Dynamic House Allocations

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Abstract

We consider the problem of repeatedly allocating a fixed quantity of a scarce good to individuals without using monetary transfers. Individuals' valuations are private information and independent draws from the same distribution over two periods. We show that expected utility at the interim stage under a detail-free dynamic mechanism is strictly greater than under a static mechanism, where a mechanism is called dynamic (static) if its future allocations (do not) depend on present reports. These results extend to many periods, heterogenous goods and valuations that are correlated over time. We also characterize the optimal, non detail-free dynamic mechanism implementing first-best in period one. For a wide range of assumptions, no such mechanism exists. Under somewhat more restrictive assumptions, we show that when first-best in period one is not implementable our detail-free mechanism is the optimal, incentive compatible mechanism among all direct incentive compatible mechanisms, including mechanisms that are not detail-free.

Keywords: House allocation, matching, mechanism design without transfers, preference intensity.

JEL: C72, C78, D02

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1 Introduction

The problem of allocating indivisible objects without monetary transfers has been studied extensively in the context of mechanism design. It has a wide array of real life applications. Those include, but are not restricted to, allocating teaching load among faculty, tasks among team members, public schools within school choice programs, and campus housing among college students.[1] Many of these applications are dynamic in nature. For example, courses are distributed anew among faculty every year; a team that shares the workload on a current project is likely to work on another project further down the road; students that are assigned a school in a choice program can apply for a new school the next year; college students who are assigned a campus housing unit can enter the lottery for housing the next year. Despite the dynamic nature of such applications, the mechanism design literature focuses exclusively on static allocation mechanisms.[2] Yet the absence of monetary transfers in static models precludes efficient distribution of resources. In this paper, we study a stylized model of a repeated allocation problem. We illustrate how the dynamic nature of the problem can be exploited to achieve unambiguous efficiency gains via a practical dynamic mechanism in the absence of monetary transfers.

The idea of using future allocation to induce incentives in the current period dates back to Townsend's seminal work in the dynamic contracting literature (Townsend, 1982).[3] We apply a similar idea in mechanism design: Consider a situation where a social planner (the chair of your department) wants to assign an indivisible good (an easy-to-teach course) among a group of agents (the faculty) once this year and once the next year. Efficiency dictates the good to be assigned in every period to an agent with a high valuation for the good. However, in the absence of monetary transfers, there is no static mechanism that gives the agents with low valuation today an incentive to opt out today. In contrast, if the social planner promises a more favorable allocation tomorrow for those who opt out today, agents with low valuations might prefer to opt out today in favor of better odds tomorrow. As a result, the good might be assigned (with a higher probability) to an agent with higher valuation. We verify this insight in a stylized dynamic allocation setup and show that a practical detail-free dynamic mechanism can improve efficiency upon static mechanisms unambiguously by increasing every agent's

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[1] Although rental rates differ for units with different qualities, those rates are predetermined and are therefore not used as an instrument for monetary transfers among students

[2] A recent exception is Ünver (2007). He studies the optimal market clearing rate in a dynamic matching model, but does not consider individual incentive constraints and cardinal utility information.

[3] It has been well known in the game theory literature even earlier that future rewards or punishments can sustain a cooperative equilibrium in a repeated game setup.

interim and ex ante payoffs.

Our dynamic house allocation problem consists of two periods, a continuum of agents and a continuum of homogenous goods to be allocated in the first period and then reallocated in the second period among the agents. An agent can consume at most one good every period. An agent's utility is additively separable over time; and his valuations for consuming a good each period are drawn from a commonly known distribution independently of other agents' valuations and independently across time. Valuations are privately observed by individuals (Section 2).

A static (allocation) mechanism allocates the goods the same way in both periods. Our benchmark is a static mechanism that allocates the goods uniformly randomly in both periods. In the absence of monetary transfers, there is no static mechanism that produces a bigger ex ante surplus than our benchmark mechanism. A dynamic mechanism conditions the allocation of the goods in the second period on the allocation of the goods in the first period. We introduce the following dynamic mechanism: In period one, individuals can either apply for the good or opt out. Those who apply for the good gain priority in the first period, those who opt out gain priority in the second period. In every period, goods are allocated first to those with priority; if there are more individuals with priority than the amount of available goods, the goods are allocated uniformly randomly among them; otherwise any remaining goods are allocated uniformly randomly among those without priority. This mechanism is simple, detail-free and practical for real-life applications. In equilibrium individuals with valuations above a cutoff value in period one apply for the good and those with valuations below the cutoff opt out (Section 3). Therefore, priority is given to those with higher valuations in period one. This increases the surplus in comparison to the surplus of the benchmark mechanism. Most importantly, in an equilibrium with an interior cutoff, *every* individual's interim expected utility is larger in comparison to the benchmark (Section 4). These results and insights easily extend to more general environments with more than two periods (Section 4), with heterogenous goods (Section 5), or with valuations correlated over time (Section 5). We further show that the conditions under which the first best in period one is implementable are quite tight and we give a sufficiency condition for our detail-free mechanism to attain the second best among all direct incentive compatible mechanisms, including non detail-free ones (Section 6).

There is a growing interest in mechanism design without monetary transfers. In particular, Casella (2005) studies a repeated game of voting over status quo versus an alternative. She shows that if agents are endowed with storable votes, then there can be efficiency gains in the equilibrium of the repeated game in comparison to the one shot game. Indeed, Casella, Gelman, and Palfrey (forthcoming) obtain efficiency gains via storable votes in laboratory experiment. An important difference to Casella's paper is that we look at a problem with a continuum of agents. This allows us to abstract from strategic interactions between agents, whereas she analyzes voting equilibria when there are two or three agents who face a repeated decision problem. As a result, whereas a Folk Theorem would apply to Casella's framework even without storable votes, we do not have the luxury to refer to the Folk theorem to achieve efficiency in our environment. Jackson and Sonnenschein (2007) propose to link several decisions that are to be made by a group of agents together, and rationing or budgeting agents' announcements about their preferences. The main contrast between our paper and Jackson and Sonnenschein's is that the latter studies optimal mechanisms for linked but independent decision problems when the number of problems becomes large while we are concerned with linked and independent decision problems when the number of these is small (two, to be exact). Börgers and Postl (2006) study a one period voting model with two agents and three alternatives. They show that no first-best mechanism can elicit preference intensities truthfully. They further study the problem of finding the second best. Similarly, Hortala-Vallve (2006) considers the possibility of achieving welfare gains by studying more elaborate voting schemes than "one man one vote" in one period model. He proposes qualitative voting, which gives every voter more than one vote. Voters can express their intensity of preferences over issues by distributing their votes over issues. He shows optimality of qualitative voting with two or three voters and two issues. As opposed to these two papers, we study a stylized repeated model, where welfare gains without monetary transfers are impossible with static mechanisms. Finally, in a one-period school choice framework, Abdulkadiroğlu, Che, and Yasuda (2007) propose a new deferred acceptance matching algorithm that breaks ties at schools preferences according to students' nonmonetary bets at schools. In their environment, heterogeneity of schools allows them to achieve efficiency gains in a one-period model. In contrast, our homogeneity assumption makes it impossible to achieve efficiency gains in a static mechanism without monetary transfers.

2 Model

We consider the problem of allocating $\alpha \in (0,1)$ units of an indivisible good to a continuum of individuals whose total mass is one without using monetary transfers. For simplicity, we refer to α as the capacity. There are two periods $t = 1, 2$. Preferences are as follows. Denote by x an individual's valuation for the good in any given period. We assume that every individual's valuation is drawn from the same distribution G on

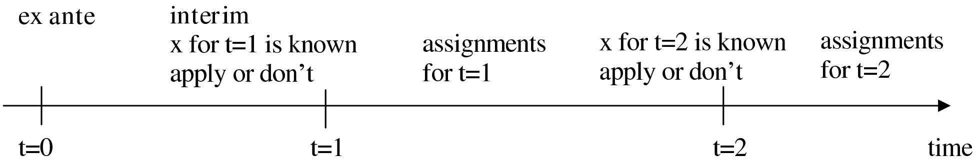

Figure 1: Timing.

$[0,1]$. These draws are independent across individuals and over time. We assume that G is strictly increasing and has no atoms on $[0,1]$. The mean and the median of G are denoted Eu and m, respectively. The valuations x are private information, and the value of the outside option is normalized to zero for every individual. Utility is additive over time, and there is no discounting, so that in period one the expected utility of an individual who is sure to get the good in both periods and whose first period draw is x is $x + Eu$. Figure 1 illustrates the timing. A house allocation problem is thus completely described by a distribution G and a capacity α, which we summarize by writing (G, α). Through most parts of the paper, we compare the following two types of *detail free* mechanisms.

Static Mechanism In every period, give each individual the opportunity to apply for the good. If there is excess demand, individuals are allocated uniformly randomly. If there is excess supply, allocate the residual capacity randomly among non-applicants. The same procedure is repeated in period two.

Observe that always applying is a dominant strategy under the static mechanism.

Dynamic Mechanism In period one, individuals can either apply for the good or opt out. Those who apply in period one have priority for the good in period one, those who opt out have priority in period two. If there is excess demand in any period, the α units are allocated uniformly randomly amongst the individuals who enjoy priority. If there is excess supply, the residual capacity is allocated uniformly randomly among the individuals with low priority.

As cutoff equilibria will play an important role under the dynamic mechanism, we provide a formal definition.

Definition 1 *The point* $x^* \in [0,1]$ *is an equilibrium cutoff point if given that the mass* μ *of others applies it is optimal (i) for an individual to apply if and only if* $x \geq x^*$ *and (ii)* $\mu = 1 - G(x^*)$. *If* $x^* \in (0,1)$, *it is called an interior equilibrium point.*

3 Equilibrium

We begin by stating and proving one of the main results of this paper:

Proposition 1 *Under the dynamic mechanism, any house allocation problem* (G, α) *has a cutoff equilibrium, and all equilibria under this mechanism are cutoff equilibria.*

Proof: We first prove the existence part. This part of the proof consists of three steps. In step 1, we derive four cases that have to be distinguished and the necessary conditions for an equilibrium for each case. In step 2, we construct an equilibrium for any $\alpha \leq \frac{1}{2}$ and in step 3, we construct an equilibrium for all $\alpha > \frac{1}{2}$.

Step 1: Consider Figure 2 to see that as a function of α and μ, which remains to be determined in steps 2 and 3, there are four cases that can occur.

Case 1: Assume $1-\mu \leq \alpha \leq \mu$. In this case, the number of non-applicants in $t=1$ is smaller than the capacity, which in turn is smaller than the number of applicants. For this case to occur in a cutoff equilibrium, the following must hold for some x:

$$U_A(x) = \frac{\alpha}{\mu}x + \frac{\alpha-(1-\mu)}{\mu}Eu = Eu = U_O(x) \qquad \Leftrightarrow \qquad x^* = \frac{1-\alpha}{\alpha}Eu,$$

where U_A (U_O) denotes the expected utility of applying (opting out). Clearly, $\frac{\partial U_A}{\partial x} = \frac{\alpha}{\mu} > 0 = \frac{\partial U_O}{\partial x}$.

Case 2: Assume $\mu \leq \alpha \leq 1-\mu$. Then, the capacity is smaller than the number of applicants in $t=1$. Consequently, no applicant in $t=1$ will get the good in $t=2$.

$$U_A(x) = x = \frac{\alpha-\mu}{1-\mu}x + \frac{\alpha}{1-\mu}Eu = U_O(x) \qquad \Leftrightarrow \qquad x^* = \frac{\alpha}{1-\alpha}Eu.$$

It is easy to see that $\frac{\partial U_A}{\partial x} = 1 > \frac{\alpha-\mu}{2-\mu} = \frac{\partial U_O}{\partial x}$.

Case 3: Assume $\alpha \leq \mu \leq 1-\alpha \Leftrightarrow \alpha \leq \min\{\mu, 1-\mu\}$. Note that in this case the capacity is smaller than the number of applicants and non-applicants. Consequently, there will be random rationing among individuals with priority in both periods:

$$U_A(x) = \frac{\alpha}{\mu}x = \frac{\alpha}{1-\mu}Eu = U_O(x) \qquad \Leftrightarrow \qquad x^* = \frac{\mu}{1-\mu}Eu.$$

Obviously, $\frac{\partial U_A}{\partial x} = \frac{\alpha}{\mu} > 0 = \frac{\partial U_O}{\partial x}$.

Case 4: Assume $\alpha \geq \mu \geq 1-\alpha \Leftrightarrow \alpha \geq \max\{\mu, 1-\mu\}$. In this case, the capacity is large enough to give a unit of the good to every individual with positive probability in both periods:

$$U_A(x) = x + \frac{\alpha-(1-\mu)}{\mu}Eu = \frac{\alpha-\mu}{1-\mu}x + Eu = U_O(x) \qquad \Leftrightarrow \qquad x^* = \frac{1-\mu}{\mu}Eu.$$

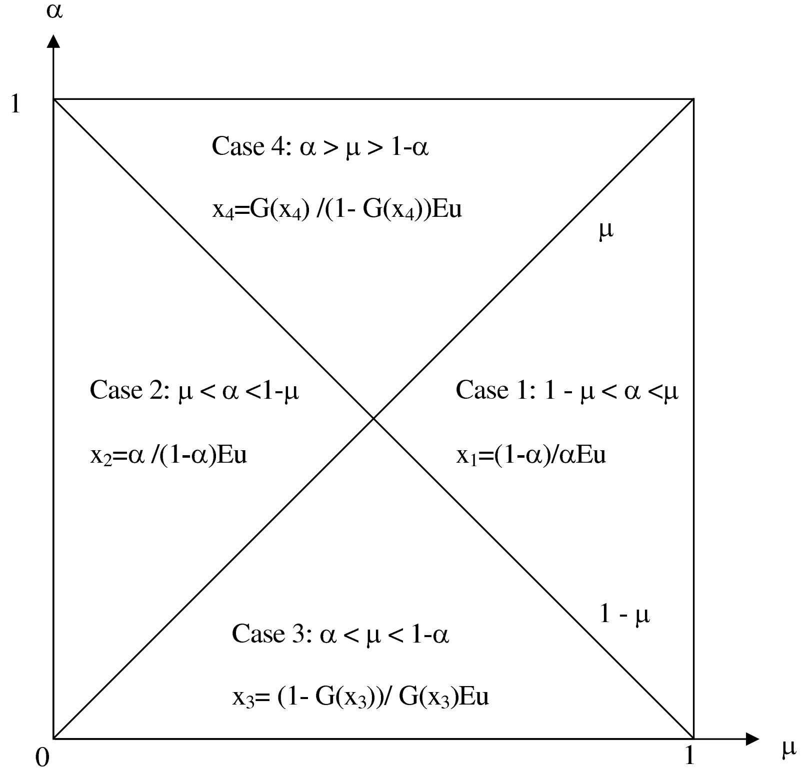

Figure 2: Four cases.

It is easy to check that $\frac{\partial U_A}{\partial x} = 1 > \frac{\alpha - \mu}{2 - \mu} = \frac{\partial U_O}{\partial x}$.

These conditions and cases can be summarized as follows:

$$\begin{aligned} x_1 &= \frac{1-\alpha}{\alpha} Eu \quad 1-\mu \leq \alpha \leq \mu \quad \text{(Case 1)} \\ x_2 &= \frac{\alpha}{1-\alpha} Eu \quad \mu \leq \alpha \leq 1-\mu \quad \text{(Case 2)} \\ x_3 &= \frac{\mu}{1-\mu} Eu \quad \alpha \leq \mu \leq 1-\alpha \quad \text{(Case 3)} \\ x_4 &= \frac{1-\mu}{\mu} Eu \quad \alpha \geq \mu \geq 1-\alpha \quad \text{(Case 4)}, \end{aligned}$$

where we have dropped the "star" and used subscripts to indicate the respective cases. Now replace μ by $1 - G(x_i)$ for $i = 1, .., 4$ to get:

$$\begin{aligned} x_1 = \frac{1-\alpha}{\alpha} Eu & \qquad G(x_1) \leq \alpha \leq 1 - G(x_1) \\ x_2 = \frac{\alpha}{1-\alpha} Eu & \qquad 1 - G(x_2) \leq \alpha \leq G(x_2) \\ x_3 = \frac{1-G(x_3)}{G(x_3)} Eu & \qquad \alpha \leq 1 - G(x_3) \leq 1 - \alpha \\ x_4 = \frac{G(x_4)}{1-G(x_4)} Eu & \qquad \alpha \geq 1 - G(x_4) \geq 1 - \alpha. \end{aligned}$$

Note that $\frac{\partial x_1}{\partial \alpha} < 0$ and $\frac{\partial x_2}{\partial \alpha} > 0$.

Step 2: The second requirement for some x^* and a given μ to constitute a cutoff equilibrium is $\mu = 1 - G(x^*)$.

It is useful to distinguish the cases with $\alpha \leq \frac{1}{2}$ and $\alpha > \frac{1}{2}$. We begin with the former and show that such a pair x^* and $\mu = 1 - G(x^*)$ always exists. For $\alpha \leq \frac{1}{2}$, the strategy of the proof is to start with an x close to the median such that the restriction $\alpha \leq 1 - G(x) \leq 1 - \alpha \Leftrightarrow \alpha \leq \min\{G(x), 1 - G(x)\}$ of a case 3 equilibrium is met. If this is not an equilibrium (i.e. if for no such x, $x = x_3$ holds), then we can either decrease x until we have an x that satisfies all of the restrictions of a case 1 equilibrium, or we can increase x until we have a case 2 equilibrium. For $\alpha > \frac{1}{2}$, the strategy of the proof is completely analogous, except that we start with an x close to the median that satisfies the restrictions $\alpha \geq 1 - G(x) \geq 1 - \alpha \Leftrightarrow \alpha \geq \max\{G(x), 1 - G(x)\}$ of a case 4 equilibrium.

Recall that m denotes the utility of the median, i.e. $G(m) = \frac{1}{2}$, and consider case 3. Clearly, for $\alpha \leq \frac{1}{2}$ there is always an x such that $\alpha \leq \min\{G(x), 1 - G(x)\}$, since we can always choose $x = m$. If in addition, $x = \frac{1-G(x)}{G(x)} Eu$ holds, we have an equilibrium. So, assume either $x > \frac{1-G(x)}{G(x)} Eu$ or $x < \frac{1-G(x)}{G(x)} Eu$ for all x for which the restriction $\alpha \leq \min\{G(x), 1 - G(x)\}$ holds. We consider the first case first. Start with an x such that $\alpha \leq \min\{G(x), 1 - G(x)\}$ holds and decrease x until $x = \tilde{x}$ with $\tilde{x} > \frac{1-G(\tilde{x})}{G(\tilde{x})} Eu$ and $\alpha = G(\tilde{x}) < 1 - G(\tilde{x})$. The last inequality holds because initially $1 - G(x) \geq \alpha$ holds and as x decreases to $\tilde{x}$, $1 - G(\tilde{x}) > \alpha$ follows. Note that $\tilde{x} > \frac{1-\alpha}{\alpha} Eu$. Now decrease x further until $\hat{x} = \frac{1-\alpha}{\alpha} Eu$. As $\hat{x} < \tilde{x}$ implies $G(\hat{x}) < \alpha < 1 - G(\hat{x})$, it follows that we have an equilibrium of the form described in case 1.

So as to complete the case where $\alpha \leq \frac{1}{2}$, assume now that for any x that satisfies the restriction $\alpha \leq \min\{G(x), 1 - G(x)\}$, $x < \frac{1-G(x)}{G(x)} Eu$ holds. Start with any such x and increase it until x equals $\tilde{x}$, which is such that $\tilde{x} < \frac{1-G(\tilde{x})}{G(\tilde{x})} Eu$ and $\alpha = 1 - G(\tilde{x})$. Note that $1 - G(\tilde{x}) = \alpha < G(\tilde{x})$, since initially both $G(x)$ and $1 - G(x)$ were larger than α and since the term $1 - G(x)$ decreases when x increases. Note also that $\tilde{x} < \frac{\alpha}{1-\alpha} Eu$. So, increase x further until $\hat{x} = \frac{\alpha}{1-\alpha} Eu$. This is clearly an equilibrium of the type analyzed in case 2, with $1 - G(\hat{x}) < \alpha < G(\hat{x})$.

Step 3: The reasoning for the case with $\alpha > \frac{1}{2}$ is almost completely analogous. Consider an x such that $\alpha > \max\{G(x), 1 - G(x)\}$. Because $\alpha > \frac{1}{2}$, we know that such x's always exist if we choose them close enough to m. If in addition for one such x, $x = \frac{G(x)}{1-G(x)} Eu$ holds, we have an equilibrium of the case 4 type. So, assume that no such x exists, i.e. whenever $\alpha > \max\{G(x), 1 - G(x)\}$ is satisfied, we either have $x > \frac{G(x)}{1-G(x)} Eu$ or $x < \frac{G(x)}{1-G(x)} Eu$. Consider first the case where $x > \frac{G(x)}{1-G(x)} Eu$ and the restriction $\alpha \geq \max\{G(x), 1-G(x)\}$ is satisfied. As we decrease x, $G(x)$ decreases and $1-G(x)$ increases. Since initially $\max\{G(x), 1-G(x)\} < \alpha$, the constraint that will become binding for some sufficiently small $\tilde{x}$ is $\alpha = 1-G(\tilde{x}) > G(\tilde{x})$. Assume that $\tilde{x} > \frac{G(\tilde{x})}{1-G(\tilde{x})}Eu = \frac{1-\alpha}{\alpha}Eu$. Clearly, as we decrease x further until $\hat{x} = \frac{1-\alpha}{\alpha}Eu$, $G(\hat{x}) < \alpha < 1-G(\tilde{x})$ holds, and we have an equilibrium of the case 1 type. Because we reach x_1 by decreasing x, starting from some $x < 1$, we know that x_1 is an interior cutoff point.

Finally, consider the case where for any x such that $\alpha > \max\{G(x), 1-G(x)\}$ holds, we have $x < \frac{G(x)}{1-G(x)}Eu$. Increase x until $\alpha = G(\tilde{x}) > 1-G(\tilde{x})$. Note that $\tilde{x} < \frac{G(\tilde{x})}{1-G(\tilde{x})}Eu = \frac{\alpha}{1-\alpha}Eu$. Increase x further up to $\hat{x} = \frac{\alpha}{1-\alpha}Eu$. Since $G(\hat{x}) > \alpha > 1-G(\hat{x})$, we have an equilibrium of the type considered under case 2. This equilibrium is interior if $\alpha < \frac{1}{1+Eu}$.

That there are no equilibria that are not cutoff follows directly from the monotonicity property we observed at the end of each of the four cases: Whenever an individual with some utility x' prefers (not) to apply, then so will all individuals with $x \geq x'$ $(x < x')$. Hence, there are no non-monotone equilibria. ■

For our welfare analysis below, the following result on the existence of interior cutoff equilibria is useful:

Proposition 2 *Sufficient conditions for a problem* (G, α) *to have an interior cutoff point are*

- *G and* α *satisfy* $\alpha \leq \frac{1}{Eu+1}$, *or*
- *G is such that* $Eu = m$, *and* α *can take any value in (0,1).*

Proof: It is again useful to divide the proof into two parts, one dealing with $\alpha \leq \frac{1}{2}$ and the other one with $\alpha > \frac{1}{2}$. To see that an interior cutoff equilibrium always exists for $\alpha \leq \frac{1}{2}$, recall that $x_1 = \frac{1-\alpha}{\alpha}Eu > 0$. For any $\alpha < \frac{1}{2}$, the only concern for the existence of an interior equilibrium is thus that $x_1 > 1$. However, since x_1 is only needed to prove equilibrium existence when x_3 is not an equilibrium cutoff point and because in this case x_1 is known to be smaller than some $x < 1$, we know that $x_1 < 1$ holds whenever x_3 is not an equilibrium cutoff point. Clearly, x_3 must be interior to satisfy the constraint $\alpha \leq \min\{G(x_3), 1-G(x_3)\}$. Thus, for $\alpha \leq \frac{1}{2}$, an interior cutoff equilibrium always exists.

The arguments that an interior cutoff equilibrium exists for $\alpha < \frac{1}{1+Eu}$ were given in step 3 of the proof of Proposition 1. Observe that $\frac{1}{Eu+1} > \frac{1}{2}$ for any strictly increasing G.

That $Eu = m$ is a sufficient condition for an interior cutoff equilibrium to exist for any $\alpha \in (0,1)$ follows directly by plugging $x_3 = m$ and $x_4 = m$ into the conditions $x_3 = \frac{1-G(x_3)}{G(x_3)}Eu$ and $x_4 = \frac{G(x_4)}{1-G(x_4)}Eu$ and the corresponding restrictions $\alpha \leq$

$\min\{G(x_3), 1-G(x_3)\}$ and $\alpha \geq \max\{G(x_4), 1-G(x_4)\}$, which yields $x_3 = x_4 = Eu = m$ and $\min\{G(x_3), 1-G(x_3)\} = \max\{G(x_4), 1-G(x_4)\} = \frac{1}{2}$. Thus, if $Eu = m$, case 3 is an interior cutoff equilibrium for $\alpha \leq \frac{1}{2}$ and for $\alpha > \frac{1}{2}$ case 4 is an interior cutoff equilibrium. ■

The following proposition characterizes completely the sets of equilibria under the dynamic mechanism.

Proposition 3 *For $\alpha \leq \frac{1}{2}$ there is a unique cutoff equilibrium, and this equilibrium is interior. For $\alpha > \frac{1}{2}$, there can be multiple cutoff equilibria, one of which may be such that all opt out.*

Proof: First, we show that for any $\alpha \in (0, \frac{1}{2})$ there is no equilibrium where all apply. Moreover, for $\alpha \leq \frac{1}{2}$, there is no equilibrium where all opt out. Second, we deal with the question of multiple cutoff equilibria. This multiplicity may occur for $\alpha > \frac{1}{2}$, but not for $\alpha \leq \frac{1}{2}$.

Step 1: Assume all apply, i.e. $\mu^* = 1$. But then, for any $\alpha \in (0, 1)$,

$$U_O(x) = Eu > \alpha(x + Eu) = U_A(x)$$

for x sufficiently close to zero.

If $\alpha \leq \frac{1}{2}$, then there is no equilibrium where all opt out because

$$U_A(x) = x > \alpha(x + Eu) = U_O(x)$$

for $x > \frac{\alpha}{1-\alpha} Eu$, which for $\alpha \leq \frac{1}{2}$ holds for any $x > Eu$. On the other hand, if $\alpha > \frac{1}{2}$, then $\frac{\alpha}{1-\alpha} Eu > 1$ may be the case. Thus, for $\alpha > \frac{1}{2}$ all opting out can be an equilibrium.

Step 2: From the previous step, we know that the only equilibria are interior cutoff equilibria for $\alpha \leq \frac{1}{2}$. We are now going to show that there is a unique cutoff equilibrium in this case by showing that the conditions of cases 1 trough 4 are incompatible.

Note first that for $\alpha = \frac{1}{2}$, all four cases are equivalent, provided their conditions are consistent with equilibrium. That is, $x_1 = x_2 = Eu$. So if case 1 and 2 are both consistent with equilibrium, it must be that $G(x_1) = G(x_2) = \frac{1}{2}$. Similarly, if case 3 or case 4 is consistent with equilibrium, the constraints for each of these two cases must hold with equality, implying $G(x_3) = G(x_4) = \frac{1}{2}$.

Consider therefore the case with $\alpha < \frac{1}{2}$. Trivially, there is no case 4 equilibrium for $\alpha < \frac{1}{2}$, so cases 1, 2 and 3 remain to be checked.

Next note that $x_1 = \frac{1-\alpha}{\alpha} Eu > \frac{\alpha}{1-\alpha} Eu = x_2$ implies $G(x_1) > G(x_2)$, which in turn implies $1 - G(x_1) < 1 - G(x_2)$. Because case 1 requires $\alpha < 1 - G(x_1)$ while case 2 requires $1 - G(x_2) < \alpha$, it follows that the two cases are mutually exclusive. Thus, we are left to check consistency of cases 1 and 3 and cases 2 and 3.

So as to see that cases 1 and 3 are incompatible, note first that $\alpha < G(x_3)$ implies $1 - \alpha > 1 - G(x_3)$, which in turn implies $x_1 = \frac{1-\alpha}{\alpha}Eu > \frac{1-G(x_3)}{G(x_3)}Eu = x_3$ and hence $G(x_1) > G(x_3)$. But $\alpha \leq G(x_3) < G(x_1) \leq \alpha$ is a contradiction. Hence, the two cases are mutually exclusive. In order to see that cases 2 and 3 are incompatible, observe that $\alpha < 1 - G(x_3)$ implies $1 - \alpha > G(x_3)$, which in turn implies $x_3 = \frac{1-G(x_3)}{G(x_3)}Eu > \frac{\alpha}{1-\alpha}Eu = x_2$. Hence, $G(x_3) > G(x_2) \Leftrightarrow 1-G(x_3) < 1-G(x_2)$. But for case 3, $\alpha \leq 1-G(x_3)$ must hold, whereas for case 2, $1 - G(x_2) \leq \alpha$ has to be satisfied, which with $1 - G(x_3) < 1 - G(x_2)$ yields the desired contradiction.

Finally, turn to the case $\alpha > \frac{1}{2}$. Clearly, a case 3 equilibrium cannot occur now. However, case 1 and case 2 are now not mutually exclusive since for $\alpha > \frac{1}{2}$, $x_1 < x_2$ follows, implying $G(x_1) < G(x_2) \Leftrightarrow 1 - G(x_1) > 1 - G(x_2)$. So, depending on G and α, $G(x_1) < \alpha < 1 - G(x_1)$ and $1 - G(x_2) < \alpha < G(x_2)$ can both hold. Moreover, either case can be consistent with case 4, depending again on G and α. ■

4 Welfare

Propositions 1 and 2 assert the existence of an interior cutoff equilibrium under fairly general conditions. We are now going to discuss the welfare properties of these cutoff equilibria.

Proposition 4 *In any interior cutoff equilibrium, every individual's expected utility at the interim stage in $t = 1$ is larger than under the static mechanism.*

Proof: Denote by $U(x)$ the expected interim utility of an individual under a static mechanism, where everybody always applies, i.e. $U(x) \equiv \alpha(x + Eu)$. Under the dynamic mechanism the expected utility of applying in $t = 1$ for any given μ is

$$U_A(x) = \min\left\{1, \frac{\alpha}{\mu}\right\} x + \max\left\{0, \frac{\alpha - (1-\mu)}{\mu}\right\} Eu$$

and the utility of opting out in $t = 1$ is

$$U_O(x) = \max\left\{0, \frac{\alpha - \mu}{1-\mu}\right\} x + \min\left\{1, \frac{\alpha}{1-\mu}\right\} Eu.$$

Case 1: If $1 - \mu < \alpha < \mu$, then $U_A(x) = \frac{\alpha}{\mu}x + \frac{\alpha-(1-\mu)}{\mu}Eu \geq \alpha(x + Eu) = U(x) \Leftrightarrow x \geq \frac{1-\alpha}{\alpha}Eu \equiv x_1$. On the other hand, $U_O(x) = Eu > \alpha(x + Eu) = U(x) \Leftrightarrow x < \frac{1-\alpha}{\alpha}Eu \equiv x_1$.

Case 2: If $\mu < \alpha < 1-\mu$, then $U_A(x) = x \geq \alpha(x+Eu) = U(x) \Leftrightarrow x \geq \frac{\alpha}{1-\alpha}Eu \equiv x_2$. For $U_O(x) > U(x)$, we need $\frac{\alpha-\mu}{1-\mu}x + \frac{\alpha}{1-\mu}Eu > \alpha(x+Eu) \Leftrightarrow x < \frac{\alpha}{1-\alpha}Eu \equiv x_2$.

Case 3: If $\alpha < \min\{\mu, 1-\mu\}$, then $U_A(x) = \frac{\alpha}{\mu}x \geq \alpha(x+Eu) = U(x) \Leftrightarrow x \geq \frac{\mu}{1-\mu}Eu \equiv x_3$. For $U_O(x) > U(x)$, we need $\frac{\alpha}{1-\mu}Eu > \alpha(x+Eu) \Leftrightarrow x < \frac{\mu}{1-\mu}Eu \equiv x_3$.

Case 4: If $\alpha > \max\{\mu, 1-\mu\}$, then $U_A(x) = x + \frac{\alpha-(1-\mu)}{\mu}Eu \geq \alpha(x+Eu) = U(x) \Leftrightarrow x \geq \frac{1-\mu}{\mu}Eu \equiv x_4$. For $U_O(x) > U(x)$, we need $U_O(x) = \frac{\alpha-\mu}{1-\mu}x + Eu > \alpha(x+Eu) = U(x) \Leftrightarrow x < \frac{1-\mu}{\mu}Eu \equiv x_4$.

Hence, in every cutoff equilibrium every individual has an expected utility at the interim stage that exceeds the expected utility under a static mechanism. ■

For the static mechanism, it is immaterial whether it is a one period or a two period problem. At the interim stage in $t = 1$, the expected utility of an individual with utility draw x is $U = \alpha(x+Eu)$. By Proposition 4, one can ask individuals at the interim stage whether they prefer the dynamic or the static mechanism and they will all prefer the dynamic mechanism. Moreover, because interim expected utility under the dynamic mechanism exceeds interim expected utility under a static mechanism for every individual, we have also shown:

Corollary 1 *Ex ante expected utility in a cutoff equilibrium is larger than under a static mechanism.*

We conclude this section with a very brief discussion of what happens if there are more than two periods. Denote by $W^{DM(2)}$ the overall welfare achieved in the equilibrium under the dynamic mechanism in the two period model. For simplicity, assume that the number of periods T is even[4] and denote by $W^{DM(T)}$ equilibrium welfare under a dynamic mechanism for the T period model.

Proposition 5 *Under the above assumptions, there is a dynamic mechanism that induces a cutoff equilibrium such that*

$$W^{DM(T)} \geq \frac{T}{2} W^{DM(2)}$$

holds.

Proof: The existence of a dynamic mechanism that induces a cutoff equilibrium follows immediately from the existence of such an equilibrium under a dynamic mechanism for the two period model: Repeat the two period mechanism $T/2$ times. The resulting equilibrium welfare will be $W^{DM(T)} = \frac{T}{2}W^{DM(2)}$. However, in general the longer time horizon may allow for even better mechanisms. Therefore, $W^{DM(T)} \geq \frac{T}{2}W^{DM(2)}$ holds. ■

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[4] If not, the following statements are true for the model confined to the first $\tilde{T} \equiv T-1$ periods.

5 Extensions

We now consider two extensions. First, we study a slightly modified model, where two goods A and B are to be allocated and individuals differ with respect to their ordinal preferences over A and B. Second, we analyze the basic model under the assumption that utilities are correlated across time.

5.1 Heterogenous Goods

So far, we have studied a problem where a fixed capacity of some good has to be allocated and individuals can opt out. It is straightforward to extend the model to incorporate the problem of allocating two goods A and B over two periods, one of which may be in excess demand. Examples include the allocation of heterogenous parking lots to employees of a firm, assigning students to two different types of schools and allocating workplaces, say, in different libraries or different parts of a library to students. Specifically, let $\alpha \in (0,1)$ be the capacity of good A and $1-\alpha$ the capacity of B. Each individual has a time invariant preference for either good A or good B. The utility of the less preferred good is normalized to zero for each individual and both periods, while the valuations of the preferred good are independent draws from the distribution $G[0,1]$. Denote by $\gamma \in (0,1)$ the mass of individuals who prefer A to B and assume that γ is a priori not known by the mechanism. Observe that whenever $\gamma \neq \alpha$ one good will be in excess demand.

We first describe an augmented dynamic mechanism that allows to infer the true ordinal preferences (i.e. γ) before individuals are asked to apply for the good or to opt out.[5] Second, we derive equilibrium behavior under this mechanism.

Dynamic Two Phase Mechanism

Phase 1: Ask all individuals i to report their ordinal preferences $\succ_i$. If i's preferred good is available (i.e. if there is no excess demand for this good), i is assigned to this good for both periods. Individuals who prefer a good with excess demand enter phase 2.

Phase 2: The mechanism announces the good that is in excess demand and the number of individuals who reported that they preferred the good with excess demand. Then the dynamic mechanism of Section 2 is applied, where opting out now corresponds to applying to the good that is not in excess demand.

Without loss of generality, assume that A is the good with excess demand, i.e. $\gamma > \alpha$.

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[5] Put differently, under this mechanism ordinal and cardinal preference statements are made in separate steps. The motivation is the same as the one of Sönmez and Ünver (2005), who observe that order and intensity of preferences cannot be both inferred when observing only a single variable ("bids").

Lemma 1 *Truth telling in phase 1 is a strictly dominant strategy.*

Proof: Consider an individual i with $A \succ_i B$ and let $\tilde{\gamma}$ be the measure of other individuals who report that they prefer A to B. If $\tilde{\gamma} < \alpha$, then by telling the truth he gets his first-best (i.e. good A in both periods, which is obviously better than lying). If $\tilde{a} \geq \alpha$, there is too little demand for B. Thus, by lying he would get B in both periods, which is the worst that can happen to him. Thus, truth telling is strictly dominant. ■

The lemma implies that $\tilde{\gamma} = \gamma$ in equilibrium. So the problem is now how to allocate the capacity α to $\gamma > \alpha$ individuals who prefer A to B. Normalize $\hat{\gamma} = 1$ and let $\hat{\alpha} \equiv \frac{\alpha}{\gamma}$.

Proposition 6 *The allocation problem $(G, \hat{\alpha})$ in phase 2 is equivalent to the one studied in Section 3. Consequently, all previous results obtain.*

5.2 Correlations

We now relax the assumption that utility draws are i.i.d. over time. We show that even with positive correlation, expected welfare under the dynamic mechanisms exceeds welfare under the static mechanism for any correlation coefficient less than one. For perfectly positively correlated utility, welfare is the same under the dynamic and under the static mechanism. We also briefly analyze the model with negative correlation.

We make the following assumptions. Total mass of individuals is one. As above, the fixed capacity of the good is α. There are two periods, and utility of opting out is zero for both periods. Utility for the good is greater than zero for all individuals and both periods. Denote by x the first period utility draw and by y the utility for the second period.

Positive Correlation There are many different ways to model positive correlation. Following Jackson and Sonnenschein (2004, Appendix 3, p. 45-6), we assume that after period one utility x is realized, period two utility y is

$$y = x$$

with probability $\rho \in [0, 1]$ and drawn from the distribution $G[0, 1]$ with probability $(1-\rho)$. Thus,

$$E[y \mid x] = \rho x + (1 - \rho)Eu.$$

We now show that a result that is closely related to Proposition 1 holds for the case with positive correlation.

Proposition 7 *Provided* $\rho \leq \min\left\{\frac{\alpha}{1-\alpha}, \frac{\frac{1-\alpha}{\alpha}-Eu}{1-Eu}\right\}$*, there is an interior cutoff equilibrium under the dynamic mechanism. A sufficient condition for the existence of an interior cutoff equilibrium for any* $\alpha \in (0,1)$ *and* $\rho \in [0,1]$ *is* $Eu = m$*.*

The proof is analogous to the one of Proposition 1 and Proposition 2. So we only sketch it.

Sketch of Proof: It is straightforward to derive the candidate equilibrium cutoffs $x_i(\rho)$ with $i = 1,..,4$:

$$\begin{aligned} x_1(\rho) &= \frac{1-\alpha}{\alpha-\rho(1-\alpha)}(1-\rho)Eu & & G(x_1) \leq \alpha \leq 1-G(x_1) \\ x_2(\rho) &= \frac{\alpha}{1-\alpha-\rho\alpha}(1-\rho)Eu & & 1-G(x_2) \leq \alpha \leq G(x_2) \\ x_3(\rho) &= \frac{1-G(x_3)}{G(x_3)-\rho(1-G(x_3))}(1-\rho)Eu & & \alpha \leq \min\{G(x_3), 1-G(x_3)\} \\ x_4(\rho) &= \frac{G(x_4)}{1-G(x_4)-\rho G(x_4)}(1-\rho)Eu & & \alpha \geq \max\{G(x_4), 1-G(x_4)\}, \end{aligned}$$

where we have dropped ρ's inside $x_i(.)$ on the right-hand side for notational ease.

Next proceed as in the case with $\rho = 0$. That is, separate the problem into the case with $\alpha \leq \frac{1}{2}$ and $\alpha > \frac{1}{2}$ and consider the former first.

Step 1. Choose x close to m, such that $\alpha \leq \min\{G(x), 1-G(x)\}$ holds. If in addition $x = \frac{1-G(x)}{G(x)-\rho(1-G(x))}(1-\rho)Eu$, we are done.

Step 2. So assume it does not hold for any x that satisfies $\alpha \leq \min\{G(x), 1-G(x)\}$. Then, for all these x's either (a) $x > \frac{1-G(x)}{G(x)-\rho(1-G(x))}(1-\rho)Eu$ or (b) $x < \frac{1-G(x)}{G(x)-\rho(1-G(x))}(1-\rho)Eu$ holds.

Consider first (a). Decrease x until $\tilde{x}$, where $G(\tilde{x}) = \alpha$ and note that

$$\tilde{x} > \frac{1-\alpha}{\alpha-\rho(1-\alpha)}(1-\rho)Eu = \frac{1-G(\tilde{x})}{G(\tilde{x})-\rho(1-G(\tilde{x}))}(1-\rho)Eu.$$

So, decrease x further until $\hat{x} = \frac{1-\alpha}{\alpha-\rho(1-\alpha)}(1-\rho)Eu > 0$ holds. That such a $\frac{1-\alpha}{\alpha-\rho(1-\alpha)}(1-\rho)Eu > 0$ exists is guaranteed by the assumption $\rho < \frac{\alpha}{1-\alpha}$.

Consider now (b). Increase x until $x = \tilde{x}$, where $\alpha = 1-G(\tilde{x})$. Note that $\tilde{x} < \frac{\alpha}{1-\alpha-\rho\alpha}(1-\rho)Eu$. So, increase x further until $\hat{x} = \frac{\alpha}{1-\alpha-\rho\alpha}(1-\rho)Eu < 1$. The inequality is satisfied because of the assumption $\rho < \frac{\frac{1-\alpha}{\alpha}-Eu}{1-Eu}$, which is satisfied for any $\alpha \leq \frac{1}{2}$ since for these α's $\frac{\frac{1-\alpha}{\alpha}-Eu}{1-Eu} > 1$ holds.

Step 3. Assume $\alpha > \frac{1}{2}$. Choose x close to m. Assume that for no such x, $x = \frac{G(x)}{1-G(x)-\rho G(x)}(1-\rho)Eu$ holds for otherwise we are done. So, either (c) $x > \frac{G(x)}{1-G(x)-\rho G(x)}(1-\rho)Eu$ or (d) $x < \frac{G(x)}{1-G(x)-\rho G(x)}(1-\rho)Eu$ holds. Consider first (c). Decrease x until $\tilde{x}$, where

$\alpha = 1 - G(\tilde{x}) > G(\tilde{x})$. Decrease x further until $\hat{x} = \frac{1-\alpha}{\alpha - \rho(1-\alpha)}(1-\rho)Eu$ holds. Because for the larger $\tilde{x}$, we had $\alpha = 1 - G(\tilde{x}) > G(\tilde{x})$, the restriction $G(\hat{x}) < \alpha < 1 - G(\hat{x})$ is clearly satisfied. Because of the assumption $\rho < \frac{\alpha}{1-\alpha}$, $\hat{x} > 0$ holds.

Finally, consider (d). Increase x in case the restriction is never satisfied until $x = \tilde{x}$, where $\alpha = G(\tilde{x})$. Increase x further until $\hat{x} = \frac{\alpha}{1-\alpha-\rho\alpha}(1-\rho)Eu$.

Like the conditions in Proposition 1, the condition $\rho \le \min\left\{\frac{\alpha}{1-\alpha}, \frac{\frac{1-\alpha}{\alpha} - Eu}{1-Eu}\right\}$ in Proposition 7 is only sufficient. To see this, consider a distribution G satisfying $Eu = m$. Then, $x_3(\rho) = x_4(\rho) = Eu$ for any $\rho \in [0,1]$. Clearly, case 3 is an equilibrium for any $\alpha \le \frac{1}{2}$, and case 4 is an equilibrium for any $\alpha > \frac{1}{2}$. Thus, there will be an interior cutoff equilibrium for any α and ρ, including those for which the condition is not met. ■

Proposition 7 generalizes Propositions 1 and 2 to the case with positive correlation. The only additional restriction we need to take care of is $x_1(\rho) > 0$, which holds if $\rho < \frac{\alpha}{1-\alpha}$. On the other hand, $x_2(\rho) < 1$ is guaranteed by $\rho < \frac{\frac{1-\alpha}{\alpha} - Eu}{1-Eu}$. This is a generalization of the condition $\alpha < \frac{1}{Eu+1}$ of Proposition 2.[6]

Next we compare expected utility in the interim stage under the dynamic mechanism with the expected interim utility under a static mechanism. The following proposition is the analog to, and a generalization of, Proposition 4 for the case with positively correlated utilities.

Proposition 8 *In the interim stage of an interior cutoff equilibrium with positive correlation, expected utility under the dynamic mechanism is larger than under the static mechanism.*

Proof: The proof mimics the one for Proposition 4. As in Proposition 4, denote by $U(x) = \alpha(x + E[y \mid x]) = \alpha((1+\rho)x + (1-\rho)Eu)$ the expected utility in the interim stage under the static mechanism. Under the dynamic mechanism, we have:

Case 1: $U_A(x) = \frac{\alpha}{\mu}x + \frac{\alpha-(1-\mu)}{\mu}E[y \mid x] \ge \alpha(x + E[y \mid x]) = U(x) \Leftrightarrow x \ge \frac{1-\alpha}{\alpha-\rho(1-\alpha)}(1-\rho)Eu = x_1(\rho)$. On the other hand, $U_O(x) = E[y \mid x] \ge \alpha(x + E[y \mid x]) = U(x) \Leftrightarrow x \le \frac{1-\alpha}{\alpha-\rho(1-\alpha)}(1-\rho)Eu = x_1(\rho)$.

Case 2: $U_A(x) = x \ge \alpha(x + E[y \mid x]) = U(x) \Leftrightarrow x \ge \frac{\alpha}{1-\alpha-\rho\alpha}(1-\rho)Eu = x_2(\rho)$. On the other hand, $U_O(x) = \frac{\alpha-\mu}{1-\mu}x + \frac{\alpha}{1-\mu}E[y \mid x] \ge \alpha(x + E[y \mid x]) = U(x) \Leftrightarrow x \le \frac{\alpha}{1-\alpha-\rho\alpha}(1-\rho)Eu = x_2(\rho)$.

Case 3: $U_A(x) = \frac{\alpha}{\mu}x \ge \alpha(x + E[y \mid x]) = U(x) \Leftrightarrow x \ge \frac{\mu}{1-\mu-\rho\mu}(1-\rho)Eu = x_3(\rho)$. On the other hand, $U_O(x) = \frac{\alpha}{1-\mu}E[y \mid x] \ge \alpha(x + E[y \mid x]) = U(x) \Leftrightarrow x \le \frac{\mu}{1-\mu-\rho\mu}(1-\rho)Eu = x_3(\rho)$.

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[6] To see this, set $\rho = 0$ in the condition of Proposition 7 to get the condition of Proposition 2.

Case 4: $U_A(x) = x + \frac{\alpha-(1-\mu)}{\mu} E[y \mid x] \geq \alpha(x + E[y \mid x]) = U(x) \Leftrightarrow x \geq \frac{\mu}{1-\mu-\rho\mu}(1-\rho)Eu = x_4(\rho)$. On the other hand, $U_O(x) = \frac{\alpha-\mu}{1-\mu} x + E[y \mid x] \geq \alpha(x + E[y \mid x]) = U(x) \Leftrightarrow x \leq \frac{\mu}{1-\mu-\rho\mu}(1-\rho)Eu = x_4(\rho)$. ■

An immediate corollary to Proposition 8 is:

Corollary 2 *Whenever an interior cutoff equilibrium exists with positive correlation, ex ante expected utility under the dynamic mechanism exceeds ex ante expected utility under a static mechanism.*

Negative Correlation Though the case with positive correlation seems relevant for many real world applications, there are also situations where period one and two utilities are reasonably thought of as negatively correlated. Consider for example the allocation of workplaces in a library to students. It is reasonable to assume that all students have on average the same workload over the full year. However, some students will have a heavy workload (and thus a strong preference for a workplace) in the first semester. On the other hand, those who have a low workload in the first semester can be quite sure to have a lot of work in the second semester. Consequently, utilities will be negatively correlated over the academic year.

We capture negative correlation by assuming that $y = 1-x$ with probability $\rho \in [0, 1]$ and drawn from $G[0, 1]$ with probability $(1-\rho)$. Moreover, the distribution $G[0, 1]$ from which utility for the good is drawn is now assumed to be symmetric, i.e. $g(x) = g(1-x)$. This assumption makes sure that the unconditional expectation of period two utility is the same as the expectation of period one utility.

We write $E_X[.]$ for the expectation taken with respect to the distribution of x. Observe the following: When $g(.)$ is symmetric, $E_X[x] = E_X[1-x] = \frac{1}{2}$. To see this, first note that $E_X[1-x] = 1 - E_X[x]$, which is immediate. Second, $E_X[x] = E_X[1-x]$. To see this, note that $E_X[x] = \int_0^M xg(x)dx = \int_0^1 (1-x)g(1-x)dx = \int_0^1 (1-x)g(x)dx$, where the last equality holds because of symmetry. But $\int_0^1 (1-x)g(x)dx = E_X[1-x]$. Together with the first observation this thus proves that $E_X[x] = \frac{1}{2}$. It is also true that the median m is equal to $\frac{1}{2} = E[x] \equiv Eu$, where we drop the subscript X if there is no danger of confusion. Moreover, $E[y \mid x] = \rho(1-x) + (1-\rho)Eu = (1+\rho)Eu - \rho x$, where the second equality follows because $Eu = \frac{1}{2}$.

We can now state the general existence result, which is straightforward to prove because G is symmetric.

Proposition 9 *With negative correlation, an interior cutoff equilibrium always exists.*

Proof: Consider the four cases of Proposition 1. Replace Eu by $E[y \mid x] = (1+\rho)Eu - \rho x$ and solve for the respective cutoff $x_i(\rho)$, $i = 1, .., 4$ to get

$$\begin{aligned}
x_1(\rho) &= \frac{1-\alpha}{\alpha + \rho(1-\alpha)}(1+\rho)Eu && G(x_1) \le \alpha \le 1 - G(x_1) \\
x_2(\rho) &= \frac{\alpha}{1-\alpha+\rho\alpha}(1+\rho)Eu && 1 - G(x_2) \le \alpha \le G(x_2) \\
x_3(\rho) &= \frac{1-G(x_3)}{G(x_3) + \rho(1-G(x_3))}(1+\rho)Eu && \alpha \le \min\{G(x_3), 1-G(x_3)\} \\
x_4(\rho) &= \frac{G(x_4)}{1-G(x_4)+\rho G(x_4)}(1+\rho)Eu && \alpha \ge \max\{G(x_4), 1-G(x_4)\}.
\end{aligned}$$

Because G is symmetric, $Eu = m$ holds. Thus, for $\alpha \le \frac{1}{2}$ case 3 is always an equilibrium, and for $\alpha > \frac{1}{2}$, case 4 is always an equilibrium. ■

Next we address interim expected utility.

Proposition 10 *With negative correlation, interim expected utility under the dynamic mechanism exceeds interim expected utility under a static mechanism.*

The proof is omitted because it is a one-to-one replication of the proofs of Propositions 4 and 8. The only additional thing that one needs to keep in mind is that $Eu = \frac{1}{2}$ because the distribution G is symmetric. The proposition also implies that ex ante expected welfare under the dynamic mechanism exceeds expected welfare under a static mechanism.[7]

6 Optimal Mechanisms

Thus far, we have not analyzed optimal mechanisms. This is what we do now. Throughout this section, we focus on the case with no correlation and we assume that the mechanism knows and may depend on the distribution G. That is, we depart from our previous approach by studying mechanisms that are not detail-free. We also depart slightly from the exposition above by assuming that there are N ex ante identical individuals and that aggregate capacity K satisfies $0 < K < N$, so that per capita capacity is $\alpha \equiv \frac{K}{N}$. Due to the revelation principle, we can focus on direct mechanisms. As before, utility draws x_i are independent across individuals and, as mentioned, independent over time.

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[7] The following seems also worth mentioning. For $\alpha = \frac{1}{2}$, the cutoff in period one is $x_1 = m$, and we have first-best in period one: $W_1^{DM} = E[x \mid x \ge m] = W_1^{FB}$. In period two, welfare is $W_2^{DM} = 2\int_0^m (\rho(1-x) + (1-\rho)E[x])\, g(x)dx$, where $2g(x)$ is the density conditional on $x \le m$. Thus, $W_2^{DM} = (1-\rho)E[x] + 2\rho\int_0^m (1-x)g(x)dx = (1-\rho)E[x] + \rho E[x \mid x \ge m]$. Therefore, overall, welfare under the dynamic mechanism is $W^{DM}(\rho) = (1+\rho)E[x \mid x \ge m] + (1-\rho)E[x]$, which for $\rho = 1$ is equal to (overall) first-best welfare $W^{FB} = 2E[x \mid x \ge m]$.

A direct mechanism in this setup asks every individual i to report his type and as a function of i's report $\hat{x}_i$ assigns the good to i in period t with probability $p_t(\hat{x}_i)$, where $t = 1, 2$. A direct mechanism is called incentive compatible if i's expected utility is maximized when reporting $\hat{x}_i = x_i$, where x_i is i's utility draw in period one. A direct incentive compatible mechanism, a mechanism for short, is said to be feasible if it respects capacity constraints, i.e. if

$$\int_0^1 p_t(x_i) g(x_i) dx_i \leq \alpha \quad \text{for} \quad t = 1, 2.$$

Observe that capacity constraints only have to be satisfied ex ante. We assume that N is large. Therefore, the aggregate capacity constraints will be satisfied if they are satisfied ex ante for every individual i.

Before deriving the optimal mechanism, we show that whenever some mechanism achieves the first-best allocation in $t = 1$, which we denote $FB(1)$, then this mechanism achieves the second-best allocation over both periods, i.e. implements the optimal incentive compatible allocation overall.

Lemma 2 *In the two-period game, the optimal incentive compatible second period allocation is a random allocation.*

Proof: In $t = 2$, the game reduces to a static game. Thus, all individuals have a dominant strategy to apply (or to report the highest utility if asked to report utilities). Consequently, based on period two reports, the allocation can only be random. On the other hand, because of the i.i.d. assumption, period one reports cannot be informative about period two utilities. Thus, there is no way to improve upon a random allocation in period two. ■

It follows immediately that:

Corollary 3 *If a mechanism establishes $FB(1)$, then it establishes second-best overall.*

6.1 Preliminaries

We approach the problem of deriving optimal, incentive compatible mechanisms by first rewriting the maximization problem in such a way that the only remaining choice variables are those that are directly relevant for welfare, and then we solve the rewritten maximization problem. Following Krishna (2002, Ch.5), we denote the expected utility of i at the interim stage when reporting $\hat{x}_i$ while the truth is x_i by

$$U_i(\hat{x}_i, x_i) = p_1(\hat{x}_i) x_i + p_2(\hat{x}_i) Eu,$$

and we denote further by

$$U_i(x_i) = p_1(x_i)x_i + p_2(x_i)Eu \tag{1}$$

the expected utility of i when reporting truthfully. Observe that because the $p_t(\hat{x}_i)$'s are probabilities and $Eu > 0$, individual rationality, which requires $U_i(x_i) \geq 0$ for all i and all $x_i \in [0,1]$, is trivially satisfied.[8]

Therefore, incentive compatibility implies

$$U_i(x_i) \geq \max_{\hat{x}_i \in [0,1]} \{p_1(\hat{x}_i)x_i + p_2(\hat{x}_i)Eu\},$$

so that $U_i(x_i)$ is the maximum over a family of affine functions. Hence, it is a convex function and therefore differentiable almost everywhere; see e.g. Krishna (2002, Ch.5).

Second, notice that

$$\begin{aligned} U_i(\hat{x}_i, x_i) &= p_1(\hat{x}_i)x_i + p_2(\hat{x}_i)Eu = p_1(\hat{x}_i)\hat{x}_i + p_2(\hat{x}_i)Eu + p_1(\hat{x}_i)(x_i - \hat{x}_i) \\ &= U_i(\hat{x}_i) + p_1(\hat{x}_i)(x_i - \hat{x}_i). \end{aligned}$$

Since IC implies $U_i(x_i) \geq U_i(\hat{x}_i, x_i)$ for all $x_i, \hat{x}_i \in [0,1]$, we have

$$U_i(x_i) - U_i(\hat{x}_i, x_i) \geq p_1(\hat{x}_i)(x_i - \hat{x}_i).$$

For $x_i > \hat{x}_i$ this implies

$$\frac{U_i(x_i) - U_i(\hat{x}_i, x_i)}{x_i - \hat{x}_i} \geq p_1(\hat{x}_i)$$

and for $x_i < \hat{x}_i$ it implies

$$\frac{U_i(x_i) - U_i(\hat{x}_i, x_i)}{x_i - \hat{x}_i} \leq p_1(\hat{x}_i).$$

Taking limits as $\hat{x}_i$ approaches x_i from below and from above, respectively, yields

$$U_i'(x_i) \geq p_1(x_i) \geq U_i'(x_i),$$

wherever $U_i'(x_i)$ exists, whence

$$U_i'(x_i) = p_1(x_i)$$

almost everywhere follows. Because $U_i(x_i)$ is convex, it holds also that $p_1(x_i)$ is (weakly) increasing. Moreover, the function $U_i'(x_i)$ is Riemann-integrable. Therefore,

$$U_i(x_i) = U_i(0) + \int_0^{x_i} p_1(t_i)dt_i. \tag{2}$$

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[8] A similar observation obtains in Börgers and Postl (2006).

Observe that $U_i(0) = p_2(0)Eu$. By the definition of $U_i(x_i)$ in (1), equality (2) implies $p_2(0)Eu + \int_0^{x_i} p_1(t_i)dt_i = p_1(x_i)x_i + p_2(x_i)Eu$ or equivalently

$$p_2(x_i) = p_2(0) + \frac{1}{Eu}\left[\int_0^{x_i} p_1(t_i)dt_i - p_1(x_i)x_i\right]. \tag{3}$$

That is, once the function $p_1(x)$ is determined, then the function $p_2(x_i)$ is also completely determined up to the constant $p_2(0)$. Therefore, (3) is the equivalent to the Revenue (or Payoff) Equivalence Theorem in mechanism design with monetary transfers. Notice further that $\int_0^{x_i} p_1(t_i)dt_i - p_1(x_i)x_i$ will be non-positive because $p_1(x_i)$ is increasing. Consequently, $p_2(x_i)$ is a decreasing function. Moreover, evaluating $p_2(x_i)$ at $x_i = 1$ yields

$$p_2(1) = p_2(0) + \frac{1}{Eu}\left[\int_0^{1} p_1(t_i)dt_i - p_1(1)\right], \tag{4}$$

which has to be no less than zero.

Next, multiplying both sides in (3) by $g(x_i)$ and integrating yields

$$\begin{aligned}\underbrace{\int_0^1 p_2(x_i)g(x_i)dx_i}_{=\alpha} &= p_2(0) + \frac{1}{Eu}\int_0^1\left[\int_0^{x_i} p_1(t_i)dt_i - p_1(x_i)x_i\right]g(x_i)dx_i \\ &= p_2(0) - \frac{1}{Eu}\int_0^1\left[x_i - \frac{1-G(x_i)}{g(x_i)}\right]p_1(x_i)g(x_i)dx_i,\end{aligned}$$

where the second line follows by reversing the order of integration in the double integral and then integrating. Observe that $\int_0^1 p_2(x_i)g(x_i)dx_i$ on the lefthand side is just the capacity constraint in period 2. Thus, the capacity constraint for $t = 2$ can be written as

$$\alpha = p_2(0) - \frac{1}{Eu}\int_0^1\left[x_i - \frac{1-G(x_i)}{g(x_i)}\right]p_1(x_i)g(x_i)dx_i. \tag{5}$$

Therefore, the ex ante expected utility of any individual under the incentive compatible mechanism $(p_1(\mathbf{x}), p_2(\mathbf{x}))$ can be written as

$$E[U_i(X)] = p_2(0)Eu + \int_0^1\int_0^{x_i} p_1(t_i)dt_i g(x_i)dx_i.$$

Reversing the order of integration in the double integral and integrating yields

$$W = E[U_i(X)] = p_2(0)Eu + \int_0^1 p_1(t_i)[1 - G(x_i)]dx_i. \tag{6}$$

This is essentially the objective functional we aim to maximize over $(p_1(\mathbf{x}), p_2(\mathbf{x}))$ subject to these being probabilities (in particular, subject to the righthand side in (4) being nonnegative) and to the two capacity constraints. Moreover, $p_1(x_i)$ has to be nondecreasing because of incentive compatibility (i.e. because $U_i(x_i)$ is convex), and $p_2(x_i)$ nonincreasing.

It is useful to rearrange (5) so that

$$p_2(0)Eu = Eu\alpha + \int_0^1 \left[x_i - \frac{1-G(x_i)}{g(x_i)}\right] p_1(x_i)g(x_i)dx_i, \tag{7}$$

which allows us to write (6) as

$$\begin{aligned} W &= Eu\alpha + \int_0^1 \left[x_i - \frac{1-G(x_i)}{g(x_i)}\right] p_1(x_i)g(x_i)dx_i + \int_0^1 p_1(t_i)[1-G(x_i)]dx_i \\ &= Eu\alpha + \int_0^1 x_i p_1(x_i)g(x_i)dx_i. \end{aligned} \tag{8}$$

Observe also that

$$\begin{aligned} p_2(1)Eu &= p_2(0)Eu + \int_0^1 p_1(t)dt - p_1(1) \\ &= Eu\alpha + \int_0^1 xg(x)p_1(x)dx + \int_0^1 G(x)p_1(x)dx - p_1(1) \\ &= W + \int_0^1 G(x)p_1(x)dx - p_1(1), \end{aligned}$$

where the second line[9] is due to (7) and the third line follows from (8). Consequently,

$$p_2(1) = \alpha + \frac{1}{Eu}\left[\int_0^1 xg(x)p_1(x)dx + \int_0^1 G(x)p_1(x)dx - p_1(1)\right].$$

Since $p_2(1) \geq 0$ has to hold, the constraint is

$$p_2(1) = \alpha + \frac{1}{Eu}\left[\int_0^1 xg(x)p_1(x)dx + \int_0^1 G(x)p_1(x)dx - p_1(1)\right] \geq 0. \tag{9}$$

So the constrained maximization problem can now be written as

$$\max_{p_1(x)\in[0,1]} W = Eu\alpha + \int_0^1 xp_1(x)g(x)dx$$

subject to (9), to

$$p_2(0) = \alpha + \frac{1}{Eu}\int_0^1 \left[x - \frac{1-G(x)}{g(x)}\right] p_1(x)g(x)dx \leq 1$$

and to the capacity constraint in $t = 1$, i.e.

$$\alpha - \int_0^1 g(x)p_1(x)dx \geq 0. \tag{10}$$

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[9] Observe in particular that (7) can be written as $p_2(0)Eu = Eu\alpha + \int_0^1 x_i p_1(x_i)g(x_i)dx_i - \int_0^1 p_1(x_i)dx_i + \int_0^1 G(x_i)p_1(x_i)dx_i$.

6.2 Implementability of $FB(1)$

The first question we want to address is when $FB(1)$ is implementable. To that end, observe that $FB(1)$ requires $p_1(x) = 1$ for all $x \geq x_1$ and $p_1(x) = 0$ for all other x, where x_1 satisfies $G(x_1) = 1 - \alpha$ or $x_1 = G^{-1}(1-\alpha)$. That is, $FB(1)$ is a two-class mechanism. Thus under a mechanism that implements $FB(1)$ the capacity constraint in period one is always satisfied. The constraint (9) becomes

$$\alpha + \frac{1}{Eu}\left[\int_{x_1}^{1} xg(x)dx + \int_{x_1}^{1} G(x)dx - 1\right] \geq 0.$$

Integrating by parts and cancelling the two terms including $\int_{x_1}^{1} G(x)dx$ and the two 1's, this becomes

$$p_2(1) = \alpha - \frac{1}{Eu}x_1 G(x_1) \geq 0.$$

Replacing x_1 by $G^{-1}(1-\alpha)$ and $G(x_1)$ by $1-\alpha$ and rearranging, this condition can be written as

$$\alpha \geq \frac{G^{-1}(1-\alpha)}{Eu + G^{-1}(1-\alpha)}. \tag{11}$$

More generally, under any two class mechanism characterized by $(p, \overline{x})$, where p is the probability for individuals with $x \geq \overline{x}$ to be assigned and zero is the probability of being assigned for all others, $p_2(0)$ as given in (7) can be written as

$$p_2(0) = \alpha + \frac{p}{Eu}\overline{x}(1 - G(\overline{x})). \tag{12}$$

This follows by applying the same arguments as for $p_2(1)$. As for the implementability of $FB(1)$, recall that $FB(1)$ is a two class mechanism with $p = 1$ and $\overline{x} = x_1 \equiv G^{-1}(1-\alpha) \Leftrightarrow G(x_1) = 1 - \alpha$. Plugging this into (12), one gets

$$p_2(0) = \alpha + \frac{1}{Eu}G^{-1}(1-\alpha)\alpha.$$

Summarizing, we have derived the following intermediate result:

Lemma 3 *Whenever $FB(1)$ is implementable, the optimal mechanism is given by*

$$p_2(0) = \alpha + \frac{1}{Eu}G^{-1}(1-\alpha)\alpha \quad \textit{and} \quad p_2(1) = \alpha - \frac{1}{Eu}x_1 G(x_1), \tag{13}$$

and by definition of $FB(1)$, $x_1 = G^{-1}(1-\alpha)$, $p_1(0) = 0$ and $p_1(1) = 1$.

Now $p_2(0) \leq 1$ holds if and only if

$$\alpha \leq \frac{Eu}{Eu + G^{-1}(1-\alpha)}. \tag{14}$$

Taking together (11) and (14), the necessary and sufficient condition for $FB(1)$ to be implementable is thus

$$\frac{G^{-1}(1-\alpha)}{Eu+G^{-1}(1-\alpha)} \leq \alpha \leq \frac{Eu}{Eu+G^{-1}(1-\alpha)}.$$

Rearranging yields

$$G^{-1}(1-\alpha) \leq \alpha[Eu+G^{-1}(1-\alpha)] \leq Eu. \tag{15}$$

Observe that the inequalities in (15) imply $1-\alpha \leq G\left(\frac{\alpha}{1-\alpha}Eu\right)$ and $1-\alpha \leq G\left(\frac{1-\alpha}{\alpha}Eu\right)$. Taken together the necessary and sufficient condition for implementability of $FB(1)$ thus is

$$1-\alpha \leq \min\left\{G\left(\frac{1-\alpha}{\alpha}Eu\right), G\left(\frac{\alpha}{1-\alpha}Eu\right)\right\}. \tag{16}$$

Assume $Eu = m$. Then the righthand side is (weakly) less than $\frac{1}{2}$ for any $\alpha \in (0,1)$ and the lefthand side is weakly larger than $\frac{1}{2}$ for $\alpha \leq \frac{1}{2}$. Thus, for $\alpha \leq \frac{1}{2}$ the condition is satisfied only for $\alpha = \frac{1}{2}$ and $Eu = m$.

Proposition 11 *For $Eu \leq m$ and $\alpha \leq \frac{1}{2}$, $FB(1)$ is implementable if and only if $\alpha = \frac{1}{2}$ and $Eu = m$.*

Note that for $Eu = m$ and $\alpha = \frac{1}{2}$, the simple dynamic mechanism has as unique cutoff equilibrium $x = Eu$ and thus implements $FB(1)$. The proposition also shows that the conditions under which $FB(1)$ is implementable are quite tight. For example, for any symmetric G and $\alpha < \frac{1}{2}$, $FB(1)$ cannot be implemented. On the other hand, the assumption $Eu \leq m$ in the proposition matters in the sense that e.g. for $G(x) = x^{1/2}$, which implies $m = \frac{1}{4} < \frac{1}{3} = Eu$, $FB(1)$ is implementable for all $\alpha \geq 0.463$.[10] However, the fact that even in this example with $m < Eu$, $FB(1)$ cannot be implemented for small α's is a reflection of very general phenomenon:

Proposition 12 *For any G, there is a $\underline{\alpha} \in (0,1]$ such that $FB(1)$ cannot be implemented for $\alpha < \underline{\alpha}$.*

Proof: For $\alpha \leq \frac{1}{2}$, the minimum on the righthand side of (16) is $G\left(\frac{\alpha}{1-\alpha}Eu\right)$, which continuously increases in α and is zero at $\alpha = 0$. The lefthand side continuously decreases in α. At $\alpha = 0$, the lefthand side is 1 while the righthand side is 0. Because of continuity, it follows that $1-\alpha > \min\left\{G\left(\frac{1-\alpha}{\alpha}Eu\right), G\left(\frac{\alpha}{1-\alpha}Eu\right)\right\}$ will hold for α greater than zero. Continue increasing α. If $1-\alpha$ and $\min\left\{G\left(\frac{1-\alpha}{\alpha}Eu\right), G\left(\frac{\alpha}{1-\alpha}Eu\right)\right\}$ intersect at some $\alpha_0 < 1$, then $\underline{\alpha} = \alpha_0$. Otherwise, $\underline{\alpha} = 1$.■

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[10] For $\alpha < \frac{1}{2}$, (16) implies $\left(\frac{\alpha}{1-\alpha}\frac{1}{3}\right)^{1/2} \geq 1-\alpha$, which holds for all $\alpha \geq 0.463$. For $\alpha \geq \frac{1}{2}$, the relevant condition is $\left(\frac{1-\alpha}{\alpha}\frac{1}{3}\right)^{1/2} \geq 1-\alpha$, which is satisfied for all $\alpha \leq 1$.

6.3 Optimal Mechanisms When $FB(1)$ Cannot be Implemented

Assume that G satisfies (i) increasing virtual cost, i.e. $x + G(x)/g(x)$ increases in x and (ii) that Case 3 is an equilibrium under the dynamic mechanism (e.g. $Eu = m$). Note that assumption (i) is the analog to standard regularity assumption that the virtual valuation is increasing.

We will temporarily assume that the constraint $p_2(0) \leq 1$ is not binding at the optimum, which of course remains to be checked at the end. Rewriting the constraint (9) yields

$$Eu\alpha - p_1(1) + \int_0^1 \left[x + \frac{G(x)}{g(x)}\right] g(x)p_1(x)dx \geq 0.$$

Therefore, the Lagrangean for our problem can be written as

$$\begin{aligned}\max_{p_1(x),\lambda,\lambda_1} \mathcal{L} &= Eu\alpha + \lambda\alpha + \lambda_1[Eu\alpha - p_1(1)] \\ &+ \int_0^1 \left[x + \lambda_1\left(x + \frac{G(x)}{g(x)}\right) - \lambda\right] p_1(x)g(x)dx,\end{aligned}$$

where $\lambda > 0$ and $\lambda_1 \geq 0$ are Lagrange multipliers.

Observe that if $\lambda_1 = 0$ holds, then one could implement $FB(1)$, provided the solution also satisfies $p_2(0) \leq 1$. So assume that $\lambda_1 > 0$ holds. Now because of the assumption that the virtual cost $x + \frac{G(x)}{g(x)}$ is increasing, the term in brackets is strictly increasing in x. Therefore, the solution is clear: Set $p_1(x) = 0$ as long as $x + \lambda_1\left(x + \frac{G(x)}{g(x)}\right) < \lambda$. Otherwise set $p_1(x)$ as large as the two constraints allow. In other words, the optimal mechanism is a two-class mechanism, which gives probability zero to the low class. Denote by $\overline{x}$ the cutoff point between the two classes and by p the probability with which individuals in the high class are assigned the good in $t = 1$.

The pair $(p, \overline{x})$ then constitutes our only choice variables. But recall that we also have exactly two constraints, (9) and (10), which for a two-class mechanism become $\alpha = p\int_{\overline{x}}^1 g(x)dx$ and $p = Eu\alpha + p\int_{\overline{x}}^1 \left[x + \frac{G(x)}{g(x)}\right] g(x)dx$. Thus, the constraints will entirely determine the solution values for p and $\overline{x}$. Integrating out in the first constraint and rearranging, one gets

$$p = \frac{\alpha}{1 - G(\overline{x})}. \tag{17}$$

Observe also that[11] $\int_{\overline{x}}^1 \left[x + \frac{G(x)}{g(x)}\right] g(x)dx = 1 - \overline{x}G(\overline{x})$ and hence the second constraint

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[11] To see this, rewrite the second equality to get $\int_{\overline{x}}^1 \left[x + \frac{G(x)}{g(x)}\right] g(x)dx = \int_{\overline{x}}^1 xg(x)dx + \int_{\overline{x}}^1 G(x)dx$. Then integrate the first integral by parts to get $\int_{\overline{x}}^1 xg(x)dx = 1 - \overline{x}G(\overline{x}) - \int_{\overline{x}}^1 G(x)dx$.

becomes

$$p = \frac{Eu\alpha}{\overline{x}G(\overline{x})}. \tag{18}$$

Equating (17) and (18) and solving for $\overline{x}$ yields

$$\overline{x} = \frac{1 - G(\overline{x})}{G(\overline{x})} Eu. \tag{19}$$

Now observe that this is exactly the same as Case 3 above, which is the unique cutoff equilibrium for $\alpha < \frac{1}{2}$ and $Eu = m$. Thus, we have shown:

Proposition 13 *Assume $\alpha \leq \frac{1}{2}$ and that G satisfies increasing virtual costs and that Case 3 is the equilibrium under the dynamic mechanism (e.g. $Eu = m$). Then the dynamic mechanism is the optimal mechanism.*

Notice that the optimal mechanism among all non-detail free mechanisms can be implemented via a detail free mechanism.

7 Conclusions

We study the potential for using detail-free dynamic mechanisms in repeated allocation problems to increase efficiency without using monetary transfers. The results are promising. We show that in any interior cutoff equilibrium every individual's interim expected utility under the simple dynamic mechanism exceeds his interim expected utility under a static mechanism. We also derive sufficient conditions for our detail-free dynamic mechanism to be the optimal mechanism among all direct incentive compatible mechanisms, including non detail-free ones. Future research on more than two periods and, eventually, more than one good seems particularly relevant and fruitful.

Appendix

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