding along the same lines as in the proof of Lemma 3, where we determined the sign of the expression $\partial^2\Pi_j/\partial k_j^2$, we obtain

$$\frac{\partial^2\Pi_j}{\partial k_j \partial k_I} = \frac{q_j \left(\sum_{s=0}^{8} (P')^s (\sum_{h=1}^{3} \kappa_{sh}(P'')^h + \kappa_{s4}P''' + \kappa_{s5}C_j''' + \kappa_{s6}C_I''' + \kappa_{s7}) \right)}{k_j^2 k_I (C_j'' - k_j P')^2 \eta^3} - R' - k_j R''. \tag{31}$$

In this expression, $\kappa_{sh} = \kappa_{sh}(q_j, k_j, q_I, k_I, C_j'', C_I'', P', P'', N)$, $s \in \{0, ..., 8\}$ and $h \in \{1, ..., 7\}$. As above, we do not specify the explicit expressions for κ_{sh} here but only spell out their respective signs. These signs are the following: For $h = \{1, 2, 3\}$ $\kappa_{sh} \leq 0$, if both s and h are either even (including 0) or odd and $\kappa_{sh} \geq 0$ if one is even and the other one is odd. κ_{s4}, κ_{s5}, $\kappa_{s6} \leq 0$ for s even (including 0) and κ_{s4}, κ_{s5}, $\kappa_{s6} \geq 0$ for s odd. Finally, $\kappa_{s7} < 0$ for s even (including 0) and $\kappa_{s7} > 0$ for s odd, implying that the numerator in the fraction is negative because P'' is negative or not too positive and P''' and C''' are positive or not too negative.

As in Lemma 3, the denominator is positive because $\eta > 0$. Therefore, the first term in (31) is negative. Because $R' > 0$ and R'' is either positive or not too negative as well, we obtain $\partial^2\Pi_j/\partial k_j \partial k_I < 0$. ■

A.5 Proof of Lemma 5

Differentiating (3) and (4) with respect to $\underline{k}$ yields

$$\frac{\partial^2\Pi_j}{\partial k_j^2}\frac{dk_j}{d\underline{k}} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j \partial k_h}\frac{dk_h}{d\underline{k}} + \frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\frac{dk_I}{d\underline{k}} = 0$$

and

$$\frac{\partial^2\Pi_I}{\partial k_I^2}\frac{dk_I}{d\underline{k}} + N\frac{\partial^2\Pi_I}{\partial k_I \partial k_j}\frac{dk_j}{d\underline{k}} + \frac{\partial^2\Pi_I}{\partial k_I \partial \underline{k}} = 0.$$

Using the fact that in equilibrium $dk_h/d\underline{k} = dk_j/d\underline{k}$ for $h, j \neq I$ we get

$$\frac{dk_j}{d\underline{k}} = \frac{\frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\frac{\partial^2\Pi_I}{\partial k_I \partial \underline{k}}}{\frac{\partial^2\Pi_j}{\partial k_j^2}\frac{\partial^2\Pi_I}{\partial k_I^2} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j \partial k_h}\frac{\partial^2\Pi_I}{\partial k_I^2} - N\frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\frac{\partial^2\Pi_I}{\partial k_I \partial k_j}} \tag{32}$$

and

$$\frac{dk_I}{d\underline{k}} = -\frac{\frac{\partial^2\Pi_I}{\partial k_I \partial \underline{k}}\left(\frac{\partial^2\Pi_j}{\partial k_j^2} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j \partial k_h}\right)}{\frac{\partial^2\Pi_j}{\partial k_j^2}\frac{\partial^2\Pi_I}{\partial k_I^2} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j \partial k_h}\frac{\partial^2\Pi_I}{\partial k_I^2} - N\frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\frac{\partial^2\Pi_I}{\partial k_I \partial k_j}}. \tag{33}$$

The terms that appear in these expressions are given by (23), (24), (28), (29), (30) and by

$$\frac{\partial^2\Pi_I}{\partial k_I \partial \underline{k}} = R' > 0.$$

From the proofs of the previous lemmas we know that all terms in (28), (29) and (30) have a negative sign. Thus, the numerators of the fractions on the right-hand side of (32) and (33) are both negative. The denominator in these fractions is the same in both equations. It is easy to show that $|\partial^2\Pi_j/\partial k_j^2| > |\partial^2\Pi_j/(\partial k_j \partial k_h)|$ which implies that

$$\frac{\partial^2\Pi_j}{\partial k_j^2}\frac{\partial^2\Pi_I}{\partial k_I^2} > \frac{\partial^2\Pi_j}{\partial k_j \partial k_h}\frac{\partial^2\Pi_I}{\partial k_I^2}. \tag{34}$$

In addition one can also easily show that $|\partial^2\Pi_I/\partial k_I^2| > |\partial^2\Pi_I/(\partial k_I \partial k_j)|$. Tedious calculations then reveal that

$$\frac{\partial^2\Pi_j}{\partial k_j \partial k_h}\frac{\partial^2\Pi_I}{\partial k_I^2} > \frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\frac{\partial^2\Pi_I}{\partial k_I \partial k_j}. \tag{35}$$

The inequalities in (34) and (35) then imply that the denominator is positive. As a consequence, we get that $dk_j/d\underline{k} < 0$ and $dk_I/d\underline{k} > 0$. ■

A.6 Proof of Proposition 1

We start with the right-hand side of (6). As mentioned in the main text, if $\underline{k} = 0$, the right-hand side of (6) simplifies to $-1/N$.

We now turn to the left-hand side of (6). From equations (32) and (33) we obtain that it is given by

$$\frac{\left(\frac{dk_j^*}{d\underline{k}}\right)}{\left(\frac{dk_I^*}{d\underline{k}}\right)} = -\frac{\frac{\partial^2 \Pi_j}{\partial k_j \partial k_I}}{\frac{\partial^2 \Pi_j}{\partial k_j^2} + (N-1)\frac{\partial^2 \Pi_j}{\partial k_j \partial k_h}} < 0, \quad h \neq j,\, h, j = 1, ..., N. \tag{36}$$

At $\underline{k} = 0$, we know that there is no difference between firm I and any of the non-integrated firms. This implies that $\partial^2 \Pi_j/(\partial k_j \partial k_h) = \partial^2 \Pi_j/(\partial k_j \partial k_I)$. Now, because all the second derivatives appearing in (36) are known to be negative (from the proof of Lemmas 3 and 4), $(dk_j/d\underline{k})\,/\,(dk_I/d\underline{k}) > -1/N$ is equivalent to $\partial^2 \Pi_j/\partial k_j^2 - \partial^2 \Pi_j/\partial k_j \partial k_I < 0$. Because at $\underline{k} = 0$ all firms are symmetric, subtracting (28) from (23) yields

$$\frac{\partial^2 \Pi_j}{\partial k_j^2} - \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} = P' N \frac{dq_h}{dk_j}\left[\frac{dq_j}{dk_j} - \frac{dq_h}{dk_j}\right]$$

$$+P' q_j \left[N \frac{d^2 q_h}{dk_j^2} - \frac{d^2 q_I}{dk_j dk_I} - (N-1)\frac{d^2 q_h}{dk_j dk_I}\right] + C_j'' \frac{q_j}{k_j^2}\left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j} - \frac{dq_h}{dk_j}\right) - R'.$$

In the proof of Lemma 1 we determined dq_j/dk_j and dq_h/dk_j. Evaluating these expressions at $q_I = q_j$ and $k_I = k_j$ we can determine $dq_j/dk_j - dq_h/dk_j$ and $dq_j/dk_j - q_j/k_j - dq_h/dk_j$ to get

$$\frac{dq_j}{dk_j} - \frac{dq_h}{dk_j} = \frac{q_j C_j''}{k_j(C_j'' - P'k_j)} \quad \text{and} \quad \left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j} - \frac{dq_h}{dk_j}\right) = \frac{q_j P'}{C_j'' - P'k_j}. \tag{37}$$

Determining the second derivatives $(d^2 q_h)/(dk_j^2)$, $(d^2 q_I)/(dk_j dk_I)$ and $(d^2 q_h)/(dk_j dk_I)$ and using (37) we obtain, after simplifying,

$$\frac{\partial^2 \Pi_j}{\partial k_j^2} - \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} = -\frac{q_j^2 P' \xi}{k_j^2 (C_j'' - (2+N)k_j P' - (1+N)k_j q_j P'')(C_j'' - P'k_j)^3} - R' < 0,$$

where

$$\xi = k_j^2 \left(q_j C_j''' N + k_j C_j''(3N+2)\right)(P')^3 +$$

$$\left(q_j k_j^2 (k_j(1+3N)C_j'' + q_j C_j''' N)P'' - k_j C_j''(3k_j(N+2)C_j'' + q_j C_j''')\right)(P')^2 +$$

$$\left(-q_j k_j C_j''(k_j C_j''(3N+4) + q_j C_j''')P'' + 5(C_j'')^3 k_j\right)P' - (C_j''')^3(C_j'' - 3q_j P'' k_j) < 0.$$

That is, $\partial^2 \Pi_j/\partial k_j^2$ is larger in absolute terms than $(\partial^2 \Pi_j)/(\partial k_j \partial k_I)$. As a consequence, $(dk_j/d\underline{k})\,/\,(dk_I/d\underline{k}) > -1/N$, which implies that the left-hand side of (6) is larger than the right-hand side. Thus, at $\underline{k} = 0$ vertical integration is procompetitive at the margin. By continuity, it follows that vertical integration is procompetitive at the margin for all $\underline{k}$ below a certain, positive threshold denoted by $\underline{k}^\star$.

■

A.7 Proof of Proposition 2

We show that for any finite N there either exists a $\underline{k}^{**} < \bar{k}$, such that vertical integration is anticompetitive at the margin for all $\underline{k} > \underline{k}^{**}$, or it is procompetitive at the margin for all $\underline{k}$ close to $\bar{k}$.

Let $\underline{k} = \bar{k}$, so that $k_j = 0$ for all $j \neq I$. We first have to determine q_j/k_j in this case. Because C_j is strictly convex, C_j' is invertible and equation (2) can be written as $q_j = k_j C_j'^{-1}(P(Q) + P'(Q)q_j)$. It follows directly that if $k_j = 0$ we also have $q_j = 0$.

Observe that the inverse $C_j'^{-1}(.)$ is strictly increasing and that it is zero if and only if its argument is zero. By using the rule of L'Hôpital we get $q_j/k_j = C_j'^{-1}(P(q_I)) > 0$, if $q_j = 0$ and $k_j = 0$. To simplify notation in the following we denote $\rho \equiv C_j'^{-1}(P(q_I))$.

We now turn to (6). The right-hand side of (6) in the case of $\underline{k} = \bar{k}$ can be written as

$$-\frac{C_I'' \frac{q_I}{k_I}}{\rho(C_I'' - k_I P')N}.$$

The left-hand side of (6) in case of $q_j = k_j = 0$ can be calculated from (32) and (33). To do so we first have to determine the second derivatives in (23), (24), (28), (29) and (30) at $q_j = k_j = 0$. From the right-hand side of (23) we know that $\partial^2\Pi_j/\partial k_j^2$ at $q_j = k_j = 0$ is given by

$$\frac{\partial^2 \Pi_j}{\partial k_j^2} = P' \frac{dq_j}{dk_j}\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right] + C_j'' \frac{q_j}{k_j^2}\left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j}\right) - 2R'. \tag{38}$$

We can then calculate dq_I/dk_j, dq_h/dk_j and dq_j/dk_j at $q_j = k_j = 0$ from (18) and (19). Taking into account that $q_j/k_j = \rho$ we get, by using the rule of L'Hôpital, that

$$\frac{dq_I}{dk_j} = -\frac{k_I \rho (P' + q_I P'')}{2k_I P' + q_I k_I P'' - C_I''}, \quad \frac{dq_h}{dk_j} = 0 \quad \text{and} \quad \frac{dq_j}{dk_j} = \rho.$$

Calculating the second term of the right-hand side in (38) at $q_j = k_j = 0$ gives us, again by using L'Hôpital's rule, that

$$C_j'' \frac{q_j}{k_j^2}\left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j}\right) = \frac{\rho^2 P'(3k_I P' + q_I k_I P'' - 2C_I'')}{2k_I P' + q_I k_I P'' - C_I''}.$$

Inserting these terms into (38) and simplifying then yields

$$\frac{\partial^2 \Pi_j}{\partial k_j^2} = 2\frac{\rho^2 P'(k_I P' - 2C_I'')}{2k_I P' + q_I k_I P'' - C_I''} - 2R'.$$

In the same way we can determine the expressions for $\partial^2\Pi_I/\partial k_I^2$, $\partial^2\Pi_j/(\partial k_j \partial k_I)$, $\partial^2\Pi_j/(\partial k_j \partial k_h)$ and $\partial^2\Pi_I/(\partial k_I \partial k_j)$ at $q_j = k_j = 0$. Inserting them in (32) and (33) and simplifying we obtain that

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{C_I'' P' \rho \frac{q_I}{k_I} + \sigma}{(N+1)\left(\rho^2 P'(C_I'' - k_I P') + \sigma\right)},$$

with $\sigma \equiv R' k_I \left(2P' k_I + P'' k_I q_I - C_I''\right) < 0$.

It follows that

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} < -\frac{C_I'' \frac{q_I}{k_I}}{\rho(C_I'' - k_I P')N}$$

if and only if

$$-\left(\frac{N}{1+N}\right)\left(\frac{C_I'' P' \rho \frac{q_I}{k_I} + \sigma}{\rho^2 P'(C_I'' - k_I P') + \sigma}\right) < -\frac{C_I'' \frac{q_I}{k_I}}{\rho(C_I'' - k_I P')}. \tag{39}$$

But the left-hand side of (39) can either be larger or smaller than the right-hand side. To see this suppose first that σ is small in absolute terms. In this case, the second term of the left-hand side is approximately the same as the right-hand side. But because $-N/(1+N) > -1$, the left-hand side is larger. By continuity, vertical integration can be procompetitive at the margin for all $\underline{k}$ close to $\bar{k}$. On the other hand, suppose that N is very large. In this case, $N/(1+N)$ is close to 1. We then have that vertical integration is anticompetitive at the margin if

$$-\frac{C_I'' P' \rho \frac{q_I}{k_I} + \sigma}{\rho^2 P'(C_I'' - k_I P') + \sigma} < -\frac{C_I'' \frac{q_I}{k_I}}{\rho(C_I'' - k_I P')}. \tag{40}$$

Obviously the left-hand side equals the right-hand side if $\sigma = 0$. But because $\sigma < 0$ and $\rho^2 P'(C_I'' - k_I P') < P' C_I'' \rho \frac{q_I}{k_I} < 0$, the inequality in (40) is fulfilled. ■

A.8 Proof of Proposition 3

Case (i): We first look at the right-hand side of (6). Differentiating it with respect to $\underline{k}$ reveals that this derivative has the same sign as

$$-C_j'' C_I'' (C_I'' - k_I P')(C_j'' - k_j P')\left(\frac{d\,(q_I/k_I)}{d\underline{k}}\frac{q_j}{k_j} - \frac{d\,(q_j/k_j)}{d\underline{k}}\frac{q_I}{k_I}\right)$$

$$- P' C_j'' C_I'' \frac{q_j}{k_j}\frac{q_I}{k_I}\left((C_j'' - k_j P')\frac{dk_I}{d\underline{k}} - (C_I'' - k_I P')\frac{dk_j}{d\underline{k}}\right) \tag{41}$$

$$-P'\frac{q_j}{k_j}\frac{q_I}{k_I}\left(\frac{dC_j''}{d\underline{k}} C_I'' k_j (C_I'' - k_I P') - \frac{dC_I''}{d\underline{k}} C_j'' k_I (C_j'' - k_j P')\right) + P''\frac{dQ}{d\underline{k}} C_j'' C_I'' \frac{q_j}{k_j}\frac{q_I}{k_I}(k_j C_I'' - k_I C_j'').$$

From Lemma 5 we know that $dk_I/d\underline{k} > 0$ and $dk_j/d\underline{k} < 0$. Because of Lemma 2 this implies that $d\left(q_I/k_I\right)/d\underline{k} < 0$ and $d\left(q_j/k_j\right)/d\underline{k} > 0$. Because $q_j/k_j > q_I/k_I$, the first term in (41) is positive. Also, because $dk_I/d\underline{k} > 0$ and $dk_j/d\underline{k} < 0$, the second term is positive as well.

Now let us turn to the third term. Because C''' is positive or not very negative, we get that $dC_j''/d\underline{k}$ is also positive or not very negative while $dC_I''/d\underline{k}$ is negative or not very positive. Therefore, the third term is either positive, or, if it is negative, then only slightly so. As a consequence, the sum of the first three terms in (41) is positive.

Now let us look at the fourth term. Because $k_j < k_I$ and $C_I'' < C_j''$ the last term in brackets is negative. Because P'' is negative or not too positive we have that for $dQ/d\underline{k} \geq 0$ the fourth term is positive or only slightly negative.

But in sum this implies that (41) is positive and thus the right-hand side of (6) is strictly increasing in $\underline{k}$ if $dQ/d\underline{k} \geq 0$.

Now we turn to the left-hand side of (6) which is given by

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{\frac{\partial^2 \Pi_j}{\partial k_j \partial k_I}}{\frac{\partial^2 \Pi_j}{\partial k_j^2} + (N-1)\frac{\partial^2 \Pi_j}{\partial k_j \partial k_h}}. \tag{42}$$

where the second derivatives of the right-hand side of (42) are given by (23), (24), and (28). We can now multiply each of the three expressions by $K/R(K)$, yielding that the next to last term in these expressions is given by the elasticity of the capacity price with respect to the capacity, $R'(K)K/R(K)$. If this elasticity is sufficiently large, i.e., if the capacity supply function is sufficiently inelastic, the three terms (23), (24), and (28) are dominated by this elasticity, implying that $(dk_j/d\underline{k})/(dk_I/d\underline{k}) = -1/(N+1)$. Thus, the left-hand side of (6) does not vary with $\underline{k}$. Because we know from the proof of Proposition 1 that the right-hand side is smaller than the left-hand side at $\underline{k} = 0$ and because the right-hand side is strictly increasing at any point of intersection, there can at most be one such point.

Case (ii): We first solve for the equilibrium in the linear-quadratic case. The profit function of the integrated firm in this case can be written as

$$\Pi_I = \left[\alpha - \beta q_I - \beta \sum_{j=1}^{N} q_j\right] q_I - \frac{c q_I^2}{2k_I} - \delta(k_I - \underline{k})(k_I + \sum_{j=1}^{N} k_j),$$

and the one of a non-integrated firm j as

$$\Pi_j = \left[\alpha - \beta q_j - \beta q_I - \beta \sum_{h=1, h\neq j}^{N} q_h\right] q_j - \frac{c q_j^2}{2k_j} - \delta k_j (k_j + k_I + \sum_{h=1, h\neq j}^{N} k_h).$$

Differentiating with respect to q_I and q_j and solving for the equilibrium quantities yields

$$q_I = \frac{\alpha\left(\beta k_j + c\right) k_I}{\beta(\beta k_j(N+2)+2c)k_I + c^2 + k_j \beta c(N+1)}$$

and

$$q_j = \frac{\alpha\left(\beta k_I + c\right) k_j}{\beta(\beta k_j(N+2)+2c)k_I + c^2 + k_j \beta c(N+1)}.$$

After substituting these quantities into the respective profit functions, we can take derivatives of Π_I with respect to k_I and of Π_j with respect to k_j.[46] The equilibrium capacity k_I is then implicitly defined by

$$\begin{aligned}&\left(c^2(c+k_j(1+N)\beta)\right)\left(\alpha^2 c(2\beta+c) - N\delta c^2\right)k_j \qquad (43)\\ &+\beta(\alpha^2\beta - 4N(1+N)\delta c^2)k_j^2 - 2N\delta c\beta^2(1+N)^2 k_j^3\big) = \sum_{t=1}^{4} k_I^t \theta_t - \theta_0 \underline{k},\end{aligned}$$

with

$$\begin{aligned}
\theta_0 &= 2\delta(\beta^2(2+N)k_j k_I + \beta(N+1)k_j c + 2\beta c k_I + c^2)^3,\\
\theta_1 &= (6\beta^4\delta N c(2+N)(1+N)^2 k_j^4 - \beta^3((2+3N)\alpha^2\beta - 4\delta c^2(1+N)(7N^2+11N+1))k_j^3\\
&\quad -2\beta^2 c((3\alpha^2\beta(N+1) - 3\delta c^2(7N^2+10N+2))k_j^2\\
&\quad -\beta c^2(3\alpha^2\beta(N+2) - 12\delta c^2(2N+1))k_j - 2c^3(\alpha^2\beta - 2\delta c^2)),\\
\theta_2 &= (6\beta^5\delta N c(1+N)(2+N)^2 k_j^4 + 6\beta^4\delta c^2(2+N)(N^2+10N+2)k_j^3\\
&\quad +24\beta^3\delta c^3(2N+3)(2N+1)k_j^2 + 12\beta^2\delta c^4(7N+6)k_j + 24\beta\delta c^5),\\
\theta_3 &= 2\beta^2\delta(k_j^2\beta^2 N(2+N) + 2\beta k_j c(4N+3) + 6c^2)(k_j\beta(N+2)+2c)^2,\\
\theta_4 &= 4\beta^3\delta(k_j\beta(N+2)+2c)^3.
\end{aligned}$$

while the equilibrium capacity q_j is implicitly defined by

$$\begin{aligned}&c^3(2k_I\beta + c)(\beta(-8c^2\delta + \alpha^2\beta k_I^2 + 2c(\alpha^2\beta - c^2\delta)k_1 + c^2\alpha^2 - 8\beta^2 c\delta k_I^3) \qquad (44)\\
&= \Big((8\beta^4 c\delta(8+3N)k_I^4 - \beta^3(\alpha^2\beta(6+N) - 16c^2\delta(7+4N))k_I^3 - \beta^2 c(\alpha^2\beta(14+3N) - 18c^2\delta(3N+4))k_I^2\\
&\quad +c^2(-k_I\alpha^2\beta^2(3N+10) + c(2(9N+10)ck_I\delta - \alpha^2(N+2))\beta + 2c^3\delta(N+1))) + \sum_{t=1}^{4} k_j^t\tau_t\Big)k_j,\end{aligned}$$

[46] As before, we have to distinguish between k_j and k_h $h \neq j$, $h, j = 1, ..., N$. Of course, in equilibrium we will have $k_h = k_j$.

with

$$
\begin{aligned}
\tau_1 &= c\beta(12\beta^4 c\delta(N+4)(N+2)k_I^4 + \beta^3(-\alpha^2\beta(2+3N) + 2c^2\delta(116N+104+27N^2))k_I^3 \\
&\quad +\beta^2 c(-3\alpha^2\beta(3+N) + 6c^2\delta(39N+28+12N^2))k_I^2 \\
&\quad +\beta c^2(-9\alpha^2\beta N + 12c^2\delta(3N+5)(N+1))k_I + c^3(((1-3N)\alpha^2)\beta + 2c^2\delta(3N+4)(N+1))), \\
\tau_2 &= 2\beta^2\delta c((2+N)k_I\beta + (N+1)c)(\beta^3(N+8)(N+2)k_I^3 + c\beta^2(8N^2+45N+40)k_I^2 \\
&\quad +2c^2\beta(5N+14)(N+1)k_I + 3c^3(N+2)(N+1)), \\
\tau_3 &= 2\beta^3\delta((2\beta^2(1+N))k_I^2 + c\beta(N+9)(N+1)k_I + c^2(N+4)(N+1))((2+N)k_I\beta + (1+N)c)^2 k_j^4, \\
\tau_4 &= 2\beta^4\delta(N+1)((N+2)k_I\beta + (N+1)c)^3.
\end{aligned}
$$

We now turn to the competitive effects of a change in $\underline{k}$. Because $Q = q_I + Nq_j$, we can insert the above explicit solutions for the quantities and differentiate Q with respect to $\underline{k}$. From this we obtain $dQ/d\underline{k} > 0$ if and only if

$$
\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} > -\frac{(k_j\beta + c)^2}{N(k_I\beta + c)^2}. \tag{45}
$$

Via differentiating (43) and (44) with respect to k_j, k_I and $\underline{k}$ and solving for $dk_j/d\underline{k}$ and $dk_I/d\underline{k}$, we can determine the left-hand side of (45). Subtracting the right-hand side from the left-hand side yields an expression that has the following structure:

$$
\sum_{u=0}^{6}\sum_{z=0}^{5} k_j^u k_I^z \upsilon_{uz}, \tag{46}
$$

where $\upsilon_{uz} = \upsilon_{uz}(\alpha, \beta, \delta, c, N)$. We do not spell out the exact expressions for υ_{uz}, $u \in \{1, ..., 6\}$, $z \in \{1, ..., 5\}$ because they are rather complicated. As will become clear, we are mainly interested in determining their signs and compare them, which can be done relatively easily.

Differentiating (46) with respect to $\underline{k}$ we get

$$
\sum_{u=1}^{6}\sum_{z=1}^{5} \upsilon_{uz}\left(zk_j^u k_I^{z-1}\frac{dk_I}{d\underline{k}} + uk_j^{u-1}k_I^z\frac{dk_j}{d\underline{k}}\right) + \sum_{z=1}^{5}\upsilon_{0z} z k_I^{z-1}\frac{dk_I}{d\underline{k}} + \sum_{u=1}^{6}\upsilon_{u0} u k_j^{u-1}\frac{dk_j}{d\underline{k}},
$$

where, from Lemma 5, $dk_j/d\underline{k} < 0$ and $dk_I/d\underline{k} > 0$.

First, one can show that all $\upsilon_{uz} > 0$ if $u > z$. Thus, the expressions $\upsilon_{uz}(zk_j^u k_I^{z-1}(dk_I/d\underline{k}) + uk_j^{u-1}k_I^z(dk_j/d\underline{k}))$ for $u > z$ are all negative. The expressions for υ_{uz} with $u < z$ can have different signs. So let us first take each term $\upsilon_{uz}(zk_j^u k_I^{z-1}(dk_I/d\underline{k}) + uk_j^{u-1}k_I^z(dk_j/d\underline{k}))$, where $z = z_a > u_a = u$. Now we compare it with the corresponding expression where $u = z_a$ and $z = u_a$. One can then show that the latter expression is larger than the former in absolute values in any comparison. Therefore, the sum of each of the comparisons is negative. Finally, we have to look at terms with $u = z$. Again, v_{uz} can be positive or negative, i.e., $v_{uz} > 0$ for $u = z = 1, 2, 3$, $v_{uz} < 0$ for $u = z = 4$ and $v_{uz} = 0$ for $u = z = 5$. Now for any of these expressions $v_{uz}(zk_j^u k_I^{z-1}(dk_I/d\underline{k}) + uk_j^{u-1}k_I^z(dk_j/d\underline{k}))$ with $u = z$ we can find a previous comparison, to which we can add the expression and the resulting sum still stays negative. Thus, equation (46) is strictly decreasing in $\underline{k}$. Because at $\underline{k} = 0$, the left-hand side of (45) is larger than the right-hand side, we know that there exists either a unique intersection or no intersection between the terms on the two sides. ■

A.9 Proof of Proposition 4

We first show that $q_j \to 0$ and $k_j \to 0$, $j \neq I$, as $N \to \infty$. Suppose to the contrary that $q_j > 0$. But because $Q = q_I + Nq_j$ and $P(Q) \leq 0$, as $N \to \infty$, the first-order condition given by (2) cannot be satisfied if $q_j > 0$, because the right-hand side is positive while the left-hand side would be negative. Therefore, $q_j \to 0$, as $N \to \infty$. Given this, suppose now that $k_j > 0$. But then in the first-order condition of the capacity stage, (3), the left-hand side would be negative while the right-hand side is zero. In order to fulfill this condition we must have $k_j \to 0$. Therefore, as $N \to \infty$, $q_j \to 0$ and $k_j \to 0$.

In the proof of Proposition 2 we already calculated the case of $q_j \to 0$ and $k_j \to 0$. In addition, for $N \to \infty$ we obtain from (39) that vertical integration is anticompetitive if

$$-\frac{C_I''P'\rho\frac{q_I}{k_I} + \sigma}{\rho^2 P'(C_I'' - k_I P') + \sigma} < -\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I'' - k_I P')},$$

where ρ and σ are defined in the proof of Proposition 2. But we already showed in this proof that the inequality is fulfilled. Therefore, vertical integration is anticompetitive if $N \to \infty$. ■

A.10 Proof of Proposition 5

From the proof of Proposition 3 we know that the left-hand side of (6), $(dk_j/d\underline{k})/(dk_I/d\underline{k})$, equals $-1/(N+1)$ if the capacity supply function $R(K)$ is sufficiently inelastic. Multiplying (6) by N, the left-hand side is given $-N/(N+1)$. Taking the derivative with respect to N, we obtain $-1/(N+1)^2 < 0$, implying that the left-hand side is decreasing with N.

We now turn to the right-hand side of (6), multiplied by N, which is given by

$$-\frac{C_I''q_I k_j(C_j'' - k_j P')}{C_j''q_j k_I(C_I'' - k_I P')}. \tag{47}$$

To take the derivative of (47) we need to determine $\partial k_j/\partial N$ from (3) and $\partial k_I/\partial N$ from (4) and use these derivatives to calculate $\partial q_j/\partial N$ and $\partial q_I/\partial N$ from (2), taking into account that R is very inelastic. Doing so yields

$$\frac{\partial k_j}{\partial N}=\frac{\partial k_I}{\partial N}=-\frac{k_j}{N+2}<0,$$

$$\frac{\partial q_j}{\partial N}=-\frac{Nq_jC_j''\left(C_I''-k_IP'\right)}{\Omega}<0 \quad \text{and} \quad \frac{\partial q_j}{\partial N}=-\frac{Nq_IC_I''\left(C_j''-k_jP'\right)}{\Omega}<0,$$

with $\Omega=(N+2)((P')^2k_jk_I(N+2)+P'(k_jk_I(Nq_j+q_I)P''-k_jC_I''(N+1)-2k_IC_j'')-P''(q_Ik_IC_j''+Nq_jk_jC_I'')+C_I''C_j'')$.

Taking the derivative of (47) and using the above expressions, we obtain, under the additional assumption that the cost function C is quadratic, which implies that $C_j''=C_I''=C''$, that this derivative is given by

$$-(k_I-k_j)(-2(P')^4k_j^2k_I^2(N+2)\left(C''\right)^2+\left(C''\right)^4P''(k_I-k_j)(q_I-q_j)$$
$$-(P')^2\left(C''\right)^3\left(C''(k_j(k_j+2k_IN)+k_I(k_I+4k_j))-2P''k_jk_I(q_jN(3k_j+k_I)+q_I(k_j+3k_I))\right)$$
$$+P'\left(C''\right)^4\left(C''(k_jN+k_I)-P''(q_jN(k_I-k_j)^2+q_I(k_I^2+k_j(2k_I-k_j))\right)$$
$$+(P')^3k_jk_I\left(C''\right)^2\left(C''(3k_j(N+1)+k_I(N+5))-2P''k_jk_I(q_I+Nq_j)\right).$$

Because P'' is negative or not too positive, the derivative is positive, implying that (47) is falling in N.

From the previous analysis we know that $\underline{k}^*$ is given by the intersection of the left-hand side and the right-hand side of (6). As shown in the proof of Proposition 3, if R' is very large, the left-hand side is constant in k while the right-hand side is increasing at the point of intersection. Now we just showed that the left-hand side is lower if N is larger while the right-hand side is larger if N is larger. But this implies that the two sides cross each other at a lower value of k if N is larger. Because $\underline{k}^*(N)$ is defined as the point of intersection, it follows that it is decreasing in N. ■

A.11 Proof of Proposition 6

We know that welfare is increasing in $\underline{k}$ if and only if (7) holds. The first term on the left-hand side of (7), $PdQ/d\underline{k}$, has the same sign as the condition for pro- or anticompetitive vertical integration. Therefore, we know that it is zero at $\underline{k}^*$. As a consequence, if the rest of the left-hand side is negative at $\underline{k}^*$, this implies that $\underline{k}^*_{WF}<\underline{k}^*$.

The term $-R(dK/d\underline{k})$ is negative because overall capacity is increasing in $\underline{k}$. Thus, if the terms

$$-NC_j\frac{dk_j}{d\underline{k}} - Nk_jC_j'\left(\frac{1}{k_j}\frac{dq_j}{d\underline{k}} - \frac{q_j}{k_j^2}\frac{dk_j}{d\underline{k}}\right) - C_I\frac{dk_I}{d\underline{k}} - k_IC_I'\left(\frac{1}{k_I}\frac{dq_I}{d\underline{k}} - \frac{q_I}{k_I^2}\frac{dk_I}{d\underline{k}}\right) \tag{48}$$

are negative at $\underline{k}^*$, we have established that $\underline{k}^*_{WF} < \underline{k}^*$. We can now use the respective expressions for the cost functions and the equilibrium values of q_j and q_I in the linear-quadratic case that we calculated in the proof of Proposition 3, case (ii). Inserting them into (48) and simplifying reveals that the sign of this expression is the same as the sign of

$$-\left[N\alpha^2(c+\beta k_I)\left(k_I^2\beta^2(2c-\beta k_j(N+2)) + k_Ic\beta(c-\beta k_j(2N+5)) + c^2(c-\beta k_j(N+1))\right)\right]\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} -$$

$$-\alpha^2\left[c+k_j\beta\right]\left[k_j^2\left(\beta^2c(N+1) - k_I\beta^3(N+2)\right) + k_jc\beta\left(c(2-N) - k_I\beta(3N+4)\right) - 2k_Ic^2\beta + c^3\right]. \tag{49}$$

From (45) we know that $dQ/d\underline{k} = 0$ if $\underline{k}$ implies equilibrium values of k_I and k_j such that

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{(k_j\beta+c)^2}{N(k_I\beta+c)^2}.$$

Inserting the last equation into (49) and simplifying gives

$$-\frac{2\alpha c\beta(k_I-k_j)(c+\beta k_j)\left(k_I\beta(2c+k_j\beta(N+2)) + c(c+k_j\beta(N+1))\right)}{(c+\beta k_I)},$$

which is negative because $k_I > k_j$ at $\underline{k}^*$. Thus, we have shown that $\underline{k}^*_{WF} < \underline{k}^*$. ■

A.12 Proof of Lemma 6

Part (a) Differentiating Π_I with respect to p_I gives

$$\alpha - 2\beta p_I + \frac{\sigma(p_j + (N-1)p_h}{N} + \frac{c\beta\left((\alpha-\beta p_I)N + \sigma(p_j+(N-1)p_h)\right)}{Nk_I}, \tag{50}$$

where p_j and p_h are the prices of two non-integrated firms. Similarly, differentiating Π_j with respect to p_j gives

$$\alpha - 2\beta p_j + \frac{\sigma(p_I + (N-1)p_h}{N} + \frac{c\beta\left((\alpha-\beta p_j)N + \sigma(p_I+(N-1)p_h)\right)}{Nk_j}. \tag{51}$$

In equilibrium, $p_h = p_j$. Inserting this into (50) and (51) and solving for p_I and p_j yields

$$p_I = \frac{\alpha(k_I+c\beta)(c\beta(\beta N+\sigma) + k_j(2\beta N+\sigma))}{\varrho_1} \tag{52}$$

and

$$p_j = \frac{\alpha(k_j + c\beta)(c\beta(\beta N + \sigma) + k_I(2\beta N + \sigma))}{\varrho_1}, \quad (53)$$

with

$$\varrho_1 = \beta^2 c^2(\beta-\sigma)(\beta N+\sigma)+k_j k_I(2\beta-\sigma)(2\beta N+\sigma)+c\beta\left((k_I + k_j)(\sigma + 2\beta N)(\beta - \sigma) + \sigma\beta(k_I + Nk_j)\right) > 0.$$

Differentiating p_I and p_j with respect to k_I yields

$$\frac{\partial p_I}{\partial k_I} = -\frac{\alpha\beta^2\sigma c((\beta N + \sigma)(c\beta + k_j) + \beta N k_j)(\beta c + k_j)}{(\varrho_1)^2} < 0$$

and

$$\frac{\partial p_j}{\partial k_I} = -\frac{\alpha\beta^2 c(\beta N + \sigma)(c\beta + k_j) + \beta N k_j)}{(\varrho_1)^2} < 0.$$

In a similar way we obtain

$$\frac{\partial p_I}{\partial k_j} = -\frac{\alpha\beta^2\sigma c((\beta N + \sigma)(c\beta + k_I) + \beta N k_I)(\beta c + k_I)}{(\varrho_1)^2} < 0,$$

$$\frac{\partial p_h}{\partial k_j} = -\frac{\alpha\beta^2\sigma c((\beta N + \sigma)(c\beta + k_I) + \beta N k_I)(\beta c + k_j)}{((\beta N + \sigma)(c\beta + k_j) + \beta N k_j)(\varrho_1)^2} < 0,$$

and

$$\frac{\partial p_j}{\partial k_j} = -\frac{\alpha\beta^2\sigma c((\beta N + \sigma)(c\beta + k_I) + \beta N k_I)\varrho_2}{((\beta N + \sigma)(c\beta + k_j) + \beta N k_j)(\varrho_1)^2} < 0,$$

with

$$\varrho_2 = \beta^2 c^2(\beta N + \sigma)((\beta - \sigma)N + \sigma) + k_i k_j(2\beta N + \sigma)((2\beta - \sigma)N + \sigma)$$

$$+c\beta\left((k_I + k_j)(\beta N((2\beta - \sigma)N + 2\sigma) - \sigma^2(N - 1)) + 2\beta\sigma N k_I\right) > 0.$$

Part (b) From part (a) we know that if firm i increases its capacity from k_i to $\hat{k}_i > k_i$, then its price falls from $p_i(k_i, \mathbf{k}_{-i})$ to $p_i(\hat{k}_i, \mathbf{k}_{-i}) < p_i(k_i, \mathbf{k}_{-i})$. Because q_i is falling in p_i, this implies $q_i(\hat{k}_i, \mathbf{k}_{-i}) > q_i(k_i, \mathbf{k}_{-i})$.

The proof can then be easily conducted by looking at the first-order condition of firm i in general form. This first-order condition is given by

$$q_i(k_i, \mathbf{k}_{-i}) = -\left[p_i - C_i'\left(\frac{q_i(k_i, \mathbf{k}_{-i})}{k_i}\right)\right]\frac{\partial q_i(k_i, \mathbf{k}_{-i})}{\partial p_i}. \quad (54)$$

Because $q_i(\hat{k}_i, \mathbf{k}_{-i}) > q_i(k_i, \mathbf{k}_{-i})$, the left-hand side of (54) is larger for $\hat{k}_i$ than for k_i.

Now we turn to the right-hand side of (54). Suppose to the contrary of the claim in part (b) of the lemma that $q_i(\hat{k}_i, \mathbf{k}_{-i})/\hat{k}_i \geq q_i(k_i, \mathbf{k}_{-i})/k_i$. Because $C_i'' > 0$, this implies that C_i' in the squared bracket is larger for $\hat{k}_i$ than for k_i. From above, we know that $p_i(\hat{k}_i, \mathbf{k}_{-i}) < p_i(k_i, \mathbf{k}_{-i})$. Therefore, the squared bracket would be smaller for $\hat{k}_i$ than for k_i. In addition, with linear demand $\partial q_i(k_i, \mathbf{k}_{-i})/\partial p_i = -(\beta + \sigma(N-1))/((\beta - \sigma)(\beta + \sigma N))$ and is therefore independent of k_i. This implies that the right-hand side would be strictly smaller for $\hat{k}_i$ than k_i, which yields the desired contradiction.

Part (c) The proof that the profit function of each firm is concave and that the equilibrium is unique proceeds in the same way as in the case of quantity competition. Because $\partial^2\Pi_i/\partial k_i^2 < 0$, by using the Implicit Function Theorem it follows that

$$\text{sign}\left\{\frac{dk_i}{dk_j}\right\} = \text{sign}\left\{\frac{\partial^2\Pi_i}{\partial k_i \partial k_j}\right\}, \quad j \neq i.$$

Determining, for example, $\partial^2\Pi_j/\partial k_j \partial k_I$ we obtain

$$\frac{\partial^2\Pi_j}{\partial k_j \partial k_I} = -\frac{\alpha^2\beta^4\sigma c\left(\beta\sigma c + \beta^2 cN + k_I(2\beta N + \sigma)\right)\varrho_3}{\left[\beta^2 N(2k_j + c\beta)(2k_I + c\beta) - \beta\sigma(N-1)(k_j + c\beta)(2k_I + c\beta) - \sigma^2(k_j + c\beta)(k_I + c\beta)\right]^4} - \delta, \tag{55}$$

with

$$\varrho_3 = \beta^2 N(2k_j + c\beta)(2k_I + c\beta)\left(\beta N - \sigma(\beta c(N-2) + k_j(N-1))\right) - \sigma^3(3k_j + c\beta)(k_j + c\beta)(k_I + c\beta)$$

$$-\beta\sigma^2\left[\beta^3 c^3(2N-1) + \beta^2 c^2\left(k_I(3N-2) + k_j(7N-1)\right) + \beta c k_j\left(k_j(5N+3) + k_I(11N-5)\right) + 8Nk_I k_j^2\right].$$

At $\sigma = 0$ we obtain that (55) equals

$$\frac{\partial^2\Pi_j}{\partial k_j \partial k_I} = -\delta < 0.$$

Similarly, at $\sigma = \beta$ we have

$$\frac{\partial^2\Pi_j}{\partial k_j \partial k_I} = -\frac{\alpha^2\beta c\left(\beta c(N+1) + k_I(2N+1)\right)\varrho_4}{\left[\beta c(k_j N + k_I) + k_I k_j(2N+1)\right]^4} - \delta,$$

with

$$\varrho_4 = \beta^2 c^2(N+1)(k_I(2N+1) - 3k_j) + \beta c k_j\left(k_I(2N^2 - 1 - N) + k_j(2N^2 + 6 - 3N)\right) + k_j^2 k_I(2N-1)(4N-3).$$

Because $N \geq 1$ and $\varrho_4 > 0$, it follows that $\partial^2\Pi_i/\partial k_i \partial k_j < 0$ at $\sigma = \beta$.

Finally, longish but routine calculations show that there exists no root for the right-hand side of (55) for $\sigma \in (0, \beta)$. It follows that $dk_j/dk_I < 0$. In exactly the same one can show that $dk_j/dk_h < 0$ and $dk_I/dk_j < 0$.

Part (d) Because the sign of dk_i/dk_j, $\forall i \neq j, i, j = I, 1, ..., N$, is the same as in the case of quantity competition, the proof of this lemma proceeds exactly in the same way as the proof of Lemma 5, and is therefore omitted. ■

A.13 Proof of Proposition 8

As explained in part (d) of the proof of Lemma 6, $dk_j/d\underline{k}$ and $dk_I/d\underline{k}$ are derived in the same way as in case of quantity competition. This yields

$$\frac{dk_j}{d\underline{k}} = \frac{\frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\frac{\partial^2\Pi_I}{\partial k_I \partial \underline{k}}}{\frac{\partial^2\Pi_j}{\partial k_j^2}\frac{\partial^2\Pi_I}{\partial k_I^2} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j \partial k_h}\frac{\partial^2\Pi_I}{\partial k_I^2} - N\frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\frac{\partial^2\Pi_I}{\partial k_I \partial k_j}}$$

and

$$\frac{dk_I}{d\underline{k}} = -\frac{\frac{\partial^2\Pi_I}{\partial k_I \partial \underline{k}}\left(\frac{\partial^2\Pi_j}{\partial k_j^2} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j \partial k_h}\right)}{\frac{\partial^2\Pi_j}{\partial k_j^2}\frac{\partial^2\Pi_I}{\partial k_I^2} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j \partial k_h}\frac{\partial^2\Pi_I}{\partial k_I^2} - N\frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\frac{\partial^2\Pi_I}{\partial k_I \partial k_j}}.$$

At $\underline{k} = 0$, we have $q_I = q_j$ and $k_I = k_j$, implying that $\partial^2\Pi_I/\partial k_I^2 = \partial^2\Pi_j/\partial k_j^2$ and $\partial^2\Pi_j/\partial k_j\partial k_h = \partial^2\Pi_j/\partial k_j\partial k_I = \partial^2\Pi_I/\partial k_I\partial k_j$ in the last expressions. Simplifying then yields

$$\frac{dk_j}{d\underline{k}} = \frac{\frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\frac{\partial^2\Pi_I}{\partial k_I \partial \underline{k}}}{\left(\frac{\partial^2\Pi_j}{\partial k_j^2} - \frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\right)\left(\frac{\partial^2\Pi_j}{\partial k_j^2} + N\frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\right)} \tag{56}$$

and

$$\frac{dk_I}{d\underline{k}} = -\frac{\frac{\partial^2\Pi_I}{\partial k_I \partial \underline{k}}\left(\frac{\partial^2\Pi_j}{\partial k_j^2} + (N-1)\frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\right)}{\left(\frac{\partial^2\Pi_j}{\partial k_j^2} - \frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\right)\left(\frac{\partial^2\Pi_j}{\partial k_j^2} + N\frac{\partial^2\Pi_j}{\partial k_j \partial k_I}\right)}. \tag{57}$$

We can calculate the expressions involved in the right-hand sides of (56) and (57) in the same way as we determined $\partial^2\Pi_j/\partial k_j\partial k_I$ in the proof of part (c) Lemma 6.

From (12) the equilibrium quantities are given by

$$q_I = \frac{1}{1+\sigma N} - \frac{1+\sigma(N-1)}{(1-\sigma)(1+\sigma N)}p_I + \frac{\sigma}{(1-\sigma)(1+\sigma N)}(Np_j)$$

for the integrated firm, and by

$$q_j = \frac{1}{1+\sigma N} - \frac{1+\sigma(N-1)}{(1-\sigma)(1+\sigma N)}p_j + \frac{\sigma}{(1-\sigma)(1+\sigma N)}(p_I + (N-1)p_j),$$

for a non-integrated firm j, with p_j and p_I given by (52) and (53), respectively. We can use these formulas to determine consumer surplus, which can be written as

$$CS = \alpha N q_j + \alpha q_I - \frac{\beta}{2}N(q_j)^2 - \frac{\beta}{2}(q_I)^2 - \sigma N q_I q_j - \sigma\frac{N(N-1)}{2}(q_j)^2 - Np_j q_j - p_I q_I + m,$$

Determining how consumer surplus changes with $\underline{k}$, we need to take the derivative of CS with respect to $\underline{k}$. However, at $\underline{k} = 0$, we have $q_I = q_j$ and $p_I = p_j$, implying that if the aggregate price level given by $(p_I + Np_j)/(N+1)$ falls, then consumer surplus rises. Therefore, we have that

$$\text{sign}\left\{\frac{dCS}{d\underline{k}}\Big|_{\underline{k}=0}\right\} = -\text{sign}\left\{\frac{d\,(p_I + Np_j)}{d\underline{k}}\Big|_{\underline{k}=0}\right\}.$$

The derivative of $p_I + Np_j$ with respect to $\underline{k}$ is given by

$$\frac{d\,(p_I + Np_j)}{d\underline{k}} = \frac{\partial p_I}{\partial k_I}\frac{dk_I}{d\underline{k}} + N\frac{\partial p_I}{\partial k_j}\frac{dk_j}{d\underline{k}} + N\left(\frac{\partial p_j}{\partial k_j}\frac{dk_j}{d\underline{k}} + \frac{\partial p_j}{\partial k_I}\frac{dk_I}{d\underline{k}} + (N-1)\frac{\partial p_j}{\partial k_h}\frac{dk_h}{d\underline{k}}\right).$$

Using $\partial p_I/\partial k_I$, $\partial p_I/\partial k_j$, $\partial p_j/\partial k_j$, $\partial p_j/\partial k_I$, and $\partial p_j/\partial k_h$ determined in part (a) of Lemma 6 and $dk_I/d\underline{k}$ and $dk_j/d\underline{k}$ determined by (56) and (57) we obtain after inserting $k_I = k_j$

$$\frac{d\,(p_I + Np_j)}{d\underline{k}} = -\frac{\alpha\beta^2 c\delta\,(\beta c(\beta-\sigma) + k_j(2\beta-\sigma))^2\,(\beta c(\beta N+\sigma) + k_j(2\beta N+\sigma))^2}{(N+2)\delta\,[\beta c(\beta-\sigma) + k_j(2\beta-\sigma))^4\,(\beta c(\beta N+\sigma) + k_j(2\beta N+\sigma)]^2 + \alpha^2\beta^2 c\varrho_5}, \tag{58}$$

with

$$\begin{aligned}\varrho_5 \;=\;& 2\beta^3 N(\beta c + 2k_j)^2\,(\beta N(\beta c + 2k_j) - \sigma(\beta c(3N-4) + 4k_j(N-1))) + 3\sigma^4 k_j(\beta c + k_j)^2\\ &+\sigma^3\beta(\beta c + k_j)\left(\beta^2 c^2(N-2) + \beta c k_j(7N-10) + 8k_j^2(N-1)\right)\\ &+\sigma^2\beta^2(\beta c + 2k_j)\left(\beta^2 c^2(N^2-5N+2) + \beta c(3N^2-18N+4) + 2k_j^2(N^2-6N+1)\right).\end{aligned}$$

The numerator of the right-hand side of (58) is positive. The first term in the denominator is also positive. Finally, tedious calculation show that ϱ_5 is positive for any $\sigma \in [0, \beta]$. Therefore, the second term in the denominator is also positive, implying that the fraction is positive and the expression in (58) is negative. Hence, at $\underline{k} = 0$, the price level falls and the overall quantity rises. As a consequence, consumer surplus rises and marginal vertical integration at $\underline{k} = 0$ is procompetitive. By continuity, vertical integration is procompetitive at the margin for all $\underline{k}$ below a certain positive threshold denoted by $\underline{k}'$. ■

A.14 Proof of Proposition 9

Taking the derivative of the consumer surplus with respect to $\underline{k}$ gives

$$\frac{\partial CS}{\partial \underline{k}} = N\frac{\partial q_j}{\partial \underline{k}} + \frac{\partial q_I}{\partial \underline{k}} - Nq_j\frac{\partial q_j}{\partial \underline{k}} - q_i\frac{\partial q_I}{\partial \underline{k}} - \sigma Nq_j\frac{\partial q_I}{\partial \underline{k}} \tag{59}$$

$$-\sigma Nq_I\frac{\partial q_j}{\partial \underline{k}} - \sigma N(N-1)q_j\frac{\partial q_j}{\partial \underline{k}} - Nq_j\frac{\partial p_j}{\partial \underline{k}} - Np_j\frac{\partial q_j}{\partial \underline{k}} - q_I\frac{\partial p_I}{\partial \underline{k}} - p_I\frac{\partial q_I}{\partial \underline{k}}.$$

In this expression

$$\frac{\partial q_j}{\partial \underline{k}} = \left(\frac{\partial q_j}{\partial p_j} + (N-1)\frac{\partial q_j}{\partial p_h}\right)\left(\frac{\partial p_j}{\partial k_j}\frac{\partial k_j}{\partial \underline{k}} + \frac{\partial p_j}{\partial k_I}\frac{\partial k_I}{\partial \underline{k}} + (N-1)\frac{\partial p_j}{\partial k_h}\frac{\partial k_h}{\partial \underline{k}}\right) + \frac{\partial q_j}{\partial p_I}\left(\frac{\partial p_I}{\partial k_I}\frac{\partial k_I}{\partial \underline{k}} + N\frac{\partial p_I}{\partial k_j}\frac{\partial k_j}{\partial \underline{k}}\right),$$ [47]

$$\frac{\partial q_I}{\partial \underline{k}} = N\frac{\partial q_I}{\partial p_j}\left(\frac{\partial p_j}{\partial k_j}\frac{\partial k_j}{\partial \underline{k}} + \frac{\partial p_j}{\partial k_I}\frac{\partial k_I}{\partial \underline{k}} + (N-1)\frac{\partial p_j}{\partial k_h}\frac{\partial k_h}{\partial \underline{k}}\right) + \frac{\partial q_I}{\partial p_I}\left(\frac{\partial p_I}{\partial k_I}\frac{\partial k_I}{\partial \underline{k}} + N\frac{\partial p_I}{\partial k_j}\frac{\partial k_j}{\partial \underline{k}}\right),$$

$$\frac{\partial p_j}{\partial \underline{k}} = \frac{\partial p_j}{\partial k_j}\frac{\partial k_j}{\partial \underline{k}} + (N-1)\frac{\partial p_j}{\partial k_h}\frac{\partial k_h}{\partial \underline{k}} + \frac{\partial p_j}{\partial k_I}\frac{\partial k_I}{\partial \underline{k}}, \quad \text{and} \quad \frac{\partial p_I}{\partial \underline{k}} = \frac{\partial p_I}{\partial k_I}\frac{\partial k_I}{\partial \underline{k}} + N\frac{\partial p_I}{\partial k_j}\frac{\partial k_j}{\partial \underline{k}}.$$

From (12) we know that

$$\frac{\partial q_j}{\partial p_j} = \frac{\partial q_I}{\partial p_I} = -\frac{\beta + \sigma(N-1)}{(\beta-\sigma)(\beta+\sigma N)} \quad \text{and} \quad \frac{\partial q_I}{\partial p_j} = \frac{\partial q_j}{\partial p_I} = \frac{\partial q_j}{\partial p_h} = \frac{\sigma}{(\beta-\sigma)(\beta+\sigma N)}.$$

We can then insert all these expressions together with $\partial p_I/\partial k_I$, $\partial p_I/\partial k_j$, $\partial p_j/\partial k_j$, $\partial p_j/\partial k_I$, and $\partial p_j/\partial k_h$ from the proof of part (a) of Lemma 6 into (59).

At $\underline{k} = \bar{k}$, we have $k_j = 0$. The equilibrium prices and quantities are therefore given by

$$p_I = \frac{\alpha(\beta c + k_I)(\beta N + \sigma)}{\beta c(\beta-\sigma)(\beta N+\sigma) + k_I\left(2\beta N(\beta-\sigma) + \sigma(2\beta-\sigma)\right)},$$

$$p_j = \frac{\alpha\left(\beta c(\beta N+\sigma) + k_I(2\beta N+\sigma)\right)}{\beta c(\beta-\sigma)(\beta N+\sigma) + k_I\left(2\beta N(\beta-\sigma) + \sigma(2\beta-\sigma)\right)},$$

$$q_I = \frac{\alpha\beta k_I(\beta N+\sigma)}{\beta c(\beta-\sigma)(\beta N+\sigma) + k_I\left(2\beta N(\beta-\sigma) + \sigma(2\beta-\sigma)\right)} \quad \text{and} \quad q_j = 0.$$

Inserting this together with $k_j = 0$ into (59) and simplifying gives

$$\frac{\partial CS}{\partial \underline{k}} = \beta^2 c^2(\beta-\sigma)(\beta N+\sigma)\varrho_6\frac{\partial k_I}{\partial \underline{k}} - N\left(\beta c(\beta N+\sigma) + k_I(2\beta N+\sigma)\right)\varrho_7\frac{\partial k_j}{\partial \underline{k}},$$

with

$$\varrho_6 \equiv (\beta N+\sigma)\Big[c\beta(\beta-\sigma)(\beta N+\sigma) + k_I\left(2\beta N(\beta-\sigma) + \sigma(2\beta-\sigma)\right)\Big] + \alpha\left[\beta^2\sigma N(\beta N+\sigma) + c\beta\sigma(2N\beta+\sigma)\right]$$
$$+\alpha k_I\Big[\beta^4 N^2 + \beta^3 N\left(N^2\sigma - (1+\sigma)N + 2\sigma\right) + \beta^2\left(N^3\sigma - N^2(\sigma^2+\sigma-1) + N\sigma(2+\sigma) + \sigma^2\right)$$
$$+\beta\sigma\left(N^3 - N^2(1+\sigma) + N(2-\sigma) + \sigma\right) + \sigma^2\Big] > 0$$

[47]Here we used that in equilibrium the indices h and j are interchangeable because the respective prices, quantities, and capacities are the same.

and

$$\varrho_7 = -(\beta-\sigma)\Bigg\{ \Big[c\beta(\beta N+\sigma)+k_I(2\beta N+\sigma)\Big]\Big[c\beta N(\beta-\sigma)(\beta N+\sigma)+k_I(2N\beta(\beta-\sigma)+\sigma(2\beta-\sigma))\Big]$$

$$-\alpha c\beta\Big[c\beta(N\beta+\sigma)+k_I\Big(\beta^3\sigma N(M-1)+\beta^2\left(N^2(4-\sigma)+N\sigma(1+\sigma)-\sigma^2\right)+\beta\sigma\left(N^2+N(5-\sigma)-\sigma\right)+2\sigma^2\Big)\Big]\Bigg\}$$

$$-\alpha k_I^2\Big[\beta^4\sigma N(2N-1)+\beta^3\left(N^2(4+2\sigma+\sigma^2)+N\sigma(3\sigma+1)-\sigma^2\right)\Big]$$

$$-\alpha k_I^2\Big[\beta^2\left(N^2(2-\sigma)+N(3-3\sigma-\sigma^2)+\sigma(1+\sigma)\right)+\beta\sigma^2\left(N^2+N(3-\sigma)-1+\sigma\right)+\sigma^3\Big]<0,$$

where the inequality sign for ϱ_6 and ϱ_7 are due to the fact that $N \geq 1$ and $\beta \geq \sigma$. Because $\partial k_I/\partial \underline{k} > 0$ and $\partial k_I/\partial \underline{k} < 0$, we obtain $\partial CS/\partial \underline{k} > 0$ at $\underline{k} = \bar{k}$. ■

B Additional Material (not intended for publication)

In this appendix (which is not intended for publication) we provide additional results. First, following Riordan (1998) we introduce an additional, exogenous cost advantage for the vertically integrated firm, and we show that all our results extend to this more general model. Second, allowing for an (identical) exogenous degree of vertical integration by all firms other than I, we show that the main insights from our analysis carry over to this richer setup. Third, we provide the omitted proofs for Section 5. Fourth, we provide the result on the incentives to integrate with differentiated Bertrand competition.

B.1 Exogenous cost advantage for the integrated firm

We now assume that the integrated firm I has a cost advantage of $\gamma \geq 0$ per unit of output.[48] Therefore, its cost function can be written as

$$c(q_I, k_I) = k_I C\left(\frac{q_I}{k_I}\right) - \gamma q_I.$$

Thus, the profit function of firm I becomes

$$\Pi_I(q_I, k_I) = P(Q)q_I - k_I C\left(\frac{q_I}{k_I}\right) + \gamma q_I - (k_I - \underline{k})R\left(K\right).$$

At the quantity stage, the first-order condition of firm I is now given by

$$P + P' q_I = C_I' - \gamma, \tag{60}$$

[48]One can also interpret γ as a quality advantage of the integrated firm's product.

while the one of a non-integrated firm is still

$$P + P' q_I = C'_j. \tag{61}$$

Because γ is a fixed cost advantage per output unit, it is easy to check that the proofs of Lemma 1 and 2 are not affected by γ.

Due to the Envelope Theorem, the first-order condition of the capacity stage is not affected by the cost advantage and is still given by (4). Therefore, the results of Lemmas 3-5 are the same as in the main text.

Competitive threshold We now show that Proposition 1 extends to the case in which firm I has a cost advantage of γ per unit of output. The proof of Proposition 1 can be seen as a special case with $\gamma = 0$. Now let us look at the case $\gamma > 0$. From (3) and (4) we know that at $\underline{k} = 0$ we have $k_I = k_j$ if $q_I = q_j$. But for $\gamma > 0$, equations (60) and (61) imply $q_I > q_j$ at $k_I = k_j$. Together with (3) and (4) this in turn implies that $k_I > k_j$. One can show that $q_I/k_I > q_j/k_j$ because $q_I > q_j$ is a first-order effect. Thus, at $\underline{k} = 0$ and $\gamma > 0$, firm I utilizes capacity more intensively. This implies that a shift in capacity to firm I is also procompetitive for $\gamma > 0$. By continuity it follows that vertical integration is procompetitive at the margin for all $\underline{k}$ below a certain, positive threshold even for $\gamma > 0$.

It is readily checked that the results of Proposition 2 to 5 are not affected by γ. In fact, the proofs of Propositions 2, 3(i), 4, and 5 do not depend on γ at all. The proof of the linear-quadratic case in Proposition 3, case (ii), is affected by γ because the formulas (43) and (44) change, but the qualitative results stays the same.

We can also provide numerical simulations to show how the competitive threshold varies with γ. Figure 5 summarizes these results. It plots the competitive threshold, here denoted $\underline{k}^*(N, \gamma)$, as a function of N and γ for the linear quadratic model for $\gamma \in \{0, 0.05, 0.1, 0.15, 0.2\}$.

Figure 5 also reveals that vertical integration is procompetitive for a larger set of $\underline{k}$ the larger is γ because increases in γ result in upward shifts of $\underline{k}^*$.[49] The intuition is that the integrated firm utilizes its capacity to a larger degree if its cost advantage is bigger. Therefore, capacity is shifted to the more efficient firm which makes vertical integration more likely to be procompetitive.

[49] Each curve $\underline{k}^*(N, \gamma)$ also exhibits a flat segment initially. This flat part corresponds to the smallest value of $\underline{k}$ such that the non-integrated competitors stop production (in our notation $\bar{\underline{k}}$). For any $\underline{k} > \bar{\underline{k}}$, vertical integration is procompetitive simply because it reduces the cost of the only active firm. The fact that the curves $\underline{k}^*(N, \gamma)$ intersect for small values of N does therefore not conflict with the statement that vertical integration is procompetitive for a larger set of $\underline{k}$ the larger is γ.

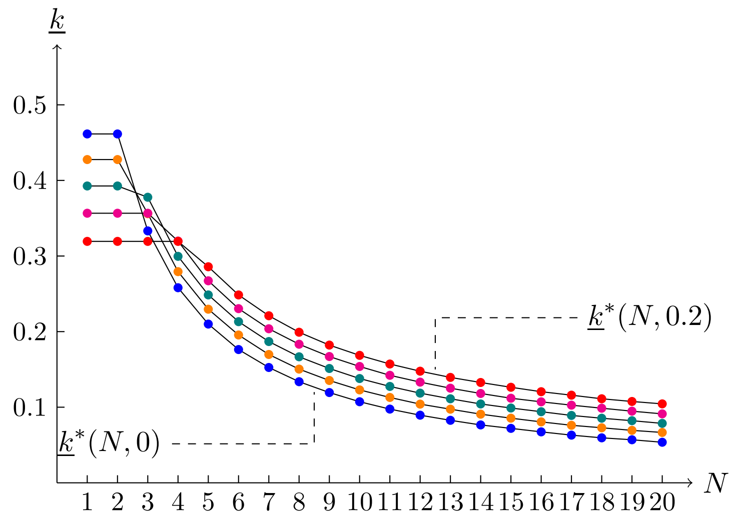


Figure 5: The competitive threshold $\underline{k}^*(N, \gamma)$ for $\gamma \in \{0, 0.05, 0.1, 0.15, 0.2\}$.

Welfare We turn to the welfare analysis. The welfare result with an exogenous cost advantage has to be modified as follows:

Proposition 10 *In the linear-quadratic case, there either exists a unique $\hat{\gamma}$ such that $\underline{k}_W^* < \underline{k}^*$ for all $\gamma < \hat{\gamma}$ and $\underline{k}_W^* > \underline{k}^*$ for all $\gamma > \hat{\gamma}$, or $\underline{k}_W^* < \underline{k}^*$ for all γ.*

Sketch of the proof: We have already shown that $\underline{k}_W^* < \underline{k}^*$ for $\gamma = 0$. So we turn to the case $\gamma > 0$. With $\gamma > 0$, equation (7) needs to be modified to

$$P\frac{dQ}{d\underline{k}} - NC_j\frac{dk_j}{d\underline{k}} - Nk_jC_j'\left(\frac{1}{k_j}\frac{dq_j}{d\underline{k}} - \frac{q_j}{k_j^2}\frac{dk_j}{d\underline{k}}\right) - C_I\frac{dk_I}{d\underline{k}} - k_IC_I'\left(\frac{1}{k_I}\frac{dq_I}{d\underline{k}} - \frac{q_I}{k_I^2}\frac{dk_I}{d\underline{k}}\right) + \gamma\frac{dq_I}{d\underline{k}} - R\frac{dK}{d\underline{k}} > 0. \tag{62}$$

Writing (62) under the linear-quadratic specification for the case of $\underline{k} = \underline{k}^*$, i.e., when $dQ/d\underline{k} = 0$, we get

$$\begin{aligned} &-\frac{k_j^2k_I^2\kappa}{(k_I\beta(2c + k_j\beta(N+2)) + c(c + k_j\beta(N+1)))^3} \\ &+ c\gamma\frac{(c + \beta k_j(N+1))(c(\alpha+\gamma) + \alpha\beta k_j + \beta\gamma k_j(N+1)) - k_I\beta N(\alpha c + \beta k_I(\alpha - \gamma))\left(\frac{dk_j}{d\underline{k}}/\frac{dk_I}{d\underline{k}}\right)}{(k_I\beta(2c + k_j\beta(N+2)) + c(c + k_j\beta(N+1)))^2} \\ &-\delta(k_I + Nk_j)(N\left(\frac{dk_j}{d\underline{k}}/\frac{dk_I}{d\underline{k}}\right) + 1), \end{aligned} \tag{63}$$

with

$$\kappa \equiv \Big[N(\alpha c + \beta k_I(\alpha - \gamma))(k_I^2\beta^2(\alpha - \gamma)(2c - \beta k_j(N+2))$$
$$+ k_Ic\beta(c(\alpha - 3\gamma) - \beta k_j(\alpha(2N+5) + \gamma(N+1))) + c^2\alpha(c - \beta k_j(N+1))\Big]\left(\frac{dk_j}{d\underline{k}}/\frac{dk_I}{d\underline{k}}\right)$$

$$+ [c(\alpha+\gamma) + k_j\alpha\beta + \beta\gamma k_j(N+1)] \Big[k_j^2(\alpha+\gamma(N+1))\left(\beta^2 c(N+1) - k_I\beta^3(N+2)\right)$$
$$+k_j c\beta\left(c(2(\alpha+\gamma) - N(\alpha-2\gamma)) - k_I\beta(\alpha(3N+4)+\gamma(N+4))\right) - 2k_I c^2\beta(\alpha+\gamma) + c^3(\alpha+\gamma)\Big].$$

From (45) we have that $(dk_j/d\underline{k})/(dk_I/d\underline{k})$ at $\underline{k} = \underline{k}^*$ is given by

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{(k_j\beta + c)(\beta(\gamma(N+1)+\alpha)k_j + c(\gamma+\alpha))}{N(k_I\beta+c)((\beta(\alpha-\gamma))k_I + c\alpha)}.$$

Inserting this into (63), differentiating the resulting expression with respect to γ and using the fact that $dk_I/d\gamma > 0$ and $dk_j/d\gamma < 0$ reveals that the expression is strictly increasing in γ. But we know already that (63) evaluated at $\underline{k} = \underline{k}^*$ is negative at $\gamma = 0$ which implies that $\underline{k}^*_{WF} < \underline{k}^*$. Therefore, we have shown there exists either a unique value of γ denoted by $\hat{\gamma}$ such that $\underline{k}^*_{WF} < \underline{k}^*$ for all $\gamma < \hat{\gamma}$ and $\underline{k}^*_{WF} > \underline{k}^*$ for all $\gamma > \hat{\gamma}$, or no such value exists because (63) turns positive only at such high values of γ at which the non-integrated firms are not active. In the latter case $\underline{k}^*_{WF} < \underline{k}^*$ for all γ. ■

B.2 Partial Integration of all Firms

In this subsection we consider the case in which all firms with whom firm I competes have, at the outset, the same (exogenous) degree of vertical integration $\underline{k}_n \geq 0$ whereas firm I's (exogenous) degree of vertical integration is as before denoted $\underline{k}$. We assume $\underline{k} \geq \underline{k}_n$ and analyze further integration by firm I.[50] It is not hard to show that none of our previous results is affected qualitatively by this change in assumptions.[51] In particular, marginal vertical integration starting at $\underline{k} = \underline{k}_n$ is always procompetitive. Moreover, vertical integration to monopoly can still be procompetitive, in which case firm I's rivals now sell their capacities to firm I or to an outside market upstream.

It is also informative to analyze how the competitive threshold, now denoted $\underline{k}^*(N, \underline{k}_n)$ to explicitly account for its dependence on $\underline{k}_n$, varies with N and $\underline{k}_n$. This threshold is such that for any smaller degree of vertical integration for firm I, i.e., for any $\underline{k} < \underline{k}^*(N, \underline{k}_n)$, vertical integration is procompetitive at the margin. We do this analysis numerically for the linear-quadratic specification, for which the results can easily be computed. Figure 6 depicts $\underline{k}^*(N, \underline{k}_n)$ for four values of $\underline{k}_n$. As one would expect based on the model with $\underline{k}_n = 0$, $\underline{k}^*(N, \underline{k}_n)$ decreases in N. Interestingly, $\underline{k}^*(N, \underline{k}_n)$ increases in $\underline{k}_n$, which re-emphasizes a theme that has emerged from this paper: Antitrust authorities should be less wary of vertical integration the

[50] So firms other than I are now more aptly called 'non-integrating' rather than 'non-integrated'.

[51] We provide a sketch of the proof at the end of the subsection.

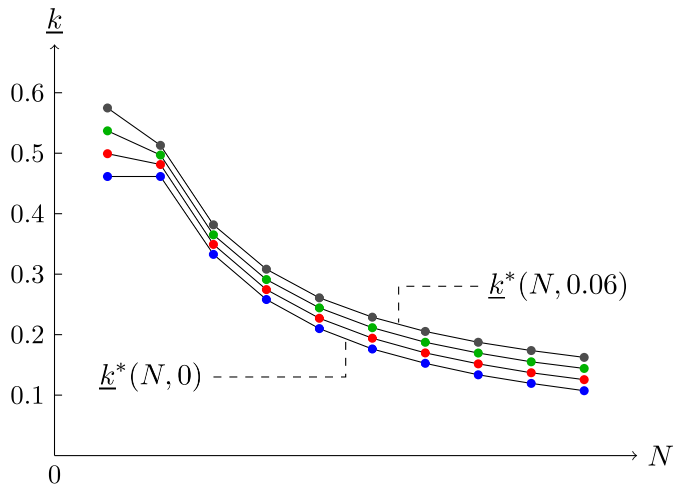


Figure 6: The competitive threshold $\underline{k}^*(N, \underline{k}_n)$ with partially integrated competitors as a function of N for $\underline{k}_n \in \{0, 0.02, 0.04, 0.06\}$.

more market power the integrating firm's competitors have. The intuition behind this result is clear: When all firms are vertically integrated to some extent, the capacity reduction of non-integrating firms following an increase in $\underline{k}$ is smaller than at $\underline{k}_n = 0$. These firms do not bear the market price, $R(K)$, on their $\underline{k}_n$ inframarginal units, which makes them less sensitive to increases in the input price. Thus, the foreclosure effect from integration is smaller when all firms are integrated to a positive degree at the outset. So we obtain again the result that in an industry with several large firms, the efficiency effect of vertical integration dominates the foreclosure effects.

Sketch of the proof: The competitive effects of vertical integration can still be evaluated using (6). If all firms are integrated, the right-hand side of (6) is the same but the left-hand side may differ.

As in the proof of Proposition 1, if all firms including firm I are vertically integrated to the same extent, (6) can be evaluated by determining $\partial^2\Pi_j/\partial k_j^2 - \partial^2\Pi_j/\partial k_j \partial k_I$. These two expressions are given by (23) and (28) but with the difference that in both expressions the last term is now given by $(k_j - \underline{k}_n)R''$ instead of $k_j R''$. However, because this change is the same in both expressions, the difference between the two is still the same as in our main analysis. Thus, the proof of Proposition 1 goes through in exactly the same way.

It is easy to check that $\underline{k}_n$ plays no role in the proof of Proposition 2 because $k_j = 0$ there, that is, all (partially) integrated firms sell their quantity to firm I or to an outside market.

The same holds for the proof of Proposition 4.

Tedious but standard calculations that closely follow those of the proof of Proposition 3 show that the arguments used there also hold if all firms are vertically integrated. This is the case because the proofs of cases (i) and (ii) of Proposition 3 depend on the equilibrium capacities and quantities and not on the degree of integration. Although the degree of integration affects the equilibrium, the calculations are very similar.

B.3 Omitted Propositions and Proofs of Section 5

The next proposition presents the result that similar statements as the ones given in Propositions 1 and 2 hold for the welfare analysis:

Proposition 11 *For any finite N, there exists a $\underline{k}_W^* > 0$ such that for all $\underline{k} < \underline{k}_W^*$ vertical integration is welfare increasing at the margin. There also either exists a $\underline{k}_W^{**} < \bar{k}$ such that for all $\underline{k} > \underline{k}_W^{**}$ vertical integration is welfare decreasing at the margin, or it is welfare increasing at the margin for any $\underline{k}$ close to $\bar{k}$.*

Sketch of the proof We start with the case where $\underline{k} = 0$. In the proof of Proposition 1 we calculated the left-hand side of (8). To determine the right-hand side of (8) we first insert $dQ/dk_I = dq_I/dk_I + N dq_j/dk_I$, $dQ_{-I}/dk_I = N dq_j/dk_I$ and $dQ/dk_j = dq_I/dk_j + dq_j/dk_j + (N-1)dq_h/dk_j$ into the right-hand side and then use equations (17), (18), (19) and (22) from the proof of Lemma 1, i.e., the derivatives of q_i with respect to k_j, $i, j = I, 1, ..., N$. Knowing that at $\underline{k} = 0$ we have $q_I = q_j$, $k_I = k_j$ and $C_I'' = C_j''$, the right-hand side simplifies to $-1/N$. But from the proof of Proposition 1 we know that the left-hand side is larger than $-1/N$ at $\underline{k} = 0$. Therefore, marginal vertical integration is welfare increasing at this point. By continuity there exists a threshold $\underline{k}_W^*$ such that vertical integration is welfare enhancing at the margin for all $\underline{k} < \underline{k}_W^*$.

Now we turn to the case where $\underline{k} = \bar{k}$. From the proof of Proposition 2 we know that in this case

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{C_I'' P' \rho \frac{q_I}{k_I} + \sigma}{(N+1)\left(\rho^2 P'(C_I'' - k_I P') + \sigma\right)}.$$

Proceeding in the same way as above to determine the right-hand side of (8) but now inserting

$\underline{k} = \bar{k}$, $q_j = k_j = 0$ yields

$$-\frac{P'q_I^2C_I'' - k_IR'\left(C_I'' - k_I(2P' + q_IP'')\right)(k_I - \bar{k})}{P'\rho q_Ik_I^2N(P' + q_IP'')} = -\frac{P'q_I^2C_I'' + \sigma(k_I - \bar{k})}{NP'\rho q_Ik_I^2(P' + q_IP'')}, \quad (64)$$

where the equality sign is due to $\sigma \equiv R'k_I(2P'k_I + P''k_Iq_I - C_I'') < 0$ as defined in the proof of Proposition 2. Subtracting (64) from $(dk_j/d\underline{k})/(dk_I/d\underline{k})$ then reveals that this difference has the same sign as

$$-k_I(1+N)\sigma^2 - \sigma P'\left[C_I''(N+1)(k_I\rho^2 + q_I^2) - k_I^2\rho\left(P'(\rho(N+1) + q_IN) + P''Nq_I^2\right)\right]$$

$$-C_I''q_I^2\rho^2(P')^2\left[C_I''(N+1) - k_IP'(2N+1) - k_Iq_INP''\right] + (1+N)\bar{k}\sigma\left[\sigma + \rho^2P'(C_I'' - k_IP')\right].$$

The first three terms in this expression are negative while the last term is positive. Therefore, if the ex ante capacity that is needed to induce the non-integrated firms to stop producing, $\bar{k}$, is small, the fourth term is small as well. In this case the expression is negative and welfare is decreasing at $\underline{k} = \bar{k}$. By continuity there then exists a $\underline{k}_W^{**}$ such that for all $\underline{k} > \underline{k}_W^{**}$ vertical integration is welfare reducing at the margin. If instead $\bar{k}$ is relatively large, the fourth term dominates the first three terms. The expression is then positive and vertical integration is welfare enhancing at the margin. ■

The next result is akin to Proposition 3:

Proposition 12 *Suppose either that (i) $R(K)$ is very inelastic or that (ii) the model is linear-quadratic. Then, for any finite N there either exists a unique $\underline{k}_W^* \in (0, \bar{k})$ such that vertical integration is welfare enhancing at the margin for all $\underline{k} < \underline{k}_W^*$ and welfare reducing at the margin for all $\underline{k} > \underline{k}_W^*$, or vertical integration is always welfare enhancing.*

Sketch of the proof Case (i): From (8) we know that vertical integration enhances welfare if

$$N\left[-P'\left(q_j\frac{dQ}{dk_j} + (N-1)q_j\frac{dq_h}{dk_j} + q_I\frac{dq_I}{dk_j}\right) + R'k_j\right]\left(\frac{dk_j}{d\underline{k}}\right) + \quad (65)$$
$$+\left(-P'\left(q_I\frac{dQ}{dk_I} + q_j\frac{dQ_{-I}}{dk_I}\right) + R'(k_I - \underline{k})\right)\left(\frac{dk_I}{d\underline{k}}\right) > 0.$$

If R is sufficiently inelastic, we can calculate $dk_j/d\underline{k}$ and $dk_I/d\underline{k}$ from (32) and (33) to get

$$\frac{dk_j}{d\underline{k}} = -\frac{1}{N+2} \quad \text{and} \quad \frac{dk_I}{d\underline{k}} = \frac{N+1}{N+2}. \quad (66)$$

Inserting this into the last expression and using the fact that R is sufficiently inelastic, i.e., R' is very large, yields $R'(-Nk_j + (N+1)(k_I - \underline{k}))/(N+2) > 0$. Differentiating the left-hand side of the last equation with respect to $\underline{k}$ and using (66) yields $-R'/(N+2)^2 < 0$. Therefore, the term that determines the sign of (65) is strictly decreasing in $\underline{k}$. Because welfare is increasing in $\underline{k}$ at $\underline{k} = 0$, there is either a unique intersection point or none.

The proof for case (ii) proceeds along the same lines as the proof of case (ii) in Proposition 3 and is therefore omitted. ■

B.4 Incentives to Integrate with Differentiated Bertrand Competition

We can show a proposition akin to Proposition 7 of Section 6:

Proposition 13 *Also in the case of differentiated price competition downstream, the acquisition of a marginal ownership stake at $\underline{k} = 0$ is always profitable for firm I.*

Proof For the same arguments as in Section 6, we obtain (dropping arguments)

$$\frac{\partial \Pi_I}{\partial \underline{k}}|_{\underline{k}=0} = N\left[p_I \frac{\partial q_I}{\partial p_j}\frac{\partial p_j}{\partial \underline{k}} - (k_I - \underline{k})\delta \frac{\partial k_j}{\partial \underline{k}}\right] + \delta K$$

and

$$\frac{\partial [\underline{k} R(K(\underline{k}))]}{\partial \underline{k}}|_{\underline{k}=0} = \delta K.$$

Given strategic complements $\partial q_I/\partial p_j > 0$ and $\partial k_j/\partial \underline{k} < 0$ by part (d) of Lemma 6, we obtain that $\partial p_j/\partial \underline{k} > 0$ is a sufficient condition for

$$\frac{\partial \Pi_I}{\partial \underline{k}}|_{\underline{k}=0} > \frac{\partial [\underline{k} R(K(\underline{k}))]}{\partial \underline{k}}|_{\underline{k}=0},$$

i.e., for some vertical integration to be profitable. We know that

$$\frac{\partial p_j}{\partial \underline{k}} = \frac{\partial p_j}{\partial k_j}\frac{dk_j}{d\underline{k}} + \frac{\partial p_j}{\partial k_I}\frac{dk_I}{d\underline{k}} + (N-1)\frac{\partial p_j}{\partial k_h}\frac{dk_h}{d\underline{k}}. \tag{67}$$

In the same way as in the proof of Proposition 8 we can insert the respective values in the right-hand side of (67), which yields

$$\text{sign}\left\{\frac{\partial p_j}{\partial \underline{k}}\right\} =$$

$$\text{sign}\Big\{\delta\left(\beta c(\beta-\sigma) + k_j(2\beta-\sigma)\right)^3\left(\beta c(\beta N-\sigma) + k_j(2\beta N-\sigma)\right)^2\left(\beta c(\beta(N+1)-\sigma) + k_j(2\beta N - \sigma(N+1))\right)$$

$$+\alpha^2\beta^2\sigma c\Big[\beta^6 c^3 N^2 + k_j^3\left(2\beta N+\sigma\right)\left(2\beta N(2\beta N-\sigma) + \sigma(2\beta-3\sigma)\right) + \beta^4 c^3\sigma(\beta N(N+2) - \sigma(N+1))$$

$$\beta c k_j^2 \left(3\beta N(4N\beta^2 + \sigma(6\beta - 5\sigma)) - 2\sigma(4N\beta^2 - 3\sigma(\beta - \sigma))\right) + \beta^2 c^2 k_j(\beta N + \sigma)(\beta N(6\beta - 5\sigma) + \sigma(5\beta - 3\sigma))\Big]\Big\}.$$

It is easy to check that for all $\sigma \in [0, \beta)$ the sign is positive. Hence, $\partial p_j / \partial \underline{k} > 0$ and vertical integration at $\underline{k} = 0$ is profitable. ■

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